THE STRANGE CASE OF CLAUDIUS PTOLEMY

R. R. Newton

Ptolemy's main work on astronomy, written in about the year 142, has been considered by almost all scholars since then to be a work of genius in which Ptolemy developed theories that represented the astronomical motions as accurately as naked-eye observations could follow them. Study shows that this is not so. As a result, much of the history of Greek astronomy must be rewritten.

In an earlier article in the *APL Technical Digest*,[1] I explained how work aimed at improving the accuracy of the Transit Navigation System led me to study ancient astronomy. That study has helped to determine the parameters that describe the tide on a global basis, parameters that are used in determining the orbits of the Transit satellites. In addition, the study of ancient astronomy tells us the rate at which the moon is receding from the earth, the rate at which the rotation of the earth is decreasing, and something about the mechanisms that affect the rotation of the earth.

Claudius Ptolemy is a prominent figure in ancient astronomy, so it was inevitable that my study of ancient astronomy would lead me to examine his work in considerable detail. This led to an unexpected result that will entail serious revisions to the history of astronomy as it is usually written.

I shall start by giving a brief survey of Greek astronomy before the time of Ptolemy and then take up the strange case of Claudius Ptolemy himself.

[1] R. R. Newton, Applied Ancient Astronomy," *APL Technical Digest* **12**, No. 1, 11–20 (1973).

A Brief Survey of Greek Astronomy[2]

The earliest Greek astronomer of whom we have firm knowledge is named Meton, who worked in Athens around the year −430.[3] Meton made careful measurements of the lengths of the seasons, of the month, and of the year. He seems to have been the first person to realize, or at least to make formal use of, the fact that 19 years are almost exactly equal to 235 months. He devised a calendar based upon this fact that was an ingenious combination of a lunar and a solar calendar.

Meton is widely credited in the literature with observing the time of the summer solstice in the year −431. However, I have shown that that "observation" was actually calculated by someone who, around the year −108, wanted to put the time of the solstice of −431 on an inscription.

[2] Most of the material in this section is taken from the first five chapters of my study *The Crime of Claudius Ptolemy,* The Johns Hopkins University Press, Baltimore (1977). This book gives an extensive list of additional references.

[3] In this paper I write years before the common era in astronomical rather than historical style. In historical style, the year before the year 1 is called 1 B.C.E., the preceding year is called 2 B.C.E., and so on. In astronomical style, the year before the year 1 is called 0, the preceding year is called —1, and so on. Thus the year —430 is the same as 431 B.C.E.

He had no better way to find the time than to calculate it from the best available theory of the sun. As it happens, the time that this unknown person calculated is in error by about 28 hours. That innocent act has caused untold grief to students of the Athenian calendar, who have assumed that the time given was a genuine observation.

The next astronomer I want to mention is Aristarchus of Samos, who was active around −280. Aristarchus studied the sizes of the sun and moon in a famous work[4] that is still extant. He concluded that the diameter of the moon is about a third the diameter of the earth and that the diameter of the sun is about seven times that of the earth. The first value is reasonably accurate, but the second is far too small. Even so, Aristarchus correctly found that the sun is much larger than the earth in volume and (presumably) in mass.[5]

Aristarchus is also famous for proposing that the sun, not the earth, is the center of the solar system. His writing on this subject has not survived, and we do not know the reasons that led him to this conclusion. We may speculate that he was led to it because of his discovery that the sun is far larger than the earth. By his estimate, the sun is more than 300 times the earth in volume; it probably seemed to him absurd to speak of a 1-pound dog wagging a 300-pound tail.

In this rapid survey, I must pass over many important figures and go directly to Hipparchus, who to me is the most important astronomer in Greek or any other antiquity. The dates of the observations credited to Hipparchus range from −161 September 27 to −127 March 23. He studied the lengths of the seasons and the length of the year. He devised a theory of the sun's motion that was not improved upon for a thousand years. He studied the lunar motion thoroughly and discovered the effect that is now called the lunar evection. He discovered the precession of the equinoxes. He made the observations for a catalogue of stars that will be discussed later in this paper. Finally, with regard to the planets, Ptolemy (who is certainly a peccable authority) says in Chapter IX.2 of his famous book[6] that Hipparchus put the observations of the planets into order and showed that they did not agree with existing planetary models. I think that Ptolemy intends to imply that Hipparchus developed nothing new in the theory of planetary motion. However, I do not think we can trust Ptolemy in such a matter and I think it is quite possible that Hipparchus did contribute to planetary theory.

Before turning to Ptolemy's work, we should note briefly the state of mathematics in his time. Many readers are acquainted with Euclid, who was a contemporary of Aristarchus and whose work on plane geometry has still not been surpassed in many ways. In addition, plane trigonometry was probably well developed by the time of Hipparchus, although it was still in a rather rudimentary condition in the time of Aristarchus. Ptolemy gives a table of the chord function[7] to a precision of three sexagesimal positions (that is, to 1 part in 216 000). Spherical trigonometry, although it plays a major role in astronomy, was slower to develop. Hipparchus apparently had to use rather awkward methods of handling problems in spherical trigonometry, but the field had been brought to a high level of development by the time of Menelaos, who wrote a major treatise on spherical trigonometry[8] around the year 100.

Ptolemy's Work on Astronomy

Claudius Ptolemy lived in Alexandria in the first part of the second century, and he should be called Hellenistic rather than Greek. Dates of astronomical observations that he claims to have

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[4] Aristarchus of Samos, *On the Sizes and Distances of the Sun and Moon* (ca. −280). There is an edition with a parallel English translation by Sir Thomas Heath in *Aristarchus of Samos,* Oxford University Press, Oxford (1913). Most histories of the subject say that Aristarchus found both the sizes and distances of the sun and moon; the English title usually applied to his work is *On the Sizes and Distances of the Sun and Moon.* Actually Aristarchus estimated the sizes but not the distances. (See pp. 172ff of *The Crime of Claudius Ptolemy.*)

[5] Greek astronomers from an early time knew that the earth is basically spherical, and they had a reasonably accurate estimate of its diameter. They probably had no idea that the earth and sun have quite different densities.

[6] C. Ptolemy, *'E Mathematike Syntaxis* (ca. 142). There is an edition by J. L. Heiberg in *C. Ptolemaei Opera Quae Exstant Omnia,* B. G. Teubner, Leipzig (1898), that is still considered the best Greek text available. There is a translation of this specific text into German by K. Manitius, B. G. Teubner, Leipzig (1913), as well as a Greek text with a parallel translation into French by N. B. Halma, Henri Grand Libraire, Paris (1813). I use the division into chapters adopted by Heiberg and Manitius, which differs sometimes from that adopted by Halma.

[7] In modern terms, chord $\alpha = 2 \sin \frac{1}{2} \alpha$.

[8] Menelaos, *Sphaerica* (ca. 100). The Greek text is not known to be extant. Abu Nasr Mansur bin Ali bin Iraq prepared an Arabic version around the year 1000. There is a German translation of this version by Max Krause under the title *Die Sphärik von Menelaos aus Alexandrien,* Weidmannsche Buchhandlung, Berlin (1936).

made range from 127 March 26[9] to 141 February 2. He wrote on many subjects: mathematics, optics, music, geography, and astronomy. Except for some minor points, I have not studied his work on any subject except astronomy.

Ptolemy's major work on astronomy[6] is usually given the name *'E Mathematike Syntaxis* in English transliterations of the Greek. Since the names now given to many ancient works were assigned by later editors or commentators, I do not know whether this is a title that Ptolemy himself chose or not. It is an old title in any event. The work is divided into thirteen portions that are usually called "books" in modern discussions of the work. Each book is usually identified by a Roman numeral and is divided into "chapters" that are identified by an Arabic numeral following the book number. Thus, for example, I referred to Chapter IX.2 a moment ago; this means the second chapter in the ninth book.

I shall frequently refer to this work of Ptolemy by the short term *Syntaxis,* although I am well aware that *Almagest* is used in much writing.[10] Further, since the word *Syntaxis* is ambiguous as it occurs in the combination *'E Mathematike Syntaxis,* I leave it untranslated.

The *Syntaxis* starts with some mathematical preliminaries that deal mainly with plane and spherical trigonometry. After them, Ptolemy takes up the obliquity of the ecliptic, the motion of the sun and of the moon, the theory of lunar and solar eclipses, the precession of the equinoxes, a catalogue of the stars, the motions of the planets, and a few miscellaneous matters such as how far from the sun a planet must get before it can be seen.

Ptolemy bases all his theories of astronomical motions on observations, and he emphasizes the importance of doing so. From that standpoint, Ptolemy's approach to astronomy is the same as that of a modern astronomer. It is important to keep this point in mind while appraising Ptolemy and his actions. Roughly half the observations that Ptolemy uses are ones he claims to have made himself; the rest are credited to astronomers earlier than himself. Some of the observations he uses were allegedly made more than eight centuries before his own time by the Babylonians. Unfortunately, most of those early Babylonian "observations" were in fact fabricated by Ptolemy; the earliest one he gives that is likely to be genuine is an observation of a lunar eclipse made on −501 November 19 (see Table 1).

Ptolemy's Claimed Solar Observations[11]

More than 250 years before the time of Ptolemy, Hipparchus determined that the length of the year is 1/300 of a day less than 365¼ days, or $365^d\ 5^h\ 55^m\ 12^s$. (This is actually too long by $6^m\ 26^s$.) Hipparchus found his length of the year by combining an observation that he made in −134 with one made by Aristarchus in −279, so he had a "base line" of only 145 years.

Ptolemy claims to have measured the times of four solstices and equinoxes in years near +135, which gave him a base line of about 414 years from Aristarchus. If he had merely made his measurements with the same accuracy that Hipparchus achieved, taking no advantage of the improvements in technique that were probably available, he would have found the year with an error of 2^m or less. Amazingly, Ptolemy's error in the length of the year is identical with that of Hipparchus. How could this happen?

Table 2 lists the solar observations that Ptolemy claims to have made.[12] The column labeled "reported time" gives the times that Ptolemy claims to have found by careful measurements made with an instrument that he describes. The column labeled "correct time" gives the times that I find by calculation from modern theory; I believe the errors in calculating these times are no more than about an hour. We see that the error in each measured equinox is about 28 hours and that the error in the measured solstice is about 36 hours.[13] I estimate that the standard deviation of error by

[9] I inadvertently gave the date of this observation as 127 March 6 on p. 342 of the first printing of *The Crime of Claudius Ptolemy.* However, I gave the date correctly on p. 317 of the same reference. The error was corrected in the second printing.

[10] *Almagest* was coined, probably about a millenium ago, by combining the definite article in Arabic with the Greek word meaning "greatest"; this combination has been anglicized to yield *Almagest.* Since I do not believe that "The Greatest" is an accurate description of Ptolemy's book, I prefer not to call it *Almagest.*

[11] Most of the information in this section is condensed from Sections V.3 and V.4 of *The Crime of Claudius Ptolemy.*

[12] All tables and figures in this paper except Fig. 1 have been reproduced from *The Crime of Claudius Ptolemy* by permission of The Johns Hopkins University Press.

[13] Calculation from the modern theory of the sun agrees well with observations that are far older than the time of Ptolemy. Thus the trouble is with Ptolemy's claimed "observations" and not with the modern theory.

Table 1

OBSERVATIONS THAT PTOLEMY ATTRIBUTES TO OTHERS

Observer or Place	*Quantity Observed*	*Date*	*Conclusion*
Meton	Summer solstice	−431 Jun 27	Fabricated
Aristarchus	Summer solstice	−279 Jun 26	Genuine
Hipparchus	Summer solstice	−134 Jun 26	Genuine
Hipparchus	Six autumnal equinoxes	−161, −158, −157, −146, −145, −142	Genuine
Hipparchus	Fourteen vernal equinoxes	−145 to −127	Genuine
Alexandria	Vernal equinox	−145 Mar 24	May be genuine
Eratosthenes	Obliquity of ecliptic	ca. −225	May be genuine
Babylon	Triad of lunar eclipses	−720 Mar 19, −719 Mar 8, −719 Sep 1	One is certainly fabricated; the other two may be
Babylon	Triad of lunar eclipses	−382 Dec 23, −381 Jun 18, −381 Dec 12	Fabricated
Alexandria	Triad of lunar eclipses	−200 Sep 22, −199 Mar 19, −199 Sep 12	Fabricated
Babylon	Lunar eclipse	−490 Apr 25	May be genuine
Alexandria	Lunar eclipse	125 Apr 5	May be genuine
Babylon	Lunar eclipse	−501 Nov 19	May be genuine
Hipparchus	Longitudes of sun and moon	−127 Aug 5	Probably fabricated
Hipparchus	Longitudes of sun and moon	−126 May 2, −126 Jul 7	Fabricated
Babylon	Lunar eclipse	−620 Apr 22	Fabricated
Babylon	Lunar eclipse	−522 Jul 16	Fabricated
Alexandria	Lunar eclipse	−173 May 1	Fabricated
Rhodes	Lunar eclipse	−140 Jan 27	Fabricated
Hipparchus	Configuration of stars	No details	Not tested
Timocharis	Longitude of α Vir	ca. −290	May be genuine
Hipparchus	Longitude of α Vir	ca. −130	May be genuine
Hipparchus	Longitude of α Leo	ca. −130	May be genuine
Timocharis or Aristyllus	Declination of 18 stars	ca. −290	Genuine
Hipparchus	Declination of 18 stars	ca. −130	Genuine
Timocharis, Agrippa, or Menelaos	Seven lunar conjunctions or occultations	ca. −290 to ca. +95	Fabricated
Dionysios	Longitude of Mercury at maximum elongation	−261 Feb 12, −261 Apr 25	May be genuine
Dionysios	Longitude of Mercury at maximum elongation	−256 May 28, −261 Aug 23	Fabricated
Alexandria	Longitude of Mercury at maximum elongation	−236 Oct 30, −244 Nov 19	Fabricated
Theon	Longitude of Mercury at maximum elongation	130 Jul 4	May be genuine
Dionysios	Longitude of Mercury	−264 Nov 15 and 19	May be genuine
Theon	Longitude of Venus at maximum elongation	132 Mar 8, 127 Oct 12, 129 May 20	May be genuine
Timocharis	Longitude of Venus	−271 Oct 12 and 16	May be genuine
Alexandria	Conjunction of Mars with β Sco	−271 Jan 18	May be genuine*
Alexandria	Jupiter occulted δ Cnc	−240 Sep 4	May be genuine
Alexandria	Conjunction of Saturn with γ Vir	−228 Mar 1	May be genuine

*However, the correct date is probably −271 Jan 16.

(Reproduced from *The Crime of Claudius Ptolemy* by permission of The Johns Hopkins University Press) the methods that Ptolemy describes should be around 3 hours.

For simplicity, let me assume that the error in each of Ptolemy's "observations" is ten standard deviations and each error is in the same direction. The probability that a set of four observations would have this property because of the chance occurrence of measurement error is about 10^{-92}.

Thus the hypothesis that the times were measured is totally unable to account for the reported times in Table 2. A different but simple hypothesis which, in contrast, accounts for the reported times exactly, to every significant figure, is that Ptolemy fabricated the equinox and solstice times that he claims emphatically to have measured with great care. We can test the hypothesis with the aid of Table 3. Let us take the third line of the table as an example.

In Book III of the *Syntaxis,* Ptolemy claims that the length of the year that Hipparchus had found, namely 1/300 of a day less than 365¼ days, is highly accurate and cannot be improved upon. One way he "shows" this is by comparing the equinox measurement that he claims to have made at 13 hours on 140 March 22 with the "starting observation" that was made at 06 hours on −145 March 24. (I shall explain the origin of the "starting observations" in a moment.) Now let us study the hypothesis that instead of *making* his "measurement", as he claims to have done, Ptolemy *fabricated* it by assuming Hipparchus's length of the year.

Table 2

PTOLEMY'S ALLEGED EQUINOX AND SOLSTICE OBSERVATIONS

Reported Time		*Correct Time*[a]	
Day	*Hour*[b]	*Day*	*Hour*[b]
132 Sep 25	14	132 Sep 24	9.9
139 Sep 26	07	139 Sep 25	2.6
140 Mar 22	13	140 Mar 21	9.4
140 Jun 25	02	140 Jun 23	14.0

[a] As calculated from modern tables.
[b] Local time at Alexandria.

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Table 3

HOW PTOLEMY'S EQUINOX AND SOLSTICE OBSERVATIONS WERE FABRICATED

Starting Observation		*Number of Years*	*Fabricated Time*		*Reported Hour*
Day	*Hour*		*Day*	*Hour*	
−146 Sep 27	00	278	132 Sep 25	13.8	14
−146 Sep 27	00	285	139 Sep 26	07.2	07
−145 Mar 24	06	285	140 Mar 22	13.2	13
−431 Jun 27	06	571	140 Jun 25	02.3	02

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Since the 16th century, astronomers have often reckoned time by using a continuous count of days (which I shall call the "day number") without attention to the passage of months and years. Day number 0.0 in the adopted system came at noon on −4712 January 1. The *Explanatory Supplement*[14] gives tables that make it easy to convert noon on any calendar date into the day number and vice versa for any date from −2000 to 2000. From those tables, we find that the day number was 1 668 179.0 at noon on −145 March 24. Hence the starting time of 06 hours on that date has the day number 1 668 178.75. When we multiply 285 by Hipparchus's value for the length of the year (365.246 666 667), we get 104 095.30 days, and when we add this to the starting day number we get 1 772 274.05 This is the day number of 1.2 hours after noon (hence at 13.2 hours) on 140 March 22. When we state the time to the nearest hour, as Ptolemy does, we get 13 hours on 140 March 22, which agrees exactly with the time that Ptolemy claims to have measured. A similar thing happens with each of Ptolemy's claimed observations. Thus our simple hypothesis accounts exactly for the situation, while the hypothesis that Ptolemy measured the equinoxes and solstices is completely unable to do so.

Thus there is no doubt (more accurately, there is 1 doubt in 10^{92}) that Ptolemy fabricated the solar observations he claims to have made with such great care.

The starting observations in the first three lines of Table 3, namely those on −146 September 27 and −145 March 24, were made by Hipparchus on the island of Rhodes. The solstice observation in the last line, the one dated −431 June 27, is the one that is traditionally attributed to Meton. I have already mentioned that this "observation"

[14] *Explanatory Supplement to The Astronomical Ephemeris and to The American Ephemeris and Nautical Almanac*, H. M. Stationery Office, London, 436–439 (1961).

was actually a calculation made long before the time of Ptolemy by an unknown person who probably had an innocent motive. I find wry amusement in the fact that Ptolemy, who is one of the supreme scientific frauds, was apparently taken in by a fabrication that in his time was more than two centuries old.

Dividing Graduated Intervals by Eye

The next topic in Ptolemy's writing that I want to take up is his star catalogue. In discussing it, we need some preliminary results about the way a person divides an interval on a graduated scale by using only the naked eye. Obtaining these results is simple but tedious.

In stating a calculated angle, Ptolemy always uses degrees, minutes, and seconds. However, when he states the results of a measurement, he almost always uses only degrees and simple fractions thereof. In his star catalogue, the only simple fractions that he uses in giving stellar positions, aside from the value 0, are $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$. For ease in writing, I shall often express these by their equivalents in minutes of arc, namely 0, 10, 15, 20, 30, 40, 45, and 50, but the reader should remember that this is not what Ptolemy actually did. Also, I shall usually omit the symbol for a minute of arc for brevity.

Now let us see what happens when a person uses a scale graduated in degrees, with the intention of estimating fractions of a degree by eye. A modern observer has been indoctrinated with the decimal system almost from birth, and I think most modern observers will use fractions whose denominator is 10. An ancient Greek observer, however, was more likely to use fractions whose denominator was 12.

Suppose that the observer did not think he could estimate fractions smaller than $\frac{1}{6}$ reliably. That is, while using fractions whose denominator was 12, he decided not to use a fraction whose numerator was smaller than 2. Thus the fractions he would end up using were those whose numerators are factors of 12 and multiples thereof (while being larger than 1). This leads to the fractions $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ (aside from 0), which are the same fractions that we find in Ptolemy's star catalogue.

These fractions should not occur with equal frequency, for a reason that we can see with the aid of Fig. 1. In Fig. 1a, we see the part of a scale from 1° to 2°, plus small amounts of adjacent intervals. The marks other than the degree marks correspond to the fractions that we have just enumerated. In Fig. 1b, I repeat the marks at 1°, 1° 30′, and 2° to keep the reader located on the scale, but I have placed these marks below the horizontal line. The marks above the line are the bisectors of the fractional intervals shown in Fig. 1a.

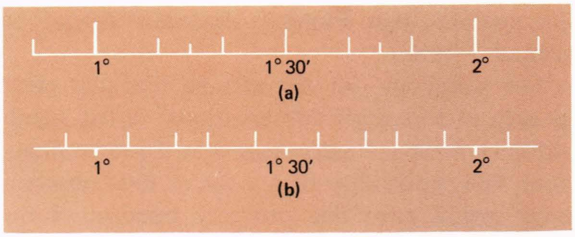

Fig. 1—The division of a degree interval into fractions. In 1a, the heaviest marks are at 1° and 2°, and lesser marks are at 0° 50′, 1° 10′, 1° 15′, 1° 20′, 1° 30′, 1° 40′ 1° 45′, 1° 50′, and 2° 10′. In 1b, the marks below the line are at 1°, 1° 30′, and 2°. The marks above the line are the bisectors of the intervals that appear in 1a.

Consider the first two marks above the line in Fig. 1b, which lie at 0° 55′ and 1° 5′. If the observer I have been talking about judges that a point lies between these marks, he will assign the fraction of a degree to be 0. Since the distance between the marks is 10′ out a total interval of 60′, the fraction 0 will occur one sixth of the time. The reader can also see that the fraction $\frac{1}{2}$ (30′) will occur one sixth of the time.

Now consider the second and third marks above the line in Fig. 1b. They lie at 1° 5′ and 1° 12′.5, and the distance between them is one eighth of a degree. If the observer judges that a point lies between these marks, he will assign it the fraction $\frac{1}{6}$ (10′), and thus the fraction 10 should occur one eighth of the time. The fractions 20, 40, and 50 should occur with the same frequency.

Finally, the reader should be able to see that the fractions 15 and 45 should occur one twelfth of the time.

The frequencies I have just derived would apply if an observer divided the intervals precisely according to the theoretical principles outlined. Actually, every observer has his own way of dividing an interval. An observer's way of dividing is called his personal equation, and we expect different observers to come up with frequencies that differ slightly from those that have been derived. However, we expect that almost every observer will assign the fractions 0 and 30 somewhat more than one sixth of the time, taking the extra occurrences from the neighboring fractions. We also expect him to assign 0 more often than 30.

Ptolemy's Star Catalogue[15]

In Chapter VII.4 of the *Syntaxis,* Ptolemy says that he has measured the coordinates of all the stars down to the sixth magnitude that are visible at Alexandria. He gives the coordinates of these stars, along with their magnitudes, in a star catalogue that occupies most of Books VII and VIII of the *Syntaxis*. Because of the precession of the equinoxes, the longitude of a star (but not its latitude) increases steadily with time, so it is necessary to specify the epoch at which the longitudes are to apply.[16] Ptolemy uses the epoch that we call 137 July 20.

Ptolemy also tells us a little about the method he claims to have used in measuring the coordinates. It amounts to measuring the position of a star relative to the moon at a specific instant, calculating the position of the moon from his lunar theory at that instant, and thence finding the position of the star. We can tell immediately that Ptolemy has lied about the way he found the coordinates in the star catalogue, and we can tell this in two different ways.

First, all the stars in Ptolemy's star catalogue were visible (that is, they were sometimes above the horizon) on the island of Rhodes and, in fact, the stars in the catalogue go all the way down to the horizon of Rhodes, which is well to the north of Alexandria. On the other hand, there are many bright stars that were visible in Alexandria but not on Rhodes, but none of these stars appears in the catalogue. That is, the stars do not go down to the horizon at Alexandria. However, they should do so if Ptolemy indeed observed all the stars that were visible there. We conclude that the observations for the catalogue were actually made on Rhodes and not in Alexandria as Ptolemy claims.

Second, the errors in the stellar positions, if they were measured in the way Ptolemy claims, are necessarily greater than the errors in Ptolemy's theory of the moon. An elementary study shows that the lunar theory has a bias of $-1^{\circ}.1$ in longitude (the sign means that the theory gives a longitude that is too small) and a standard deviation of more than $36'$ about the biased longitude. When we compare the longitudes in the star catalogue with those calculated from modern data, we find that they have the same bias of $-1^{\circ}.1$, which agrees with what we expect if Ptolemy used his lunar theory in finding the stellar coordinates. However, the standard deviation of the tabulated longitudes about the biased position is only $22'$, which is much less than the corresponding error in the lunar theory. This is impossible if Ptolemy did what he claims.

In summary, Ptolemy did not use the moon in finding the coordinates in the star catalogue, as he claims to have done, and the basic measurements were made on Rhodes, not in Alexandria. Is it possible to discover, this long after the event, how Ptolemy did find the coordinates in the catalogue?

As it turns out, we can reconstruct Ptolemy's method by studying the frequency with which fractions of a degree occur in the catalogue. The situation is shown in Fig. 2. In Fig. 2a, the solid line shows the frequency with which the various fractions occur in the latitude. As I have shown elsewhere,[17] this frequency distribution agrees well with the results found in the preceding section, including the prediction that 0 and 30 should occur more than one sixth of the time, with 0 predominating. The only peculiarity about the distribution of the latitude fractions is that 45 occurs somewhat less often than it should.

The broken line in Fig. 2a gives the distribution of the fractions that occur in the longitudes. The latitude and longitude fractions should have the same distribution, except for ordinary statistical fluctuations, but it is clear from the figure that they do not. There are two notable points about the longitude fractions: (a) there are no longitudes with the fraction 45 and only a trifling number with the fraction 15,[18] and (b) the longitude fractions have a maximum at 40, not at 0 as they should.

We may say flatly that the distribution of the longitude fractions could not have come from any set of measured values whatsoever. In other

[15] Most of the information in this section is found in Sections IX.6 and IX.7 of *The Crime of Claudius Ptolemy*.

[16] In fact, both the latitudes and longitudes of stars change with time. However, the change in latitude is too slow to have been discovered by Ptolemy's time.

[17] See pp. 245ff in *The Crime of Claudius Ptolemy*.

[18] The longitudes with the fraction 15 are so few that they probably result from copying errors in the texts that have come down to us.

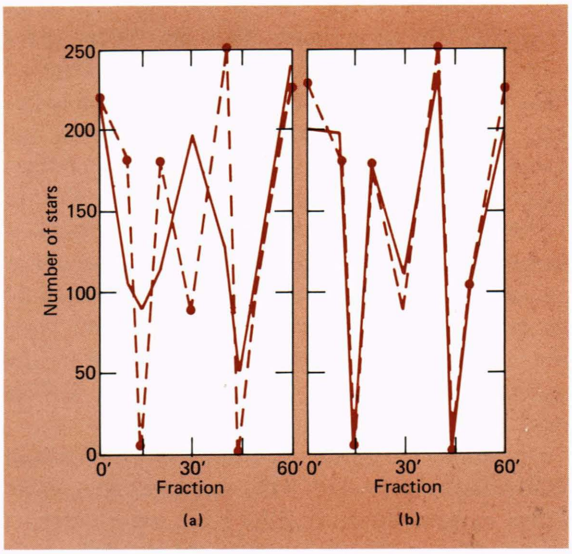

Fig. 2—The solid line in 2a shows the distribution of the fractional values in the latitudes that appear in Ptolemy's star catalogue, while the broken line shows the distribution for the longitudes. The solid line in 2b shows the distribution that would be expected if Ptolemy plagiarized the star catalogue by a method described in the text. The broken line in 2b repeats the distribution for the longitudes for comparison. (This figure is reproduced from *The Crime of Claudius Ptolemy* by permission of The Johns Hopkins University Press.)

words, the longitude values are calculated, not measured. For the method by which Ptolemy calculated the longitudes, I postulate the following:

1. He added an angle of the form N° 40′, in which N is some integer to be determined, to a set of measured longitudes.
2. He realized that adding this amount to a longitude whose fraction was 15 gave him the fraction value 55 and that adding to the fraction 45 gave him the fraction 25. Since the rules of the game did not allow these fractions, Ptolemy symmetrically changed 15 to 20 and 45 to 40 before adding N° 40′, so that an original fraction 15 became 0 and an original fraction 45 became 20.[19] I assume that the personal equation for the original longitudes was the same as that for the latitudes. This is reasonable, since the latitudes and longitudes were presumably measured by the same observer originally.

It is now straightforward to calculate the distribution that the longitude fractions have if my postulates and assumption are correct. The solid line in Fig. 2b shows the distribution that would result if Ptolemy calculated the longitudes by the method described, while the broken line shows the actual distribution. The agreement is almost exact, and the discrepancies are about what we expect from normal statistical fluctuations.

Thus Ptolemy did not measure the positions shown in the star catalogue as he claims to have done. Instead, he plagiarized a star catalogue that had been prepared by some other astronomer, but he added N° 40′ to the longitudes in doing so. Next we asked whether we can learn anything about the original catalogue that he plagiarized.

It would take too much space to demonstrate how we learned about the original table, so I shall merely summarize the results. We find that the integer N is 2. We also find, within surprisingly tight tolerances, that the observations for the original catalogue were made and the catalogue was prepared between the years −138 and −122. For brevity, I shall say that it was prepared near the year −130.

We already know that the observations for the star catalogue were made on Rhodes, and now we learn that they were made near the year −130. We also know from evidence that predates and hence is independent of Ptolemy that Hipparchus prepared a star catalogue on Rhodes near the year −130. It is unlikely that two astronomers working on Rhodes prepared star catalogues at essentially the same time. In other words, Ptolemy plagiarized the star catalogue of Hipparchus.

Thus we should no longer refer to the star catalogue in the *Syntaxis* as Ptolemy's. Instead, we should call it Hipparchus's. In particular, if we subtract 2° 40′ from the longitudes as they appear in the *Syntaxis,* while leaving the latitudes alone, we have rediscovered Hipparchus's star catalogue, which has long been considered to be lost.[20]

It is personally gratifying to have been able to rediscover Hipparchus's catalogue. While it is important and indeed necessary both for history and for modern astronomy to prove that Ptolemy's *Syntaxis* is a gigantic fraud, doing so is essentially a negative and destructive work. However, it is a

[19] I thank Mr. Dennis Rawlins of San Diego for suggesting in private correspondence that Ptolemy made these symmetric changes before the addition, instead of making what look like arbitrary changes after the addition, as I wrote in my book.

[20] We must make a minor qualification to this statement. Hipparchus's catalogue undoubtedly had some longitudes with the fractions 15 and 45, but we can no longer find which ones they were. In other words, when we reconstruct the catalogue, we make an error of 5′ in a few longitudes. Since the standard deviation of error in the longitudes is 22′, this additional error, though unfortunate, is not serious.

positive accomplishment to recover Hipparchus's catalogue, which is about 270 years earlier than the *Syntaxis* and probably the oldest star catalogue in existence.

The Extent of Ptolemy's Fraud

I have analyzed all the observations in the *Syntaxis* in great detail. There is not enough space here to present any more of the analysis, but it is important to state the results: Of all the observations that Ptolemy claims to have made, every one that he uses in his analysis and that can be tested is fraudulent. There are no exceptions to this statement.

Although there are no exceptions, there are two qualifications in this statement that should be explained. First, in deriving some of his parameters, Ptolemy uses only observations that he claims to have made himself; second, he uses only as many observations as parameters. Thus there is no way to test the observations to see if they are genuine. However, the size of the errors in the observations suggests that they are fraudulent.

Furthermore, one special set of eighteen observations is presented by Ptolemy.[21] He uses six of them formally in his analysis; he uses the other twelve only as camouflage to make it appear that he has chosen the set of six at random from a larger set. Interestingly, the twelve observations he does not use formally turn out to be genuine. This proves among other things that Ptolemy's actions in fabricating data were knowing and deliberate and hence that the word "fraud" is justified.

Unfortunately the statement in the first paragraph of this section by no means covers the full extent of Ptolemy's fraud. In addition to using observations that he claims he made himself, Ptolemy uses many others that he claims were made by earlier astronomers. About a third prove to be fraudulent. In other words, not only has Ptolemy knowingly fabricated observations he claims as his own, he has also deliberately falsified history by inventing observations supposedly made by earlier astronomers. We can show that some of the remaining observations are genuine, but we can reach no conclusion about many of them.

Table 4 summarizes the observations that Ptolemy claims to have made himself while Table 1 summarizes those he claims were made by earlier astronomers. Of those he claims to have made himself, the configuration of the stars called α Cnc, β Cnc, and α CMi may be fabricated or it may be genuine; however, Ptolemy makes no use of this configuration in developing his theories. The declinations of 12 stars that are genuine have already been mentioned, but they are not used either. Among the observations that Ptolemy claims were made by others, most of the solar observations have independent confirmation, so they are genuine. The same remark applies to the two sets of stellar declinations measured about -290 and -130. I have proved that many of the observations in Table 1 are fabricated, as the table indicates. The observations marked "may be genuine" are ones that I have not had an opportunity to study in detail.

The lesson is now clear. We cannot accept any statement in the *Syntaxis* as evidence, either for history or for astronomy. This does not mean that every statement in the *Syntaxis* is false; every liar tells the truth sometimes. It does mean that we can accept only those statements that have independent verification, and therefore are not actually using Ptolemy's statements as evidence on their own merits.

This conclusion has further unhappy implications. In addition to giving many "observations", Ptolemy makes many statements about past events that relate both to the history of astronomy and to more general history; in some important matters he is the only witness we have. We must now discard all these elements of previously accepted history and use only evidence that is strictly independent of Ptolemy. Since the *Syntaxis* was accepted as an important source book of history for about 1800 years, Ptolemy's statements have been woven into the very fabric of standard astronomical history. That fabric has now come unravelled; reweaving it will be a formidable job for historians.

The Reception of Ptolemy's Work

Most of the observations that Ptolemy claims to have made himself show errors of about 1°, and his theories of motion show errors of about the same size or larger. These errors are far larger than we expect even for observations made with the naked eye. This means that competent ob-

[21] There are measurements of the declinations of certain stars that Ptolemy uses in deriving his seriously wrong value of the precession of the equinoxes (Chapter VII.3 of the *Syntaxis*).

Table 4

OBSERVATIONS THAT PTOLEMY CLAIMS TO HAVE MADE HIMSELF

Quantity Observed	*Dates*	*Error*	*Conclusion*
Autumnal equinoxes	132 Sep 25, 139 Sep 26	28^h	Fabricated
Vernal equinox	140 Mar 22	28^h	Fabricated
Summer solstice	140 Jun 25	36^h	Fabricated
Angle between tropics	Several unspecified dates	21′	Fabricated
Latitude of Alexandria	No details	15′	Fabricated
Triad of lunar eclipses	133 May 6, 134 Oct 20, 136 Mar 6		Fabricated
Longitudes of sun and moon	139 Feb 9	>1°	Fabricated
Longitudes of sun, moon, and Regulus	139 Feb 23	>1°	Fabricated
Inclination of lunar orbit	Several unspecified dates	~16′	Fabricated
Meridian altitude of moon	135 Oct 1	41′	Fabricated
Relation between apparent diameters of sun and moon	No details	1′20″	Fabricated
Configuration of α Cnc, β Cnc, α CMi	No details		May be fabricated
Other stellar alignments	No details		Not investigated
Longitudes in star catalogue	No details	1°.1	Fabricated
Longitudes in star catalogue	No details	21′	Fabricated
Declination of 12 stars stated but not used	No details	7′	Genuine
Declinations of 6 stars used to find precession	No details	20′	Fabricated
Conjunction of moon with each planet	138 Dec 16, 138 Dec 22, 139 May 17, 139 May 30, 139 Jul 11	40′	Fabricated
Seven longitudes of Mercury at maximum elongation	132 Feb 2 to 141 Feb 2	1°	6 are fabricated; 1 cannot be tested
Two longitudes of Venus at maximum elongation	140 Jul 30, 136 Dec 25	1°	1 is fabricated; 1 cannot be tested
Longitude of Venus at maximum elongation	136 Nov 18	1°.5	Fabricated
Two longitudes of Venus at maximum elongation	134 Feb 18, 140 Feb 18	0°.5 1°.5	1 is fabricated; 1 cannot be tested
Three longitudes at opposition for each outer planet	127 Mar 26 to 139 May 27	~1°	1 is fabricated for each planet; the others cannot be tested

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servations made in Ptolemy's own time would have shown up his work for the fabrication that it is, and it should never have been given a place in the scholarly literature.

In view of this, I am often asked: Why did the *Syntaxis* come to be accepted as a great work? Why did astronomers not reject it immediately? These are questions to which I have no good answer, although I have made a few suggestions in Section XIII.10 of *The Crime of Claudius Ptolemy*. The main point I want to make in closing is that the usual question, as I have just posed it, is too narrow. The *Syntaxis* was accepted as a great work in antiquity, in the Middle Ages, and by most scholars today. Why?

Scholars have used two main devices for maintaining the greatness of the *Syntaxis*. One is to exaggerate Ptolemy's achievements; the other is to ignore evidence that does not uphold his greatness. Here are examples of both.

In a famous history of astronomy before the modern period, J. L. E. Dreyer[22] writes with regard to the *Syntaxis*: "Nearly in every detail (except the variation of distance of the moon) it represented geometrically [the movements of the

[22] J. L. E. Dreyer, *History of the Planetary Systems from Thales to Kepler* (1905). This has been republished under the title *A History of Astronomy from Thales to Kepler* by Dover Publications, New York (1953). The quotation is on p. 200 of the Dover edition.

sun, moon, and planets] almost as closely as the simple instruments then in use enabled observers to follow them . . ." I can find no basis for Dreyer's statement. We saw a moment ago that the standard deviation in the longitude of a star in Hipparchus's catalogue is about 22′. Ancient observations show about the same level of error for the planets and rather less error for the sun and moon, which are brighter and can be located more accurately.

We also saw that the standard deviation in Ptolemy's theory for the longitude of the moon is greater than 35′, aside from the bias in it, and the error in his solar theory is greater than 1°. The errors in Ptolemy's planetary theories are comparable with the accuracy of observation (Table XIII.3 in *The Crime of Claudius Ptolemy*) for the outer planets, but they are more than 1° for Venus and almost 3° for Mercury. There is simply no quantitative basis for Dreyer's statement. Yet almost every recent history of ancient astronomy makes a similar claim for Ptolemy's accuracy.

Now let us turn to the matter of ignoring unfavorable evidence. The process begins with ignoring what Ptolemy himself writes. For example, Dreyer admits that Ptolemy's lunar theory does a poor job of representing the variation of distance to the moon,[23] most other writers on the subject make the same admission. They then disregard how Ptolemy actually handles the problem. Neugebauer writes: "This discrepancy is silently ignored by Ptolemy, though he could not have doubted that the actual geocentric distances of the moon were very different from what his model required."[24]

Ptolemy does not "silently ignore" this discrepancy. On the contrary, he spends many pages of the *Syntaxis* in proving that his model gives exactly the correct variation of the lunar distance, and he fabricates the observation dated 135 October 1 in Table 4 in order to do so. I have discussed Ptolemy's treatment of the variation of the lunar distance at length in Section VIII.5 of *The Crime of Claudius Ptolemy*.

Adverse evidence, when reported, has been widely ignored. Around 1800, a number of scholars became suspicious of the integrity of Ptolemy's work, but as far as I know only one of them was able to go beyond suspicion and find definite proof of fraud. J. B. J. Delambre showed[25] that some of Ptolemy's solar observations were fabricated, and he did so by exactly the method that I use in this paper. (He did not investigate all the observations, but he showed that all those he investigated were fabricated.)

Delambre's proof was of potentially great importance for the field of ancient astronomy, and it should have led to a thorough analysis of the *Syntaxis* by his method. Instead, it has been totally ignored, so far as I can find out. I have never seen a published reference to this work of Delambre except in my own writing, and the *Syntaxis* has remained enshrined in the literature as the greatest astronomical work of antiquity.

Amazingly, J. P. Britton did almost the same thing as Delambre, but 148 years later. In his doctoral dissertation,[26] Britton independently studied Ptolemy's equinox observations by using Delambre's method exactly; he found that all of them were fabricated. This was by far the most important finding in the dissertation, and it should have been pursued vigorously. Instead, it has been ignored even in the paper that Britton himself published[27] on the basis of his dissertation. I can say the same thing about Britton's finding that I did about Delambre's: I have never seen a published reference to it except in my own writing.

It remains to be seen whether *The Crime of Claudius Ptolemy* will suffer the same fate as the books of Delambre and Britton, or whether the irrefutable proof of Ptolemy's fraud, which has been in the literature for more than 150 years, will continue to be ignored.[28]

Acknowledgment

I thank my colleague B. B. Holland, of APL, for a careful reading of this paper and for many helpful discussions about it.

[23] Ptolemy's lunar theory requires the lunar distance to vary by a factor of almost two, but the correct variation is by about 5% from the mean.

[24] O. Neugebauer, *The Exact Sciences in Antiquity*, 2nd Edition, Brown University Press, Providence, R.I., 195 (1957).

[25] J. B. J. Delambre, *Histoire de l'Astronomie du Moyen Âge*, Chez Mme. Veuve Courcier, Paris, lxviii (1819).

[26] J. P. Britton, "On the Quality of Solar and Lunar Observations and Parameters in Ptolemy's *Almagest*," a dissertation submitted to Yale University (1967).

[27] J. P. Britton, "Ptolemy's Determination of the Obliquity of the Ecliptic," *Centaurus* **14**, 29–41 (1969).

[28] B. R. Goldstein, "Casting Doubt on Ptolemy," a review of *The Crime of Claudius Ptolemy*, *Science* **199**, 872 (1978).