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BOETHIUS' THE PRINCIPLES OF MUSIC, AN INTRODUCTION, TRANSLATION, AND COMMENTARY.

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BOETHIUS' THE PRINCIPLES OF MUSIC, AN INTRODUCTION, TRANSLATION, AND COMMENTARY

by

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PREFACE

Boethius' De Institutione Musica, or The Principles of Music as it will be translated in this work, has probably affected musical thought as much as any other single theoretical work in the history of Western music. This work, the roots of which reach back at least six centuries before Christ, was used as one of the chief sources of musical theory for over a millennium, and, in fact, was used as a textbook in music at Oxford as late as the eighteenth century. Yet the only published modern translation of this work is that of Oscar Paul,[1] a translation which is far from accurate and which contains far less than adequate

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1. Oscar Paul, Boethius and die Griechische Harmonik (Leipzig: Leuchart, 1872). Robert Einter, Quellen-Lexikon der Musiker und Musikgelerten (Graz, Austria: Akademische Druck-und Verlagsanstalt (reprint of 1898 edition), 1959), vol. 2, p. 91 refers to two sixteenth century translations of Boethius' musical work, an unpublished Italian translation in the Liceo in Bologna, and a Dutch translation published in Leyden, 1585. F. J. Fetis, in his Biographie Universelle des Musiciens (Paris: 1880), vol. I, pp. 466-467, claimed to have completed a French translation, but this translation has since been lost.

critical notes. Roger Bragard, one of the leading contemporary scholars with regard to Boethius' musical thought, completed translating the first two Books of *De Institutione Musica* into French as his doctoral dissertation in 1926, but, besides being incomplete, the work is only available in handwritten form at the library of the University of Liège.[2] Three chapters of Book I are translated in Strunk's *Source Readings in Music History*,[3] but these three passages serve only as a superficial introduction to the thoroughly developed theory of music contained in *The Principles of Music*. Thus there seems little need for extended justifications of an English translation of this work, especially a translation accompanied by critical notes citing Boethius' Greek and Latin sources wherever possible. For although the role of Boethius has been universally defined as being the means whereby Classical theories of music were transmitted to the Middle Ages, nevertheless no previous

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2. Roger Bragard, *Les sources du De Institutione Musica de Boèce* (unpublished Doctoral dissertation, Université de Liège, Faculté de Philosophie et Lettres, 1926).

3. Oliver Strunk, *Source Readings in Music History from Classical Antiquity through the Romantic Era* (New York: W. W. Norton & Company, Inc., 1950), pp. 79-86.

study of this work has identified precisely what these sources were and thereby exactly what was transmitted.

In the time of Boethius and during the Middle Ages and Renaissance, music was considered one of the four liberal mathematical disciplines, or *quadrivium*. The term *quadrivium* was introduced into Latin literature by the first chapter of Boethius' *De Institutione Arithmetica*, and consequently an understanding of music's place in the *quadrivium* disciplines is essential to an intelligible reading of *The Principles of Music*. For this reason the first chapter of *The Principles of Arithmetic* has also been translated in this study and placed as a preface to the translation of *The Principles of Music*.

The commentary following the translation is concerned with the four essential problems of *The Principles of Music*: the sources, the aesthetics, the mathematics, and the musical theory. In that these are problems related to the treatise at hand, the first four chapters of commentary will limit themselves to the contents of *The Principles of Music* rather than attempting to comment upon what various modern scholars have written about these contents. The last chapter of commentary is an attempt to relate certain premises of Boethius' musical thought to the Western musical tradition.

Friedlein's edition of De Institutione Musica has formed the basic text from which the following translation was made.[4] After having compared Friedlein's edition with several manuscripts not used by him, I am satisfied that the text of his edition is basically sound. The great weakness of Friedlein's edition lies in the poor presentation of Boethius' charts and diagrams. Consequently several charts from a manuscript now in the Cologne Stadtarchiv will be included in the following translation. The charts drawn by the translator are based more on charts contained in the Cologne manuscript and manuscripts in the Bibliotheque Royale, Brussels, than on the charts in Friedlein's edition.[5]

The stylistic goal of the following translation has been to render Boethius in as clear and simple an English as is possible considering the subject matter. This task was often complicated by Boethius' rhetorical style of late

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4. Anicii Manlii Torquati Boetii De Institutione Arithmetica libro duo, De Institutione Musica libri quinque. Accedit Geometria quae fertur Boetii, ed. Godofredus Friedlein (Leipzig: Teubner, 1867).

5. See list of MSS, pp. 457-460.

Latin. An attempt has been made to retain a certain rhetorical quality in the translation, but the complex structures of the late Latin text have been simplified to render them intelligible in English. The footnotes of the translation serve primarily three purposes: to cite Boethius' Greek and Latin sources, to clarify certain ambiguities concerning the theory being presented, and to give the cross-references necessary for an intelligible reading of the work.

I would like to express my most sincere gratitude to Professor Robert L. Weaver for his guidance, careful reading, and numerous suggestions in the preparation of this thesis. His patient advice, criticism, and encouragement have contributed immeasurably to the formation of this study. I would like to acknowledge the vision of The School of Music, George Peabody College for Teachers, in instituting the "Music and the Humanities" program of graduate study in music. Without this program I could never have had the privilege of working under such scholars as Professor Francis L. Newton of Vanderbilt University's Department of Classical Languages and Literatures and Professors Donald W. Sherburne and John J. Compton of

Vanderbilt University's Department of Philosophy. I would like to thank the German Fulbright Commission and the staff of the Musikwissenschaftliches Institut, University of Cologne, especially Professors Heinrich Hueschen and Karl Gustav Fellerer, for a year of invaluable research in the preparation of this study. I offer a very special thanks to Miss Joyce Maples, my Student Assistant; her assistance in preparation and proof reading of the final copy of this work far exceeded the call of duty. Finally, I would like to thank my wife for typing the first drafts of this work and for her ever present encouragement and moral support in bringing this study to completion.

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INTRODUCTION

Anicius Manlius Severinus Boethius, the son of an extablished patriarchal family of Rome, was born around 480.[1] Having been orphaned at an early age, he was taken into the household of Symmachus, leader of the Roman Senate (caput Senatus) and, though a descendent of the famous pagan antagonist to Ambrose, a pillar of the Catholic Church. The relationship between Boethius and Symmachus appears to have been richer than merely that of a guardian to a stepson; for together the two fought as political allies in the tumultuous years which were to follow, and Boethius even addressed two of his works to Symmachus.[2]

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1. Concerning the biography of Boethius in more specific detail see: Helen M. Barrett, Boethius, Some Aspects of His Times and Work (Cambridge: Cambridge University Press, 1940); Howard Rollin Patch, The Tradition of Boethius (New York: Oxford University Press, 1935), pp. 1-19; and Edward Kennard Rand, Founders of the Middle Ages (Cambridge: Harvard University Press, 1929), pp. 135-180. The following brief biography is based primarily on sources contemporary with Boethius, and they will be cited as they appear.

2. Boethius' De Institutione Arithmetica and Quomodo Trinitas unus Deus ac non tres dii are addressed specifically to his father-in-law.

These ties were no doubt strengthened when Boethius married Rusticiana, the daughter of Symmachus.

Boethius, being of noble birth and living in the household of an influential Roman family, was naturally educated in the liberal disciplines of classical Rome. One tradition, based on a statement in a letter by Cassiodorus,[3] holds that Boethius studied in Athens, a creditable tradition considering Boethius' ability in Greek and his thorough knowledge of Greek sources. In any case he had mastered Greek as well as the Greek sources of the basic liberal arts at an early age from whence he turned his attention to philosophy in the persons of Plato and Aristotle. At this time Boethius' expressed purpose was to translate all of Aristotle and Plato into Latin and to write therewith commentaries demonstrating the essential agreement between these two philosophers.[4] He completed translating much of Aristotle during this period, along with a translation of Porphyry's Introduction to the

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3. Cassiodorus Variae i. 45 (Migne Patrologia Latina (Paris: 1865) lxix. 539 C).

4. Boethius De Interpretatione ii. 2 (Migne Patrologia Latina lxiv. 433 D).

Categories and a commentary on Cicero's Topics. Most of these works are related to various problems of logic, and they remained the basic Latin tests in logic for over half a millennium.

The fact that Boethius left no translations of Plato does not imply that Boethius was not reading Plato, for it is probably a result of reading and heeding Plato that he never finished his mammoth philosophical project. One of the autobiographical moments of The Consolation of Philosophy reveals that Boethius decided to leave the quietude of his study and enter public administration following the admonition of Plato that students of wisdom should govern.[5] Thus in 510 Boethius was elected Consul and began a very notable public career under the rule of King Theodoric the Ostrogoth. Boethius further justifies his decision in a passage of his work on the Categories, written the year of his consulate:

> Although the cares of my consular office prevent me from devoting my entire attention to these studies, yet it seems to me a sort of public service to instruct my fellow-citizens in the products of reasoned investigation. Nor shall I deserve ill of my country

---

5. The Consolation of Philosophy i. 4. 18-25.

> in this attempt. In far-distant ages, other cities transferred to our state alone the lordship and sovereignty of the world; I am glad to assume the remaining task of educating our present society in the spirit of Greek philosophy. Wherefore this is verily a part of my consular duty, since it has always been a Roman habit to take whatever was beautiful or praiseworthy throughout the world and to add to its lustre by imitation. So then, to my task.[6]

Thus Boethius, like Cicero before him, considered public service the direct responsibility of a philosopher; and this sixth century Roman, one of the last Romans, strove to follow in the steps of his illustrious predecessor.

In these early years of his philosophical and public career Boethius received constant recognition for his wisdom and skill. Ennodius (c. 473-521), Bishop of Ticinum, praised Boethius as the bearer of the torch of ancient learning and states that Boethius possessed at the threshhold of his life what his elders had hardly achieved at the end of their lives.[7] Cassiodorus similarly praised him as the transmitter of ancient learning:

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6. _In Categorias Aristotelis_ ii. _init._ (Migne: _Pat. Lat._ lxiv. 201 D), translation by Rand _Founders of the Middle Ages_, p. 158.

7. Ennodius _Opera_ ed. F. Vogel, in _Auctores Antiquissimi_ vii. (Berlin: 1885, _Monumenta Germaniae Historia_), _Epistola_ vii. 13. 236.

> In your translations Pythagoras the musician, Ptolemy the astronomer, Nicomachus the arithmetician, and Euclid the geometer are read by Italians; Plato the theologian and Aristotle the logician debate in the Roman tongue; and you have given Archimedes the mechanician back to the Sicilians in Latin.[8]

Boethius was not only praised as a scholar, but his practical skill and judgment were likewise recognized. In the above cited letter from Cassiodorus[9] Boethius is asked to construct a water clock and a sundial for Gundobad, King of the Burgundians and brother-in-law to King Theodoric. Elsewhere Cassiodorus requests Boethius' aid in solving a problem relating to some coins of less than proper weight.[10] Finally Boethius' learning and judgment in music were recognized and required in the following letter:

> Since the king of the Franks, attracted by the fame of our banquets, has with earnest prayers besought us to send him a harper (citharaedus), our only hope of executing his commission lies in you, whom we know to be accomplished in musical learning. For it will be easy for you to choose a well-skilled man, having yourself been able to attain to that high and abstruse study.

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8. Cassiodorus Variae i. 45 (Migne: Pat. Lat. xvix. 529 C).
9. Ibid.
10. Cassiodorus Variae i. 10 (Migne: Pat. Lat. xvix. 514, 516).

Cassiodorus proceeds in the letter to speculate concerning the ethos of music based on classical mythology and Scriptures, and after some further discussion of the modes and the harmony of the spheres, he concludes as follows:

> But since we have made this pleasing digression (because it is always agreeable to talk about learning with learned men) let your wisdom choose out for us the best harper of the day, for the purpose that we have mentioned: herein will you accomplish a task somewhat like that of Orpheus, when he with sweet sounds tamed the fierce hearts of savage creatures. The thanks which we owe you will be expressed by liberal compensation, for you obey our rule, and to the utmost of your power render it illustrious by your attainments.[11]

These years of good fortune reached their apex in the year 522 when Boethius' two sons, still in their youth, were admitted to the Consul,[12] and when, later in the same year, Boethius became Theodoric's Magister Officiorum in Ravenna.

The question of Boethius' Christianity has not yet been mentioned, but there can be little doubt concerning this question considering the period in which he lived and five of his last works which treat tenets of the

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11. Ibid. ii. 40 (Migne Pat. Lat. lxix. 570,573), translation by Thomas Hodgkin Theodoric the Goth (New York: G. P. Pubnam's Sons, 1902), pp. 195-196.

12. Cf. The Consolation of Philosophy ii. 3. 25-35.

orthodox faith. Nevertheless The Consolation of Philosophy makes no reference to the Christian faith, and this has created some doubt concerning the authenticity of Boethius' Opuscula Sacra. Today, however, these works are considered part of Boethius' mature writings, and thus Boethius' Christianity and Catholicity are readily admitted.[13]

Boethius' Christianity, especially his defense of the orthodox doctrine of the trinity, probably had much to do with his fall from favor with Theodoric the Arian. But this is not the place to re-examine the complex intrigues which led to the imprisonment and death of Boethius at the hands of Theodoric.[14] The following passage from the sixth century Anonymus Valesii serves to portray the tragic trial, sentence and death of Boethius.

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13. Concerning the authenticity of the Opuscula sacra see Boethius, The Theological Tractates with an English Translation, The Consolation of Philosophy with the English Translation of "I.T." (1609), revised ed. H. F. Steward and E. K. Rand (London and New York: G. P. Putnam's Sons, Loeb Classical Library, 1918) pp. x, xiii.

14. A most comprehensive account of the issues and intrigues relating to Boethius' trial and death are found in Hodgkin Theodoric the Goth (cf. above n. 11) pp. 256-280.

> From this event the devil found occasion to subvert the man [Theodoric] who had hitherto governed the State well and without giving cause to complaint. . .
>
> The king began to show anger against the Romans whenever there was opportunity. Cyprian, who was then Referendarius and afterwards Count of the Sacred Largesses and Master of Offices, driven by greed, laid an information against Albinus the Patrician that he had sent letters to the Emperor Justin hostile to Theodoric's rule. Upon being summoned before the Court, Albinus denied the accusation and then Boethius the Patrician, who was Master of Offices, said in the King's presence: "False is the information of Cyprian, but if Albinus did it, both I and the whole Senate did it with one accord. It is false, my lord, Oh King." Then Cyprian with hesitation brought forward false witnesses not only against Albinus but also against his defender Boethius. But the King was laying a trap for the Romans and seeking how he might kill them; he put more confidence in the false witnesses than in the Senators. Then Albinus and Boethius were taken into custody to the baptistry of the Church.
>
> But the King summoned Eusebius, Prefect of the city of Tricinum, and, without giving Boethius a hearing, passed sentence upon him. The King soon afterwards caused him to be killed on the Calventian territory where he was held in custody. He was tortured for a very long time by a cord that was twisted round his forehead so that his eyes started from his head. Then at last amidst his torments he was killed with a club.[15]

While in prison Boethius wrote The Consolation of Philosophy, a work that was without rival during the Middle

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15. Anonymus Valesii, ed. R. Cessi in new edition of L. A. Muratori Rerum Italicarum Scriptores (Citta di Castello: 1913), sec. 83 and secs. 85,87, translation by Barrett Boethius, Some Aspects of His Times and Work (see above n. 3), pp. 58-59.

Ages. His death in 524 or 525 was viewed as that of a Christian martyr to many of his contemporaries, as was that of his father-in-law who followed him in death about a year later. Although Boethius' name has never appeared in the Martyrologium Romanum and he is thus not an official saint, nevertheless, the citizens of Pavia, in whose cathedral the bones of Boethius are kept, celebrate the feast day of St. Severinus Boethius on the twenty-third day of October each year.[16]

Whether considered a universal saint or not, Dante placed Boethius in Paradise and referred to him as l'anima santa:

> Now, an [sic] thou let thy mental vision run
> from light to light, as I record their praise,
> thou art left thirsting for the eighth bright sun.
>
> Within it on all good delights to gaze
> the saintly soul, which unto ears that cease
> mishearing shows the world's deceitful ways.

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16. See Rand Founders of the Middle Ages (cf. above n. 3), pp. 178, 179 and p. 344, n. 80.

> Down in Cieldauro, where it found release,
> it left its tortured body, thence to soar
> from martyrdom and exile to this peace.[17]

The Middle Ages recognized Boethius the musician as the instructor who would educate the ears of their age to "cease mishearing"; for Honorius of Autun, in his *De Animae Exilio et Patria*, considers Boethius the instructor in the city of Music, where a chorus of men and boys sing praises to God in low and high voices:

> *In hac urbe per Boetii doctrinam hinc chorus viris gravibus, inde puerilis acutis vocibus Deo jubilat.*[18]

The works of Boethius fall into four basic classifications: the mathematical, the logical, the theological and *The Consolation of Philosophy*. Three works survive in

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17. Dante *The Divine Comedy*, text with translation by Geoffrey L. Bickersteth (Cambridge: Harvard University Press, 1965), *Paradise*, Canto x. 121 -129:

> Or se tu l'occhio de la mente trani
> di luce in luce dietro le mie lode,
> gia de l'ottava con sete rimani.
> Per vedere ogni ben dentro vi gode
> l'anima santa che il mondo fallace
> fa manifesto a chi di lei ben ode.
> Lo corpo ond'ella fu cacciata giace
> giuso in Cieldauro, ed essa da martiro
> e da esilio venne a questa pace.

18. Honorius *De Animae Exilio et Patria* vi. (Migne, *Pat. Lat.* clxii 1244 C).

the first classification:

De institutione arithmetica libri duo

De institutione musica libri quinque

De geometria libri duo ex Euclide translati[19]

A fourth work on astronomy probably completed this classification. Cassiodorus referred to Boethius as having translated "Ptolemy the astronomer,"[20] and a letter of Gerbert written from Mantua, 983, refers to a *De Astrologia*, containing eight books, written by Boethius.[21] If such a work was written, it does not seem to have survived to the present day.

The logical works form the second and largest classification within Boethius' works:

In Porphyrii Isagogen Commentarii[22]

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19. *Anicii Manlii Torquati Boetii De Institutione arithmetica libro duo, De institutione musica libri quinque. Accedit geometria quae fertur Boetii*, ed. Godofred Friedlein (Leipzig: Teubner, 1867).

20. Cf. pp. 4,5. n. 10.

21. Julien Havet *Lettres de Gerbert* (Paris: 1889) *Epistola* viii. p. 6 f.: ". . . *id est VIII volumina Boetii de astrologia, praeclarissima quoque figurarum gometriae* [sic]. . . "

22. *Anicii Manlii Severini Boethii In Isagogen Porphyrii Commentarii*, ed. S. Brandt (Leipzig: 1906). *Corpus Scriptorum Ecclesiasticorum Latinorum* xlviii.

Commentarii in Librum Aristotelis Περὶ ἑρμηνείας[23]

In Aristotelis Praedicamenta commentariorum libri duo

Liber de Divisionibus

Liber de Definitionibus

Introductio ad Categoricos Syllogismos

De Syllogismo Categorico libri duo

De Syllogismo Hypothetico libri duo

In Ciceronis Topica Commentariorum libri sex

De differentiis Topicis libri quatour.[24]

The following five works generally known as the Opuscula Sacra form Boethius' theological works:

Quomodo Trinitas unus Deus ac non tres Dii (De Trinitate)

Utrum Pater et Filius et Spiritus Sanctus de Divinitate Substantialiter Praedicentur

Quomodo Substantiae in eo quod sint Bonae sint cum non sint Substantialia Bona

De Fide Catholica

Contra Eutychen et Nestorium[25]

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23. Boethius *Commentarii in Librum Aristotelis* Περὶ ἑρμηνείας, ed. C. Meiser (Leipzig: Teubner, 1877).

24. The last eight of these works are to be found in Migne *Patrologia Latina* (1865) lxiv.

25. Boethius *The Theological Tractates* . . . with an English translation by H. F. Stewart and E. K. Rand (Cf. above n. 14).

Finally there is The Consolation of Philosophy,[26] a work as unique among Boethius' works as it is in the general history of philosophical literature.

The chronology of Boethius' works is basically identical with the outline just presented.[27] The mathematical works were thus completed at the beginning of Boethius' literary productivity, probably between 500 and 506. That these works were his first is confirmed in the letter of dedication to Symmachus which precedes the De Institutione Arithmetica; for there he specifically calls them his "primitiae."[28] Since arithmetic was considered to hold "the first principle and position of a mother" to the other mathematical disciplines,[29] it was probably the subject of his first work. Thereafter followed The Principles of Music and the works on geometry and astronomy.

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26. Anicii Manlii Severini Boethii Philosophiae Libri Quinque, ed. R. Peiper (Leipzig: Teubner, 1871).

27. Concerning specific chronology of Boethius' works see Samuel Brandt "Entstehungszeit und zeitliche Folge der Werke des Boethius," Philologus lxii. (1903) pp. 141, 154 and 234, 275.

28. De Institutione Arithmetica, ed. Friedlein (see above n. 19), p. 5: "Ita et laboris mei primitias doctissimo indicio consecrabis et non maiore censebitur auctor merito quam probator."

29. See The Principles of Arithmetic I. i. p.

The Principles of Music and the other mathematical works are thus products of a comparatively young scholar. They can indeed be considered a personal record of the young scholar's studies in the Classical sources of these disciplines; for the mathematical works are little more than loose and very limitedly expanded translations of the Greek sources on these disciplines which were available in Boethius' time. Boethius the Scholar does not give way to Boethius the Philosopher until the mature commentaries which accompany the logical works: and even there he has been accused of maintaining a certain skeptical suspension of judgment.[30] The only sure doctrinal commitment to be found in *The Principles of Music* is that against Aristoxenus,

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30. The text and translation of the following verse by Godfrey of St. Victor is given by Rand, *The Founders of the Middle Ages* (see above n. 1), p. 144:

> Assidet Boetius stupens de hac lite
> Audiens quid hic et hic asserat perite,
> Et quid cui faveat non discernit rite,
> Nec praesumit solvere litem definite.
>
> (Sits Boethius quite stunned by this disputation,
> Listening to this and that subtle explanation,
> But to side with this or that shows no inclination
> Nor presumes to give the case sure adjudication.)

and this is because the sources from which he was working shared this commitment.

The fact that The Principles of Music is the product of a young Boethius who has not yet come to a thorough confrontation with Aristotle is essential to an understanding of the character and contents of this work. One needs to read very few pages of the work to realize that this is not a typically inductive theoretical work basing its speculation on practical problems such as performance, analysis, or composition. Such a work would be the product of one working in the Aristotelian tradition. The Principles of Music, on the other hand, is a strictly deductive work, beginning from the premise that only the forms, the quantities, the proportions acoustically present in the corporeal substance of sound are knowable; and the value, the harmony, of these proportions is judged according to a priori mathematical laws of related numbers abstracted from sound. This work is thus thoroughly grounded in the Platonic and Neo-Pythagorean tradition of deductive reasoning rather than in the Aristotelian tradition of inductive reason and classification.

The influence of the Platonic tradition on The Principles of Music is further seen in the expressed pedagogical purpose of these mathematical works. These disciplines are not treated as ends in themselves but rather are considered as a preparation for the study of philosophy. They are described as the "quadrivium," the four-way path "by which one should come to those places where the more excellent mind, having been delivered from our senses, is led to the certainty of intelligence."[31] These places of which Boethius is speaking are in the land of Platonic philosophy where pure forms, essences, exist in and of themselves and cease to "suffer radical change through participation in the corporeal."[32] Music and the other mathematical disciplines are thus not a part of philosophy proper as they would be in the Aristotelian tradition,[33] but they are rather pedagogical preparations for the ascent to pure philosophy in the Platonic sense of the word.[34]

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31. See The Principles of Arithmetic I. i. p. 27.
32. Ibid., p. 24.
33. Aristotle Metaphysics 1026a considers theoretical physics, mathematics (thus music theory), and metaphysics or theology as integral parts of philosophy. Boethius assumed this position himself after he became thoroughly versed in Aristotle; see De Trinitate ii.
34. This concept of the Liberal Arts is found in Plato Republic vii. 523-534.

The Principles of Music was thus written more for the student who aspired to philosophy than for the practicing musician, even though it was the practicing musician's most authoritative theory text for almost a thousand years. The form of the treatise is not that of an isagoge, introducing the student to the general field of music by defining music, discussing its invention, its uses and effects, its divisions, its instruments, and concluding with a brief exposition of musical proportions.[35] The general form of the work is rather that of the classical *protreptikos*, or "exhortation," exhorting the student to study music so that he can master that type of essence which consists of harmoniously related quantity or multitude, contemplated and judged by the reason alone; for thereafter he will be more able to reason concerning the pure and incorporeal essences of philosophy.[36]

The hortatory character of Boethius' mathematical works is most clearly seen in the "Introduction" to *The*

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35. The *De Musica* of Isidore of Seville's *Etymologie* iii. 15-23 (ed. W. M. Lindsay, Oxford: The Claredon Press, 1911), is a classic example of the isagoge form.

36. Concerning the *protreptikos* form and the Platonic orientation of *The Principles of Music* see Leo Schrade, "Music in the Philosophy of Boethius," *The Musical Quarterly*, XXXIII (1947) 188-200.

Principles of Arithmetic, and for this reason this "Introduction" will serve as a preface to the following translation of The Principles of Music. Boethius continues his exhortation in the "Introduction" to Book I of The Principles of Music; for there he urges the student to cease being merely affected by the pleasures of music and to search out reasons why he is so affected. Book I continues to present a rather comprehensive picture of all subjects to be discussed in the remainder of the books, discussing them rather dogmatically; for the student is to take the theory set forth in the first book on faith.[37] The remaining four books thus have the task of logically demonstrating the basic contents of Book I.

The first chapters of Book II serve as a review of sections from The Principles of Arithmetic which contain the mathematical theory necessary for musical reasoning. The last chapters of this book apply this theory to musical consonances and thereby logically demonstrate their mathematical proportions.

Book III continues to employ the mathematical theory

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37. See The Principles of Music I. xxxiii. pp. 100-101.

of Book II, but here to prove the ratios of intervals smaller than the tone. The theories of various Greek thinkers concerning these intervals are presented and discussed in terms of their mathematical consistency.

The first two chapters of Book IV present a translation of the Euclidian _Sectio Canonis_ as a review of the first three books. Thereafter the basic elements of Greek notation are discussed, and the complete gamut of the Greater Perfect System is mathematically demonstrated by means of a monochord division. The closing chapters of this book discuss the Greek octave species and tropes.

The concluding book of _The Principles of Music_ begins a paraphrased translation of Ptolemy, _Harmonics_, Book I, but the last eleven chapters of this concluding book were never completed. Boethius may have intended to continue and translate rather freely the complete work of Ptolemy, for Boethius promises future discussions concerning several topics which he never completed; and a full translation of Ptolemy's _Harmonics_ would have completed these references. If this was Boethius' original plan, _The Principles of Music_ would probably have encompassed seven books rather than the existing five.

The medieval students who mastered these five books of music theory were indeed well equipped to pursue the study of philosophy; for they would have thereby learned to abstract mathematical forms from aural perception and to reason concerning these forms as unchangeable, incorporeal essences. On the other hand, the student of this work saw these forms or essences as the determining factor in molding a "harmonious" aural experience. Therefore, The Principles of Music had an effect on the practice of music in direct proportion to its effect of exhorting students to the study of philosophy; for as they accepted the mathematical proportions as the truth, the beauty of harmonious forms, they necessarily demanded these forms to be present in the music they heard. This is particularly true of the medieval monk who studied The Principles of Music as a text in the Liberal Arts preparing for the study of theology, but sang mass and the office hours daily as part of the monastic discipline.

The Principles of Music must thus be read primarily as a protreptikos to the study of philosophy. Nevertheless the practical implications of the mathematical theory presented in the work can by no means be ignored if its proper place in the history of music and ideas is to be fully understood.

PART I

TRANSLATION

THE PRINCIPLES OF ARITHMETIC

BOOK I

I. Introduction, concerning the division of mathematics.[1]

Among all men of ancient authority who flourished through the purer reason of the mind under the leadership of Pythagoras, it was considered manifestly certain that no one was to go forth in the study of philosophy unless excellence and nobility were investigated by means of a certain four-way path (*quadrivium*)[2] which led to such

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1. Boethius' *De institutione arithmetica* is a very free translation of Nicomachus' *Eisagoge arithmetica*. This introduction, an introduction which is nessary to the full understanding of Boethius' *De institutione musica*, includes the basic contents of the first five chapters of Nicomachus' work. *Cf. Nichomachus of Gerasa: Introduction to Arithmetic*, trans. into English by Martin Luther D'Ooge, with studies in Greek arithmetic by Frank E. Robins and Louis C. Karpinski (New York: The MacMillan Company, 1926), pp. 181-189; and Commentary, Chapter I, p. 334.

2. Although the basic concept of a "four-way path" or "*quadrivium*" is found in the text of Nicomachus, the term itself is a contribution of Boethius. Moreover, this is the origin of the term in Western education, as well as one of the primary sources for the very concept.

knowledge. This fourway path will not hamper the skill of right reasoning. For wisdom, the comprehension of reality, is of things which partake in immutable substance. Moreover we say that these things are those which neither grow under tension nor shrink when reconsidered, things which are not susceptible to change through variations, but which always preserve themselves in their appropriate virtue (vis), supported through the aid of their own nature. These things are quantities, qualities, forms, magnitudes, minima, equalities, conditions, actualities (actus), dispositions, places, times, and whatever is found united in some way with bodies. Indeed these things themselves are incorporeal in nature and thrive by reason of their immutable substance, but they suffer radical change through participation in the corporeal, and through contact with variable things they change in veritable inconsistency. Therefore, since, as has been said, nature has alloted these things immutable substance and virtue (vis), they can truly and properly be said "to be." Therefore wisdom professes knowledge of these things which are in and of themselves, and which are called "essences."

There are two divisions of essences: the one is continuous and not divided by any limits, as is a tree, a stone, and all parts of this universe which are properly called "magnitudes;" the other division of essences is discontinuous with itself, and divided into parts, and just as a union is reduced by heaps into one, so is the grass, the populace, a chorus, a mass of similar objects, and whatsoever thing whose parts are terminated by their own extremities and set apart from others by a limit. "Multitude" is the proper name for these things. Moreover, some multitudes are in and of themselves, such as 3 or 4, or a square, or any number which, as it were, is independent. Others are indeed ascertained through themselves, but are referred to something else, such as a duple, a half, a sesquialter, or a sesquitertian; and anything is of such a nature that exists through its relation to something else and not in and of itself. On the other hand, some magnitudes are stationary and void of motion, while others are mobile, always turned in rotation and not still at any time. Therefore arithmetic explores that multitude which exists in and of itself, whereas the appropriate admixtures of musical modulation become fully acquainted with that multitude which is related to something. Geometry professes knowledge of immobile magnitude, whereas the skill of the astronomical discipline claims knowledge of mobile magnitude.

If the searcher should deprive himself of these four parts, he will not be able to find truth; and indeed, without this approach to knowledge he should not become properly wise concerning any truth. For wisdom, knowledge and complete comprehension, concerns itself with those things which truly are; and he who rejects these disciplines, these narrow ways to wisdom, I declare to him that he has no right to philosophize. For since philosophy is the love of wisdom, he would have despised philosophy when he rejected these disciplines.

I think it should be added that the entive potency of multitude, having progressed from one term, grows in an infinitely increasing progression. Magnitude, on the other hand, begins in a finite quantity, but admits no end to its division; for it submits to infinitessimal sectioning of its body. Thus philosophy willingly rejects this infinity and indeterminate potential of nature. For infinity can neither be unified through knowledge nor grasped by the mind. But hence reason takes unto itself those very things in which it can practice an adroit investigation of truth; for it assumes a term of finite quantity from a plurality of infinite multitude, and commits defined spaces unto itself for cognition, having rejected a section of interminable magnitude. It is therefore clear that if anyone neglect these disciplines, he would irrecoverably lose all the teaching of philosophy.

This then is that four-way path (quadrivium) by which one should come to those places where the more excellent mind, having been delivered from our senses, is led to the certainty of intelligence. Indeed there are degrees and fixed steps by which one is able to progress and ascend, so that the eye of that more worthy soul, as Plato said,[3] might be saved and established free from corporeal bodies having many eyes. Then that eye might strive to investigate and see truth through the light of truth alone. I would say that these disciplines will illumine this eye cast down and made destitute by the corporeal senses.

Which then of these disciplines ought to be studied first unless it is that one which holds the first principle and position of a mother, as it were, to the others? This one is indeed arithmetic; for it is prior to all the others,

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3. Plato Republic 527 D.

not only because God the Creator of the great universe considered arithmetic first as the model of his reasoning and created all according to it, having rationally forged all things through numbers of assigned order to find concordance, but also because arithmetic is prior by nature. For if that which is prior by nature be abolished, the posterior things are also abolished; if the posterior things perish, no substantial change takes place in the prior. Animal, for example, is prior to man; for if you abolish animal, the nature of man is also abolished at the same time; but if you should abolish man, animal will not perish. Conversely those things are posterior which infer something else in themselves, whereas the prior, as has been said, imply nothing of the posterior. If you say "man," for example, you will also denote animal at the same time; for that which is man is animal. If you say "animal," you would not be implying the species of man at the same time; for animal is not the same thing as man.

The same is seen to occur in arithmetic and geometry; for if you abolish numbers, whence does the triangle, the quadrangle, or any other thing considered in geometry receive its name? But if you abolish quadrangle and triangle, and if all geometry be consumed, "three" and "four" and the names of other numbers will not perish. Likewise when I will have said any geometric form, the name of numbers is implied at the same time; when I will have said numbers, I in no way have named any geometric forms.

It can be easily proved that the power of numbers is prior to music, not only because things which exist in themselves are prior to those which are considered in relation to others, but also because musical modulation itself is discussed in terms of numerical names. The same can thus occur in the case of music that has already been discussed in terms of geometry; for diatessaron, diapente, and diapason take their names from those of antecedent numbers. Moreover, the proportion of these same sounds in relation to each other is found in none other than numbers; for the sound of the diapason consonance is drawn together according to a duple proportion in numbers. The diatessaron of modulation is composed of the epitrita relationship; that which is called diapente is joined together through the hemiolic mean; and the epogdous in numbers is the tone in music. But let me not pursue only this point which the rest of this work will prove, namely that arithmetic without any doubt is a prior discipline.

Arithmetic is prior to the science of spheres and astronomy in so far as these other two disciplines by nature precede this third one. For in astronomy there are circles, the sphere, the center, parallel circles, and the middle axis, all of which are part of the geometric discipline. Why does this show the senior power of geometry? Because all motion is posterior to stillness, and motionlessness is by nature prior. Astronomy concerns itself with things in motion, whereas geometry considers things which are immobile. Moreover, the very motion of the stars is resounded in harmonic modulations. Wherefore it is certain that the power of music, which without doubt is naturally superseded by that of arithmetic, precedes the courses of the stars in authority. However, all the courses of the stars and all astronomical reasoning were properly established according to the very nature of numbers; for through arithmetical reason we collect risings and settings, we keep watch on the fast and slow velocities of the wandering stars, and we come to know the waning and multiple variations of the moon. Wherefore, since the power of arithmetic is clearly prior, we should begin our study with this discipline.

THE PRINCIPLES OF MUSIC

BOOK I.

I. <u>Introduction: That music is related to us by nature, and that it can ennoble or debase our character.</u>

An ability to perceive through the senses is so spontaneously and naturally present in certain living creatures that an animal without these senses cannot be imagined. But a knowledge and clear perception of these senses themselves is not so easily acquired, even with an investigation of the mind. It is obvious that we use our senses in perceiving sensible objects. But what is the exact nature of these senses in connection with which we carry out our actions? And what is the actual property of these objects sensed? The answers to these questions are not so obvious; and they cannot become clear to anyone unless the contemplation of these things is guided by a comprehensive investigation of reality.

Now sight is present in all mortals. But whether we see by images coming to the eye or by rays sent out from the eye to the object seen, this problem is in doubt to the learned, although the common man is not conscious of doubt.

Again if someone sees a triangle or square, he can easily identify it by sight. But what is the essence of a triangle or a square? This he must learn from a mathematician.

The same thing can be said of the other senses, especially concerning aural perception. For the sense of hearing can apprehend sounds in such a way that it not only judges them and recognizes their differences, but it very often takes pleasure in them if they are in the form of sweet and well-ordered modes, whereas it finds displeasure if the sounds heard are unordered and incoherent. Thus it follows that, since there are four mathematical disciplines, the others are concerned with the investigation of truth, whereas music is related not only to speculation but to morality as well. For nothing is more consistent with human nature than to be soothed by sweet modes and disturbed by their opposites. And this affective quality of music is not peculiar to certain professions or ages, but it is common to all professions; and infants, youths and old people as well are so naturally attuned to the musical modes by a certain spontaneous affection that there is no age at all that is not delighted by sweet song. Thus we can begin to understand that apt doctrine of Plato which holds that the soul of the universe is united by a musical concord.[1] For when we compare that which is coherently and harmoniously joined together within our own being with that which is coherently and harmoniously joined together in sound--that is, that which gives us pleasure--so we come to recognize that we ourselves are united according to this same principle of similarity. For similarity is pleasing, whereas dissimilarity is unpleasant and contrary.

From this same principle radical changes in one's character also occur. A lascivious mind takes pleasure in the more lascivious modes or is often softened and moved upon hearing them. On the other hand, a more violent mind finds pleasure in the more exciting modes or will become excited when it hears them. This is the reason that the musical modes were named after certain peoples, such as the "Lydian" mode, and the "Phrygian" mode; for the modes are named after the people that find pleasure in them. A people will find pleasure in a mode resembling its own character, and thus a sensitive people cannot be united by or find pleasure in a severe mode, nor a severe people in

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1. Plato Timaeus 37 A.

a sensitive mode. But, as has been said, similarity causes love and pleasure. Thus Plato held that we should be extremely cautious in this matter, lest some change in music of good moral character should occur.[2] He also said that there is no greater ruin for the morals of a community than the gradual perversion of a prudent and modest music.[3] For the minds of those hearing the perverted music immediately submit to it, little by little depart from their character, and retain no vestige of justice or honesty. This will occur if either the lascivious modes bring something immodest into the minds of the people or if the more violent modes implant something warlike and savage.

For there is no greater path whereby instruction comes to the mind than through the ear. Therefore when rhythms and modes enter the mind by this path, there can be no doubt that they affect and remold the mind into their own character. This fact can be recognized in various peoples. For those peoples which have a more violent nature delight in the more severe modes of the Thracians. Gentler

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2. Plato _Republic_ 424 B.
3. _Ibid._ 424 C.

peoples, on the other hand, delight in more moderate modes, although in these times this almost never occurs. Indeed today the human race is lascivious and effeminate (molle), and thus it is entertained totally by the representational and theatrical modes. Music was prudent and modest when it was performed on simple instruments; but since it has come to be performed in various ways with many changes, it has lost its mode of gravity and virtue, and having almost fallen into a state of disgrace, it preserves almost nothing of its ancient splendor. For this reason Plato prescribed that boys must not be trained in all modes but only in those which are vigorous and simple.[4] Moreover, it should be especially remembered that if some melody or mode is altered in some way, even if this alteration is only the slightest change, the fresh change will not be immediately noticed; but after some time it will cause a great difference and will sink down through the ears into the soul itself. Thus Plato held that the state ought to see that only music of the highest moral character and

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4. Ibid. 399 C.

prudency be composed, and that it should be modest, simple and masculine, rather than effeminate, violent or fickle.[5]

The Lacedaemonians took great care to preserve this type of music when they hired, at great expense to themselves, the Cretan, Thaletas of Gortyn,[6] who trained their sons in the art and discipline of music. In fact, this was a custom among ancient people that was observed for a considerable length of time. Thus when Timotheus of Melesia[7] added a string to those which were already established and made the music more complex, the Lacedaemonians expelled him from their city with an official decree. I have inserted this decree here in the original Greek words; but one should take care to notice that the letter *sigma* is changed to the letter *rho* in the Spartan language.

ΕΠΕΙΔΗ ΤΙΜΟΘΕΟΡ Ο ΜΙΛΗCΙΟΡ ΠΑΡΑΓΙΝΟΜΕΝΟΡ

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6. Thaletas of Gortyne in Crete, famous Greek musician, flourished between the times of Terpander and Timotheus of Melesia. Pseudo-Plutarch *De Musica* 1146 C, relates that Thaletas appeared in Lacedaemonia upon advice of an oracle.

7. Timotheus of Melesia, the most infamous musician of ancient times because of his various innovations. See Pausanius iii. 12, and Pseudo-Plutarch *De Musica* 1135 C-D, 1141 C-1142 B, 1142 C. Athenaeus 636 E, presents a different version of Timotheus' expulsion from Lacedaemonia, wherein he vindicates himself through a rather ingenius means.

ΕΝ ΤΑΝ ΑΜΕΤΕΡΑΝ ΠΟΛΙΝ ΤΑΜ ΠΑΛΙΝ ΜΩΑΝ ΑΤΙ-
ΜΑCΔΕ ΚΑΙ ΤΑΝ ΔΙΑ ΤΑΝ ΕΠΤΑ ΧΟΡΔΑΝ ΚΙΑΡΙΞΙΝ
ΑΡΟCΤΡΕΦΟΜΕΝΟΡ ΠΟΛΥΦΩΝΙΑΝ ΕΙCΑΓΟΝ ΛΥΜΑΙ-
ΝΕΤΑΙ ΤΑΡ ΑΚΟΑΡ ΤΩΝ ΝΕΩΝ ΔΙΑ ΤΕ ΤΑΡ ΠΟΛΥΧΟΡ-
ΔΙΑΡ ΚΑΙ ΤΑΡ ΚΕΝΟΤΑΤΟΡ ΤΩ ΜΕΛΕΟΡ ΚΑΙ ΠΟΙ-
ΚΙΛΑΝ ΑΝΤΙ ΑΠΛΟΑΡ ΚΑΙ ΤΕΤΑΓΜΕΝΑΡ ΑΜΦΕΙΝΝΥ-
ΤΑΙ ΤΑΝ ΜΩΑΝ ΕΠΙ ΧΡΟΜΑΤΟΡ CΥΝΕΙCΤΑΜΕΝΟΡ
ΤΑΝ ΤΩ ΜΕΛΕΟΡ ΔΙΑCΚΕΥΑΝ ΑΝΤΙ ΤΑΡ ΕΝΑΡΜΟΝΙΩ
ΠΟΤ ΤΑΝ ΑΝΤΙCΤΡΟΦΟΝ ΑΜΟΙΒ ΠΑΡΑΚΛΗΘΕΙC ΔΕ
ΚΑΙ ΕΝ ΤΟΝ ΑΓΩΝΑ ΤΑΡ ΕΛΕΥCΙΝΑΡ ΔΑΜΑΤΡΟΡ
ΑΠΡΕΠΗ ΔΙΕCΚΕΥΑCΑΤΟ ΤΑΝ ΤΩ ΜΥΘΩ ΔΙΑCΚΕΥΑΝ
ΤΑΝ ΤΑΡ CΕΜΕΛΑΡ ΟΔΥΝΑΡ ΟΥΚ ΕΝΔΙΚΑ ΤΟΡ
ΝΕΩΡ ΔΙΔΑΚΚΗ ΔΕΔΟΧΘΑΙ ΦΑ ΠΕΡΙ ΤΟΥΤΟΙΝ
ΤΩΡ ΒΑCΙΛΕΑΡ ΚΑΙ ΤΩΡ ΕΦΟΡΩΡ ΜΕΜΨΑΤΤΑΙ
ΤΙΜΟΘΕΟΝ ΕΠΑΝΑΓΚΑΞΑΙ ΔΕ ΚΑΙ ΤΑΝ ΕΝΔΕΚΑ
ΧΟΡΔΑΝ ΕΚΤΑΜΟΝΤΑΡ ΥΠΟΛΙΠΟΜΕΝΩΡ ΤΑΡ
ΕΠΤΑ ΟΠΟΡ ΕΚΑCΤΟΡ ΤΟ ΤΑΡ ΠΟΛΙΟΡ ΒΑΡΟΡ
ΟΡΟΝ ΕΥΛΑΒΗΤΑΙ ΕΝ ΤΑΝ CΠΑΡΤΑΝ ΕΠΙΦΕΡΕΝ
ΤΙ ΤΩΝ ΜΗ ΚΑΛΟΝ ΕΟΝΤΩΝ ΜΗ ΠΟΤΕ ΤΑΡΑΡΡΕΤΑΙ
ΚΛΕΟΡ ΑΓΩΝΩΝ.

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8. Wilamowitz-Moellendorf, Timothius: Der Perser (Leipzig: 1903) questions the authenticity of this text, placing it in the second century B. C. The following Latin translation of the text, with minor variances, appears in most medieval MSS written interlinearly with the Greek text: *Haec est autem interpretatio illorum. Quoniam Timotheus*

Now this text decrees the following: The Spartans were indignant with Timotheus the Milesian because he had had a detrimental influence on the characters (anima) of the boys he taught by introducing complexity into the music; and thus he interfered with their virtuous temperament.

Milesius adveniens in nostram civitatem antiquam, propriam civitatem spernens, septem chordarum cytharam subvertit, modulationem multarum vocum introducens demollivit (sic) auditus iuvenum, multas chordas et novam modulationem genuit et variam propriam simpla et ordinata circumvenit in cromaticum genus constituens, quod est mollius, divisionem pro enarmonico faciens conversionem mutuam, vocatus autem in agonem eleusinae matris turpitudinem divulgavit dispersione, enim partus non undecies novus doctrina edocuit: de talibus reges et rhetores accusabant Timotheum. Addidit autem et undecimam chordam, extendens enim superfluas relicta VII. chordarum cythara. Hoc ut singularis civis gravis videns timuit in Spartam indicare aliquid inconvenientium extentarum ne forte perturbaret gloriam certaminum. "But this is the interpretation of those words. Since Timothius of Milesia, coming into our ancient city, forsaking the character of the city, cast aspersions on the seven-stringed kithara, and, introducing a modulation of many strings, ruined the hearing of youths; and he abused our own way, having been simple and orderly; for he begot many strings and a new type of modulation, consisting of the chromatic genus which is more effeminate, creating a great revolutionary division on behalf of the enharmonic genus; thus he was called into the Eleusian Assembly which publicly pronounced him a disgrace to his mother because of his division; for dogma did not teach the new strings he created: because of such reasons the rulers and orators reproached Timotheus. And moreover, having abandoned the seven-stringed kithara, he added an eleventh string, thus promulgating excesses. One grave citizen, upon seeing this, feared to disclose anything of such inharmonious extensions in Sparta, lest perchance it alarm a great controversy.

Moreover, he was guilty of changing the harmony which he found in a temperate state into the chromatic genus, which is fickle. Thus the zeal for music among the Spartans was so great that they thought it took possession of the soul itself.

It is common knowledge that song has calmed rages many times and that it has often worked wonders on affections of either the body or the spirit. For who does not know that Pythagoras calmed a drunk adolescent of Taormine who had become incited under the influence of the Phrygian mode, and that Pythagoras further restored this boy to his rightful senses, all by means of a spondaic melody? For one night this frenzied youth was about to set fire to the house of a rival who had locked himself in the house with a whore. Now that same night Pythagoras was out contemplating the course of the heavens, as was his usual custom. When he learned that this youth under the influence of the Phrygian mode would not be stopped from his crime, even by the admonitions of his friends, he ordered that the mode be changed; and thus Pythagoras restored the frenzied mind of of the boy to a state of absolute calm.[9] Marcus Tullius tells this story in somewhat different words in his book, <u>De consiliis suis</u>,[10] but the story is as follows:

> But I will compare the ridiculous with the sublime, since there is some similarity between them. The story is told that one time certain youths became aroused by the music of the tibia, as can happen, and they were about to break in the door of a chaste woman. Pythagorus then admonished the tibia player to perform a spondaic melody. When this was done, the slowness of the tempo and the dignity of the performer caused the raging fury of these boys to subside.

But to give briefly some similar examples, Terpander[11] and Arion[12] of Methymna saved the citizens of Lesbos and Ionia from very serious illnesses by the aid of song. Moreover, in this same way Ismenias the Theban[13] is said to have cured all the maladies of the many Boeotians, who

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9. Concerning story of frenzied youth cf. Quintilian i. 10. 32, and Sextus <u>Empiricus Adversum Musicos</u> vi. 8.

10. Cicero's <u>De consiliis suis</u> is lost. The same work is cited by Augustine <u>Contra Julianum</u> v. 5. 23 (Migne <u>Patrologia Latina</u> xliv. 797.

11. Terpander of Methymna, flourished between 700 and 650 B.C., musician and composer of lyric poetry. Pseudo-Plutarch <u>De Musica</u> 1132 C, attributes to him the invention of the lyric nomes, and (1132 E) credits him with winning the Pythian games four times. Cf. Strabo xiii. 2. 4.

12. Arion, also of Methymna, roughly contemporary with Terpander. See Strabo, <u>Ibid</u>., Herodotus I. 23, and Hyginus <u>Fabula</u> 194, concerning Arion, his music, and the dauphin legend.

13. Ismenias the Theban, identified as a flutist in Lucian <u>The Ignorant Book-Collector</u> 5.

were suffering from sciatica. Similarly it is said that Empedocles had the mode of singing altered when an infuriated youth attacked one of his guests with a sword for having insulted his father; and by this means he tempered the wrath of the youth.

The power of the musical discipline was so evident to the ancient students of philosophy that the Pythagoreans would employ certain melodies when they wanted to forget their daily cares in sleep, and, upon hearing these, a mild and quiet slumber would fall upon them.[14] In the same manner, upon awakening, they would purge the stupor and confusion of sleep with certain other melodies; for these ancients knew that the total structure of our soul and body consists of musical harmony. For the very pulse of the heart itself is determined by the state and disposition of the body. Democritus is said to have told this to the physician Hippocrates, who came to treat Democritus when he was being held in custody by his fellow townsmen because they thought he was a lunatic.

But why have I said all this? Because there can be

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14. Concerning Pythagoreans' use of music for sleep see Plutarch _Isis et Osiris_ xxxi. 384, Quintilian ix. 4. 12, and Censorinus _De Die Natali_ xii. 4.

no doubt that the unity of our body and soul seems to be somehow determined by the same proportions that join together and unite the harmonious inflections of music, as our subsequent discussion will demonstrate. Hence it happens that sweet melodies even delight infants, whereas a harsh and rough sound will interrupt their pleasure. Indeed this reaction to various types of music is experienced by both sexes, and by people of all ages; for although they may differ in their actions, they are nevertheless united as one in the pleasure of music.

Why is it that those mourning in tears express their lamentation through music? This is espe~ially the case with women, who, as it were, make the cause of their weeping sweet through a song. The ancients even had the custom of letting a tibia lead funeral processions, as these lines of Statius testify:

> The tibia, whose practice it is to lead forth the youthful dead, utters its mournful note from a curving horn.[15]

And someone who cannot sing particularly well will nevertheless sing to himself, not because it is pleasant

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15. Statius _Thebaid_ vi. 120-121.

for him to hear what he sings but because it is a delight to express certain inward pleasures which originate in the soul, regardless of the manner in which they are expressed. Is it not clearly evident that the morale of soldiers is built up by the music of trumpets? If it is true that fury and wrath can be brought forth out of a peaceful state of mind, then there is no doubt that a more temperate mode can calm the raging and excessive desire of a perturbed mind. How does it happen that when someone hears a pleasant song with his ears and mind, also his body involuntarily responds with some motion similar to that of the song? And how does it happen that this same person can enjoy some melody he has already heard merely by recalling it in his memory? Thus from all these examples it appears to be beyond doubt that music is so naturally a part of us that we cannot be without it, even if we so wished.

For this reason the power of the mind ought to be directed toward fully understanding by knowledge what is inherent in us through nature. Thus just as erudite scholars are not satisfied by merely seeing colors and forms without also investigating their properties, so musicians should not be satisfied by merely finding pleasure in music without knowing by what musical proportions these sounds are put together.

II. <u>That there are three types of music, and concerning the power of music</u>.

It seems that one discussing the musical discipline should discuss, to begin with, the kinds of music which we know to be contained in this study. Indeed there are three types of music. The first type is the music of the universe (*musica mundana*), the second type, that of the human being (musica humana), and the third type is that which is created by certain instruments (*musica instrumentis constituta*), such as the kithara, or tibia or other instruments which produce melodies.

Now the first type, that is the music of the universe, is best observed in those things which one perceives in heaven itself, or in the structure of the elements, or in the diversity of the seasons.[16] How could it possibly be

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16. Concerning music of the universe see Plato Timaeus 35-36, *Laws* 889 B-C. (a) Harmony of the heavens: Pliny ii. 84; Cicero *De Re Publica* vi. 18. 18; Pseudo-Plutarch *De Musica* 1147; Nicomachus *Manual of Harmony* (in Jan, *Musici Scriptores Graeci* [Hildesheim: Georg Olms (reprint of 1895 edition), 1962], pp. 237-255), iii (JanS. 241-42); Censorinus *De Die Natali* xii; Macrobius

that such a swift heavenly machine should move silently in its course? And although we ourselves hear no sound--and indeed there are many causes for this phenomenon[17]--it is nevertheless impossible that such a fast motion should produce absolutely no sound, especially since the orbits of the stars are joined by such a harmony that nothing so perfectly structured, so perfectly united, can be imagined. For some stars drift higher, others lower, and they are all moved with such an equal amount of energy that a fixed order of their courses is reckoned through their diverse inequalities. Thus there must be some fixed order of musical modulation in this celestial motion.

Moreover if a certain harmony does not join together the diversities and contrary qualities of the four elements, how is it possible for them to unite in one body machine? But all this diversity produces a variety of both seasons

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<u>In Somnium Scipionis</u> ii. 1. 2, iv. 1-6; Ptolemy <u>Harmonics</u> iii. 10-16. 104-111. (b) Harmony of elements: Plato <u>Symposium</u> 188 A; <u>Timaeus</u> 32 C; Macrobius <u>In Somnium Scipionis</u> i. 5. 25. (c) harmony of the seasons: Plato <u>Symposium</u> 188 A.

17. Cf. Cicero <u>De Re Publica</u> vi. 18. 19; and Macrobius ii. 4. 14.

and fruits, so that the year in the final analysis achieves a coherent unity. Now if you would imagine one of these things that gives such a diversity to everything taken away, then they would all seem to fall apart and preserve none of their "consonance." Moreover, just as the lower strings are not tuned too low, lest they descend to a pitch that would be inaudible, and the higher strings are not tuned too high, lest they break under the excessive tension, but rather all the strings are coherently and harmoniously tuned, so we discern in the universal music that nothing can be excessive; for if it were, it would destroy something else. Everything either bears its own fruit or aids other things in bearing theirs. For what winter confines, spring releases, summer heats, and autumn ripens, and so the seasons in turn either bring forth their own fruit, or give aid to the others in bringing forth their own. But these things ought to be discussed later more studiously.[18]

Now one comes to understand the music of the human being[19] by examining his own being. For what unites the

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18. Boethius never returns to this topic.

19. Concerning the music of the human being see Plato _Phaedro_ 86; _Republic_ 442-443; _Laws_ 653 B; Cicero _Tusculan Disputations_ i. 10; Pseudo-Plutarch _De Musica_ 1140 B; Ptolemy _Harmonics_ iii. 5-7. 95-100.

incorporeal existence of the reason with the body except a certain harmony (coaptatio) and, as it were, a careful tuning of low and high pitches in such a way that they produce one consonance? What unites the parts of man's soul, which, according to Aristotle, is composed of a rational and irrational part?[20] In what way are the elements of man's body related to each other or what holds together the various parts of his body in an established order? But these things also will be discussed later.[21]

Now the third type of music is that which is said to be found in various instruments. The governing element in this music is either tension, as in strings, or breath, as in the tibia or those instruments which are activated by water, or a certain percussion, as in those instruments consisting of concave brass which one beats and thus produces various pitches.

Thus it seems that I ought to discuss the music of

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20. Boethius is here probably refering to Aristotle De Anima 432 A. 30. Boethius significantly does not cite De Anima 407 B-408 B, where Aristotle refutes the notion that the soul is a harmony.

21. Boethius never returns to this topic.

instruments in this first book. But this introduction should suffice. Now the elements of music themselves must be discussed.

III. Concerning the sounds and elements of music.

Consonance, which governs all musical modulation cannot be made without sound; sound indeed is not produced without some pulsation and percussion; furthermore, pulsation and percussion cannot exist by any means unless motion precedes them. For if all things were immobile, one thing could not concur with another, so that one thing would be moved by another. With all things remaining still and motion being absent, it is evident that no sound would be made. For that reason sound is defined as a percussion of the air, which percussion remains undissolved until it reaches the ear.[22]

Some motions are faster, others slower, and of these motions, some are less frequent, and others are more frequent. Now if someone regards continuous motion, then it is necessary that he observe either speed or slowness; moreover, if

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22. Cf. Nicomachus Manual iv. (JanS. 242).

someone moves his hand, he will move it either more or less frequently. And indeed if the motion be slow and less frequent, low sounds will necessarily be caused by the slow and less frequent pulsation. On the other hand, if the motions be fast and more frequent, high sounds will necessarily be produced. Thus also when a string is stretched more, it sounds high, if loosened, low. For when it is tighter, it produces a faster pulse, and vibrates more quickly, and strikes the air more frequently and densely. On the other hand, a more relaxed string causes a loose and slow pulsation; and less frequent because of this weak striking of the air, it does not vibrate long.

But one should not think that every time a string is struck, only one sound is produced, or only one percussion is present in these sounds; but as often as the air is moved, the vibrating string will have struck it. But since the great speeds of sounds are connected, one hears no interruption, and one sound, either low or high, affects the sense. Yet each sound consists of many, the low indeed of the slower and less frequent, the high of the fast and more frequent. It is like the cone which people call a "top." Someone adorns it diligently and applies one stripe of red or some other color to it. Now when one can spin it quickly, the whole cone seems dyed with red color, not because the whole thing is thus, but because the velocity of the red stripe overcomes the clear parts, and they are not allowed to appear. But these things will be treated later.[23]

Therefore since high sounds are incited from the more frequent and faster motions, the low ones from the slower and less frequent, it is evident that a high sound grows from a low one by some increase in motion, whereas by a decrease in motion, a low sound descends from a high one. For a high sound consists of more motion than a low one. However, the plurality makes the difference in these matters; for the plurality necessarily consists of a certain numerical quantity, and every smaller quantity is considered to a larger quantity as a number compared to a number. Now of these things which are compared according to number, some are equal, others unequal.[24] Thus some sounds are also equal; others indeed are different by virtue of an inequality. But in these sounds which do not

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23. See Book I. xxxi. p. 98-99 , Book IV. i. pp. 211-212.
24. cf. De institutione arithmetica i. 21.

harmonize by any inequality, there is no consonance at all. For a consonance is a concord which reduces mutually dissimilar voices into one.

IV. Concerning the kinds of inequality.

Indeed those things which are inequalities hold within themselves five changes relating to kinds of inequality. For one is transcended by another either by a multiple, or by a singular part, or by parts, or by a multiple and a part, or by a multiple and parts. Thus the first type of inequality is called "multiple." In the multiple type the larger number contains the whole smaller number within itself either twice, or three times, or four times, and so forth; and nothing is either lacking or superfluous. It is called either "duple," "triple," or "quadruple," and proceeds to infinity in this manner. But the second type of inequality is that which is called "superparticular"; in this type the larger number contains the whole smaller number within itself, and some single part of it; and that part is either a half, as three to two, and is called the "sesquialter" proportion, or a third, as four to three, and is called the "sesquitertian" proportion. And so in this manner in subsequent numbers, some part in addition to the smaller number is contained by the larger number. In the third kind of inequality the larger number contains the total lesser number within itself and besides several of its parts. And if it contains two parts more, it is called the "superbipartient" proportion, as five is to three; whereas if it contains three parts more, it is called "supertripartient," as seven is to four. And thus in the same manner, the analogy can be extended to other numbers. In the fourth type of inequality, which is a combination of the multiple and the superparticular, the larger number has within itself the lesser number either twice, or three times, or some other number of times, plus one other part of it. And if it contains it and a half part, it is called "duple supersesquialter," as five is to two; whereas if the lesser number is contained twice, plus a third part of it, it is called "duple supersesquitertian," as seven is to three. But if the lesser number is contained three times, plus a half part of it, it is called "triple supersesquialter," as seven is to two. And in the same way, the names "multiple" and "superparticular" are varied in other proportions. In the fifth type of inequality, that called "multiple superpartient," the larger number has the lesser number within itself more than once, plus more than one certain part of it. And if the larger number contains the smaller number twice and besides two parts of it, it will be called "duple superbipartient," as three is to eight; and again "triple superbipartient," as three is to eleven. Since we elucidated diligently concerning these things in our books entitled *De institutione arithmetica*,[25] we now explain them cursorily and briefly.

V. *Which kinds of inequality pertain to consonance*.

Of these types of inequalities, the last two are abandoned, since they are mixed from the others. Indeed one ought to speculate within the first three types. Multiple thus seems to hold the greater authority for consonances; however, superparticular seems to hold the next place. The superpartient is separated from the consonance of harmony, as was seen by various theorists except for Ptolemy.[26]

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25. *De institutione arithmetica* i. 22-31.
26. Ptolemy *Harmonics* i. 7. 15.

VI. Why multiplicity and superparticularity are considered in relation to consonances.

And now those things which are simple by nature are demonstrated by fitting comparison. Since lowness and highness consist in quantity, these things which can preserve a property of discrete quantity seem most perfectly to serve the nature of harmony. For indeed since some quantity is discrete and other continuous, then that which is discrete is finite at its smallest, but it proceeds through larger quantities into infinity. For unity is finite in the smallest things, and the mode of plurality is increased into infinity in the same manner as a number which begins from a finite unity increases infinitely. On the other hand, that which is continuous is indeed finite as a whole, but it is infinitely divisable. For a line which is continuous is always divided into an infinite number of parts, even though its total size is only two feet or some other definite number. Therefore a number always grows into infinity, whereas a continuous quantity is divided into infinity. Therefore, since multiplicity can be infinitely increased, it most retains the nature of number. Superparticularity, on the other hand, since it reduces the smaller into infinity, preserves the property of continuous quantity. It reduces the smaller quantity since it always contains it and a part of it--either half, a third, a fourth, or a fifth. Indeed the very part signified by the larger number decreases. For when a third is designated by three, and a fourth by four, since four surpasses three, the fourth rather than the third is found smaller. But superpartient proportions depart from simplicity in a particular manner; for two, three or four contain parts over and above unity, and, departing from simplicity, it flourishes to a certain plurality of parts. Furthermore, all multiplicity contains itself in completeness, for a duple has the total lesser quantity twice, whereas the triple contains the total lesser quantity thrice, and so on in the same manner. Superparticular proportions, on the other hand, retain nothing complete, but exceed by either a half, a third, a fourth, or a fifth; but nevertheless it achieves a division into singular and simple parts. Superpartient proportions, however, neither retain completeness nor admit singular parts, and thus, according to Pythagoras, they apply least to musical consonance. Nevertheless

Ptolemy placed even this type of proportion among consonances, as I will show later.[27]

VII. Which proportions are fitted to which musical consonances.

Now this ought to be known: all musical consonance consists of either duple, triple, quadruple, sesquialter, or sesquitertian proportion. And that which will be called "sesquitertian" in number, will be "diatessaron" in sounds; that which will be duple in proportions, will be diapason in consonances; the triple, diapason and diapente; the quadruple, bisdiapason.[28] Now let the above be said generally and without particulars; later the complete rationale of proportions will be elucidated.

VIII. What sound is, what interval is, and what consonance is.

Thus sound is the melodious inflection of the voice,

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27. See Ptolemy Ibid., and Book V. vii. pp. 304-306. To be exact, Ptolemy does not place superpartient proportions among consonances, but rather a multiple superbipartient proportion: 8:3; this is the proportion of a diapason and diatessaron, an interval which Ptolemy holds to be consonant.

28. Diatessaron = fourth; diapente = fifth; diapason = octave; diapason and diapente = octave and fifth, or a twelfth; bisdiapason = double octave, or fifteenth.

that is, fitted to song in a single tune (intensio).[29] Indeed we do not wish to define sound generally now, but that which is called phthongos in Greek, called this from the similarity of speaking, that is φθεγγεσθαι.

Interval is the distance of a high and low sound.[30]

Consonance is a mixture of high and low sound falling uniformly and pleasantly on the ears.[31]

Dissonance, on the other hand, is the harsh and unpleasant percussion of two sounds ill-mixed with each other comming to the ear.[32] For while they are unwilling to mix, and both strive by some means to come together, since each interferes with the other, both are transmitted to the sense unpleasantly.

IX. Not all judgment ought to be given to the senses, but more ought to be believed from reason; concerning the fallacy of the senses in this matter.

But concerning these things we propose thusly, that we should not grant all judgment to the senses, although every first principle of this art is derived from the sense

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29. Cf. Nicomachus Manual xii. (JanS. 261).
30. Ibid.
31. Ibid. (JanS. 262).
32. Ibid.

of hearing. For if nothing were heard, there would not have existed any argument concerning sound whatsoever. But the sense of hearing holds the first principle in a particular way, and is, as it were, an exhortation; indeed, the perfection and power of recognition consequently consists in reason, which, holding itself to fixed rules, never falters by any error. For what need is there of speaking further concerning the error of the senses when this same faculty of sensing is neither equal in all men, nor at all times equal within the same man? Therefore anyone vainly puts his trust in a changing judgment, since he aspires to seek in truth. For this reason the Pythagoreans take a certain middle path.[33] For they measure consonances themselves with the ear, but the distances by which consonances differ among themselves they do not intrust to the ears, the judgments of which are unclear, but to rules and reason--as though the sense were an obedient servant, and the reason were truly a commanding judge. For although

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33. The text here actually exaggerates the moderation of the Pythagoreans, especially those of the earlier school. cf. Book V. iii. pp. 299-300, and Pseudo-Plutarch _De Musica_ 1144 F.

the movements of almost every art, and of life itself, are introduced by impression of the senses, no certain judgment, no comprehension of truth is in these impressions if the arbitration of reason is lacking. The sense itself is dulled equally in the greatest and least of things. For it is not possible to sense the least things because of the smallness of the sensible things themselves, and the sense is also confounded by the greatest things. In the case of sounds, for example, the ear grasps them with difficulty if they are very slight, but if they are very great, the ear is deafened by the intensity of the noise.

X. In what manner Pythagoras investigated the proportions of consonances.

Thus this was mainly the reason that Pythagoras, having forsaken aural judgment, turned to reason. He did not trust the human ears, which are subject to radical change, in part by nature, in part even by external accidents, and the perceptions of which are even affected by one's age. Nor did he rely on instruments, in conjunction with which much diversity and inconsistency often originate. Take strings, for example; either the more humid air deadens the vibrations, or more arid air excites them; either a larger string renders a low sound, or a smaller one returns to a high sound; or by some other means, something would subject the stability of the prior pitch to radical change. And since the case would be the same in other instruments, Pythagoras placed a minimum of trust in these inconclusive things. And, being curious for some time, he sought a way to establish in his mind, by reason, firmly and consistently, the principles of consonances. In the meantime, by certain divine will, when he passed the workshops of blacksmiths, he overheard the beating hammers somehow resound one consonance from the diverse sounds. Thus in the presence of that which he had long sought, he approached the work amazed. And considering for a while, he thought the strength of the hammers caused the diversity of sounds. Thus, in order to test this theory more clearly, he commanded the men to exchange hammers among themselves. But the property of sounds was not contingent on the muscles of the men, but rather it followed the exchanged hammers. Thus when he observed this, he examined the weight of the hammers. And since perchance there were five hammers, one was found to weigh twice as much as another, and these two resounded a diapason consonance. The one which had weighed twice as much as a second formed the sesquitertian relation of a third, with which naturally it produced a diatessaron. He found the one which weighed twice as much as a second to be the sesquialter relation of a fourth, which was related to it by a diapente consonance. Those two, to which the above one of double weight was proved to be sesquitertian and sesquialter relation, were discovered in turn to be related by the sesquioctave proportion. The fifth hammer, which was dissonant with all, was rejected.

Therefore, since musical consonances before Pythagoras were called in part diapason, in part diapente, and in part diatessaron (which is the smallest consonance), Pythagoras first ascertained in this way by what proportion these consonances of sound were united. And in order that what was said might be clearer for the sake of discourse, the weights of the hammers were written underneath in numbers: 12, 9, 8, 6. Thus the hammers which weighed 12 and 6 pounds resounded, in the duple proportion, the diapason consonance. The hammer of 12 pounds with that of 9, and the hammer of 8 pounds with that of 6 were united by a diatessaron consonance according to the *epitrita*[34] proportion. Indeed the one of 9 pounds with that of 6, as well as those of 12 and 8 intermingled the diapente consonance. The one of 9 pounds with that of 8 resounded the tone according to the sesquioctave proportion.[35]

XI. *In what various manners the proportions of consonances were tested by Pythagoras.*

Hence having returned home, Pythagoras carefully thought by various examination that perhaps the whole rationale of harmonies consisted in these proportions. First he fitted equal weights to strings and discerned by ear their consonances. Then he substituted pipes, using some twice as long as others, as well as fitting in the other proportions. Thus he made his belief complete by various experiments. Also, following this way of measuring, he placed measured weights of water in glasses, and then struck these glasses prepared with various weights

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34. *Epitrita*: Greek for sesquitertian.

35. Concerning Pythagoras' discovery of these musical proportions and his experiments related in the following chapter, see Nicomachus *Manual* vi. (JanS. 254-48); Gaudentios *Introduction to Harmony* xi. (JanS. 340-41); Jamblicos *Vita de Pythagora* i. 24; Macrobius ii. 1. 9-14.

with a small twig or copper rod; and he rejoiced to find nothing in conflict. Being thus led, he turned to the length and density of strings that he might examine them. And thus he found the ruler concerning which we will speak later.[36] This thing with which we measure the size of strings and sounds is called a "ruler," not because it is made of wood, but because this ruler is fixed and firm under the study of anyone, so that it does not deceive anyone inquiring with inexact information.

XII. Concerning the division of voices and their explanation.

But enough concerning this. We should think about the different types of voices. All voice is either συνεχης which is continuous, or διαστηματικη, which it is called when it is interrupted by intervals.[37] And voice is continuous as when speaking, or when reciting a prose oration, we run on continuously. For the voice then hastens, not lingering in high or low sounds, but running over the words as fast as possible; and the impetus of the continuous voice is

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36. Book IV. v-xii. pp. 229-263.

37. Cf. Nicomachus Manual ii. (JanS. 238), and Book V. v-vi. pp. 302-304.

is occupied with preparing the sense and molding the words. However, διαστηματικη is that voice which we interrupt in singing, in which we submit less to words than to melody. And this particular voice is slower, and produces a certain interval, not of silence, but rather of slow and inflected song.

To these, as Albinus[38] asserted, is added a third different type which can include an intermediate voice, such as when we recite heroic poems, not in a continuous inflection as in prose, nor in an interrupted and slower manner of the voice as in song.

XIII. That human nature limits the boundlessness of voices.

But the voice which is continuous and the voice with which we traverse song are indeed naturally infinite. For by accepted consideration, no limit is placed either on running over words or rising to high pitches or sinking to low ones; but human nature imposes its own limitation on both of these. For the human breath places a limiting point on continuous voice beyond which it cannot possibly exceed.

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38. The musical writings of Albinus, also cited by Cassiodorus, De musica (MPL. lxx. 1212), are lost. cf. Martianus Capella De nuptiis ix. "de voce." See Commentary, Chapter I, pp. 346-347.

For every person speaks in a continuous manner as long as his breath allows. Moreover, the nature of man places a limiting point on the diastematic voice which determines one's high and low voice. For everyone can ascend high or descend low as much as the type of his voice by nature allows.[39]

XIV. How one hears.

At this time we should discuss how one hears. Now a sound is usually produced in the manner that a stone thrown from a distance, falls into a pond of quiet water. For first it causes a wave in a very small circle; then it disperses clusters of waves into larger circles, and so on until the motion, exhausted by the spreading out of the waves, dies away. And the latter and wider wave is always diffused by a weaker impulse. Now if something can impede the spreading waves, the same motion is immediately reversed, and just as at the center from where it started, it is spread in circles by these same undulations. Thus the sound is diffused and, at the same time, heard by everyone standing

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39. Cf. Nicomachus *Manual* ii. (JanS. 239-240).

around it. And the sound is more obscure to him who stands at some distance, since the wave of activated air comes to him weaker.

XV. <u>Concerning the order of theory, that is, the order of speculation</u>.

Now it seems that we should say this about these propositions: all song is produced within genera, and within these genera, the harmonic discipline theorizes. Moreover, these are the genera: the diatonic, the chromatic, and the enharmonic. Thus, if we first discuss the tetrachords and in what manner the great number of strings happened to come into being--to which now even more are added--thereafter these genera should be explained. This will be done after we first consider by what proportions musical harmonies are combined.

XVI. <u>Concerning the proportions of consonances and the tone and semitone</u>.

If a note is higher or lower from another note by a duple proportion, a diapason consonance will be made. If the note is higher or lower by a sesquialter, sesquitertian, or sesquioctave relation, then a diapente or diatessaron consonance, or a tone is produced. Likewise, if a diapason, such as 4:2, and a diapente, such as 6:4 be joined together, a triple proportion, 6:2, produces a consonance which is a diapason and diapente. But if two diapasons are combined, such as 2:4 and 4:8, then a quadruple consonance will be produced which is the bisdiapason. But if a sesquialter and sesquitertian, that is, a diapente and diatessaron, be joined, such as 3:2 and 4:3, a duple consonance, of course a diapason, is formed; for 4:3 brings about a sesquitertian proportion, whereas 3:2 are joined by a sesquialter relation. Thus the 4 opposed to the 2 results in a duple relation. Now the sesquialter proportion creates a diapente consonance, and the sesquitertian a diatessaron, but the duple proportion produces a diapason. Thus a diatessaron plus a diapente makes one diapason consonance.

Moreover, a tone cannot be divided into equal parts; why, however, we will explain later.[40] For now it is sufficient to know that a tone is never divided into two equal parts. So that this might be very easily demonstrated,

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40. See Book III i-ii. pp. 171-178.

let 8 and 9 represent the sesquioctave proportion. But no mediating number naturally falls between these. Thus let us multiply them by 2; and 8 twice makes 16, 9 twice, 18. Now one number naturally falls between 16 and 18, which is of course 17; and these numbers in order are 16, 17, 18. Therefore 16 and 18 combined retain the sesquioctave proportion, and thus the tone. But the middle number does not equally divide this proportion. For compared to 16, 17 contains in itself the total 16, plus 1/16 part of it, that is, one unit. Now if to this number, that is 17, the third number, 18, be compared, 18 contains the total 17 and 1/17 part of it. Therefore 17 does not surpass the smaller number and is not surpassed by the larger number by the same parts. And the smaller part is one-seventeenth, the larger part one-sixteenth. But both of these are called semitones, not because these intermediate semitones are equal at all, but because something is usually called "semi" which does not come to a whole. But in the case of these two divisions, one semitone is called "major," and the other "minor."

XVII. <u>In what prime numbers a semitone exists</u>.

At this time I will clearly explain what an integral semitone is and in what prime number it exists. For that which was just said about the division of the whole tone [18:17:16] has nothing to do with our wanting to show the proportions of semitones, but it applies more to our assertion that a tone cannot be divided into equal parts.

The diatessaron is a consonance of four pitches, and indeed of three intervals. Moreover it consists of two tones and an incomplete semitone. Let this description be considered: 192, 216, 243, 256. For if the number 192 be compared with 256, a sesquitertian proportion will be made and it will resound a diatessaron consonance. But if 216 be compared with 192, the proportion is the sesquioctave, for the difference between them is 24, which is an eighth part of 192. Therefore it is a tone. Moreover, if 243 be compared to 216, the proportion will be another sesquioctave; for the difference between them, 27, proves to be an eighth part of 216. The comparison of 256 with 243 now remains. The difference of these is 13, which multiplied 8 times, will not appear to function as a denominator of 243. Therefore this proportion is not that of a semitone,[41] but less than a semitone. For it would be thought to be a whole semitone by rule if the difference of these numbers, which is 13, multiplied by 8, could equal a denominator of 243; and thus the comparison of 243 with 256 is less than a semitone.

XVIII. _That the distance between a diatessaron and a diapente is a tone._

The diapente consonance has five pitches, four intervals, three tones and a minor semitone. Again let the number 192 be assumed and its sesquialter be taken. This number should therefore be 288, which would make a diapente consonance with 192. Now let the numbers which were related to 192 above be placed between these two numbers, that is, 216, 243, and 256; and let the description be formed in this manner: 192, 216, 243, 256, 288. Now in the above argument 192 and 256 were proved to contain two tones and a semitone. Therefore the comparison of 256 and 288 remains. The difference between 256 and 288 is 32, which is an eighth part of 256, and this is a

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41. I.e., not an exact _half_ of a tone. Concerning this series of numbers and multiples thereof see Nicomachus _Fragment II_ (JanS. 267-271).

sesquioctave proportion, that is, a tone. Thus the diapente is proved to consist of three tones and a semitone.

Now a short while ago the diatessaron consonance was derived from the numbers 192 and 256. But now the distance of a diapente from this same 192 is increased to 288. Thus the diatessaron consonance is separated from the diapente by that proportion which lies between the numbers 256 and 288, and this is a tone. Thus the diapente transcends the diatessaron by a tone.

XIX. That the diapason consists of five tones and two semitones.

The diapason consonance consists of five tones and two semitones, the two semitones of which nevertheless do not completely make one tone. For since it was demonstrated that a diapason consisted of a diatessaron and a diapente, and it was moreover proved that a diatessaron consisted of two tones and a semitone, a diapente of three tones and a semitone. These joined together at the same time produce five tones. But since these same two semitones were not true half tones, their conjunction does not add up to a whole tone; yet added together they exceed one semitone. But the rationale of these two semitones, as well as how they were ascertained in this musical consonance, will be later thoroughly explained.[42]

Meanwhile credulity must be summoned to the present disputations to support mediocre intelligence; but an indeed firm and complete faith should be summoned, since each thing will be made clear by proper demonstration. These things being said, we will discuss for a while the strings of the kithara, and also how they were added, since this determines their names. For the knowledge which is to be grasped subsequently will be made easy by having first become acquainted with these strings.

XX. Concerning the additions of strings and their names.

Nicomachus[43] said that in the beginning there was a simple music, since it was performed on four strings; and

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42. See Book II. xxxi. pp.167-170, Book III. ii. pp. 176-178, and Book V. ix. pp. 307-309, xiv. pp. 315-317.

43. This chapter is the most difficult passage in Boethius to relate to his Greek sources. Boethius here, citing Nicomachus, states that the original kithara had four strings, that it remained in this state until the time of Orpheus, and that it was invented by Mercury. Nicomachus Fragment I (JanS. 266) agrees so far as Mercury (Hermes) is concerned; but he states that the original kithara had seven strings, and was given as such to Orpheus. The Thracian

music remained in this state up to the time of Orpheus. Indeed the first and fourth strings resounded a diapason consonance; the middle strings, moreover, resounded a diapente and a diatessaron with these outer strings. Thus there was nothing unconsonant in these strings. As you can plainly see, it was an initation of the universal harmony, which consists of the four elements. The inventor of these four strings is said to have been Mercury.[44]

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women, after having killed Orpheus, cast the kithara into the sea, from whence it was delivered to Antissa in Lesbos. Certain fishermen found it and gave it to Terpander, who, after having improved it, claimed to have invented it himself. Nicomachus makes no mention of Torebus or Hyagnis, nor does he attribute the addition of the seventh string to Terpander.

Furthermore, Boethius credits Lycaon of Samos with the addition of the eighth string, thus forming two disjunct tetrachords. Nicomachus Manual v. (JanS. 244-45), however, attributes this addition to Pythagoras himself, made so that the system might include the harmonious octave.

To confuse the matter further, Boethius and Nicomachus Fragment IV (JanS. 274) agree concerning the addition of the ninth, tenth, and eleventh strings, although Nicomachus attributes the ninth string to Theophrastus of Periotes rather than Prophrastus. For further discussion of Boethius' Nicomachus source, see Chapter I of the Commentary, pp. 338-365, and Chapter IV, pp. 422-425, concerning the logic of these early additions.

44. Concerning Mercury and the invention of the kithara, cf. Homer Hymn to Hermes 30-55; Diodorus Siculus i. 16. 2; Horace Carmina i. 10; and Nicomachus Fragment I (JanS. 268).

Later Toroebus,[45] the son of Atyes, king of the Lydians, added the fifth string. Hyagnis[46] the Phrygian added the sixth string to these. But the seventh string was added by Terpander[47] of Lesbia, obviously an image of the seven planets. And the lowest of these seven strings was that one called "hypate," as the larger and more honorable; and thus they also called it "Jupiter hypatos." They also called it "counsel," because of the height of its rank, and it was attributed to Saturn because of its slow motion and low sound. The second string was the parhypate, for it was placed next to the hypate. The third string was called lichanos, since that finger is called "lichanos" which we call "index." The Greeks call it lichanos after its "tongue like articulation." And since, when playing the kithara, the index finger (which is the lichanos) was found at that string which was third from the hypate, thus this string itself was also called lichanos. The fourth string is

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45. Toroebus the Lydian, credited with the invention of the Lydian mode: Pseudo-Plutarch De Musica 1136 C.

46. Hyagnis the Phrygian, credited with the invention of the Phrygian mode: Athenaeus xiv. 624.

47. Terpander of Lesbos, credited with the addition of a "Dorian neate:" Pseudo-Plutarch De Musica 1140 F; cf. quote from Terpander cited in Strabo xiii. 2. 4.

called "mese," since it is always in the middle of these seven. The fifth string is the paramese, since it is placed next to the middle one. Moreover, the seventh is called "nete," since it is low like the "neate."[48] And the sixth string is located between this nete and the paramese; it is called "paranete," since it is next to the nete. Since the parameme is the third note from the nete, it is also signified by the word "trite." Thus this description may be given:

[e]	HYPATE
[f]	PARHYPATE
[g]	LICHANOS
[a]	MESE
[b♭]	PARAMESE OR TRITE
[c]	PARANETE
[d]	NETE

Lycaon of Samos[49] added the eighth string to these;

48. Neate: the last, lowest, the extreme; the lowest string with regard to its position on the kithara.

49. Nicomachus Manual v. (JanS. 244-45) attributes the addition of this string to Pythagoras himself. This Lycaon of Samos is in all likelihood Lycon of Tarentum (although the latter's origins is by no means certain). A Pythagorean work is accredited to Lycon, and he is considered one of the last early Greek philosophers to call himself a Pythagorean; see Diels Die Fragmente der Vorsokratiker, ed. Kranz (9th ed.; Berlin: Weidmann, 1960), I. 57. pp. 445-446.

and it fitted between the paramese (also called trite) and the paranete, so that this new string itself became the third string from the nete. Thus that note which was placed after the mese became called only paramese, and it lost the name of trite after this note was placed between the paramese and paranete. The new note was rightly called the trite, since it was located third from the nete. Thus the eight strings according to Lycaon's addition are as follows:

[e]	HYPATE
[f]	PARHYPATE
[g]	LICHANOS
[a]	MESE
[b]	PARAMESE
[c]	TRITE
[d]	PARANETE
[e]	NETE

In the above dispositions of seven and eight strings, the seven string disposition is called "synemmenon," since it is conjunct, whereas the eight string disposition is called "diezeugmenon," since it is disjunct. For in the seven string disposition there is one tetrachord, that of the hypate, parhypate, lichanos, and mese, and a second, that of the mese, paramese, paranete, and nete. Since the mese is counted in the second as well as the first, the two tetrachords are joined together by the mese. But in the eight string disposition, since there are eight notes, the first four--the hypate, parhypate, lichanos, and mese--make one tetrachord. Now a complete tetrachord, disjunct from the first one, is begun from the paramese, proceed through the trite and paranete, and ends on the nete. Here there is disjunction, which is called "diazeuxis," and the distance of a tone stands between the mese and paramese. Although the mese is not in the middle, it still retains its name; for one middle position cannot be found. The eight string disposition always yields two central strings.

Prophrastus of Periotes[50] then added a string at the larger end of the kithara, and this total produced a nine string disposition. Since this string was added above the hypate, it was called the "hyper hypaton" as long as the kithara had nine strings. Now, however, since other strings have been added, it is called the "lichanos hypaton," for in

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50. Cf. Nicomachus Fragment IV (JanS. 274), which reads Theophrastus of Periotes.

the present order and playing of the kithara, this note is played by the index finger--called lichanos. But this will become apparent later. The order of the nine string disposition is comprised as follows:

[d]	HYPERHYPATE
[e]	HYPATE
[f]	PARHYPATE
[g]	LICHANOS
[a]	MESE
[b]	PARAMESE
[c]	TRITE
[d]	PARANETE
[e]	NETE

Histiaeus of Colophon[51] added a tenth string at the larger end of the kithara, and Timothius of Milesia[52] added an eleventh string. Since these strings were added above the hypate and parhypate, they were called "hypate hypaton," as they were the largest of the largest [with regard to size], the lowest of the lowest [with regard to pitch], and the highest of the highest [with regard to position].

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51. Nicomachus Ibid.
52. Ibid.; cf. Pausanius iii. 12.

But the first of these eleven strings was called the "hypate hypaton," whereas the second was called "parhypate hypaton," since it was located next to the hypate hypaton. The third note, which was just discussed in the nine string disposition, was then named the "lichanos hypaton." The fifth string kept its old name, hypate; the sixth, lichanos (also having its old name); the seventh string, mese; the eighth, paramese; the ninth, trite; the tenth paranete; and the eleventh, nete.

Therefore, the hypate hypaton, parhypate hypaton, lichanos hypaton, and hypate form one tetrachord, and the hypate, parhypate, lichanos, and mese form another; and these two tetrachords are conjunct. The paramese, trite, paranete, and nete form a third tetrachord. But since between the first tetrachord--hypate hypaton, parhypate hypaton, lichanos hypaton, and hypate--and the last tetrachord--paramese, trite, paranete, and nete--a middle tetrachord is now positioned--that is, the hypate, parhypate, lichanos, and mese, this total middle tetrachord became called "meson," since it is in the middle. Thus after these additions these notes became called the hypate meson, parhypate meson, lichanos meson, and mese. Since the meson tetrachord is disjunct from the lowest tetrachord, that of the nete (the disjunction occurring between the mese and paramese), the lowest tetrachord became called disjunct, that is diezeugmenon. With this addition the strings became called paramese diezeugmenon, trite diezeugmenon, paranete diezeugmenon, and nete diezeugmenon, as the following diagram illustrates:

[b]	HYPATE HYPATON
[c]	PARHYPATE HYPATON
[d]	LICHANOS HYPATON
[e]	HYPATE MESON
[f]	PARHYPATE MESON
[g]	LICHANOS MESON
[a]	MESE
[b]	PARAMESE DIEZEUGMENON
[c]	TRITE DIEZEUGMENON
[d]	PARANETE DIEZEUGMENON
[e]	NETE DIEZEUGMENON

Thus here the paramese is disjunct from the mese, and for that reason this tetrachord is called "diezeugmenon."

But if the paramese is taken away, and the mese, trite, paranete, and nete remain, then the third tetrachord will be conjunct, that is, synemmenon. Thus that tetrachord will be called "synemmenon," in this manner:

[b]	HYPATE HYPATON
[c]	PARHYPATE HYPATON
[d]	LICHANOS HYPATON
[e]	HYPATE MESON
[f]	PARHYPATE MESON
[g]	LICHANOS MESON
[a]	MESE SYNEMMENON
[$b^{\flat}$]	TRITE SYNEMMENON
[c]	PARANETE SYNEMMENON
[d]	NETE SYNEMMENON

But in the above eleven note disposition, the mese, which was originally thusly named because of its middle location, came close to the nete, but was quite distant from the opposite hypate. Thus, since it did not retain its proper place, one other tetrachord was added above the nete diezeugmenon. This tetrachord was called the "hyperboleon," since its notes surpassed the earlier established nete with regard to pitch. The arrangement became thus:

[b] HYPATE HYPATON

[c] PARHYPATE HYPATON

[d] LICHANOS HYPATON

[e] HYPATE MESON

[f] PARHYPATE MESON

[g] LICHANOS MESON

[a] MESE

[b] PARAMESE

[c] TRITE DIEZEUGMENON

[d] PARANETE DIEZEUGMENON

[e] NETE DIEZEUGMENON

[f] TRITE HYPERBOLEON

[g] PARANETE HYPERBOLEON

[a] NETE HYPERBOLEON

But again, since the mese was not in the middle position, but rather closer to the hypate, a note was added above the hypate hypaton, which is called the "proslambanomenos."[53] However, other people call this note the "prosmelodos." This note is a full tone from the hypate hypaton. And it, the proslambanomenos, is an octave from

53. Cf. Nicomachus Manual xi (JanS. 256).

the mese, resounding a diapason consonance with it. It also resounds a diatessaron, that is a fourth, with the lichanos hypaton. The lichanos hypaton resounds a diapente, that is a fifth, with the mese. The mese in turn is one tone from the paramese, and it produces a diapente consonance, a fifth, with the nete diezeugmenon. The nete diezeugmenon produces a diatessaron consonance, a fourth, with the nete hyperboleon. Finally, the proslambanomenos resounds a bisdiapason consonance with the nete hyperboleon.

[a] PROSLAMBANOMENOS (or prosmelodos)
[b] HYPATE HYPATON
[c] PARHYPATE HYPATON
[d] LICHANOS HYPATON
[e] HYPATE MESON
[f] PARHYPATE MESON
[g] LICHANOS MESON
[a] MESE
[b] PARAMESE
[c] TRITE DIEZEUGMENON
[d] PARANETE DIEZEUGMENON
[e] NETE DIEZEUGMENON
[f] TRITE HYPERBOLEON
[g] PARANETE HYPERBOLEON
[a] NETE HYPERBOLEON

XXI. Concerning the genera of song.

Now that these things have been explained, we should discuss the genera of melodies. There are indeed three genera: the diatonic, the chromatic, and the enharmonic. The diatonic genus is somewhat more severe and natural (durius et naturalius), whereas the chromatic departs from natural inflection and becomes more sensual (mollius); finally the enharmonic genus is beautifully and fittingly yoked together. Since there are five tetrachords, the hypaton, the meson, the synemmenon, the diezeugmenon, and the hyperboleon, the singing voice progresses semitone, whole tone, and whole tone according to the diatonic. This occurs in the first tetrachord, likewise in the second, and all the others. It is called diatonic because of this progression; for it proceeds through tone after tone. However, the chromatic genus, which is called "colored," since it is the first alteration from the above inflection, is sung through a semitone, a semitone, and a trihemitone; for the whole diatessaron consonance contains two tones and a semitone, but no more. This word, that is, chromatic, is derived from "superfacies" (surfaces), which, when they are altered, turn into another color. The enharmonic genus is even more closely joined, for it is sung in all tetrachords through diesis, diesis, and a double tone. A diesis is half of a semitone. Thus this description of the progression through all the tetrachords in the three genera may be given:

Diatonic		
S	T	T

Chromatic		
S	S	Trihemitone

Enharmonic		
D	D	2T

XXII. Concerning the order and names of the notes in the three genera.

Now the order of all the notes ought to be clarified, for they are changed in the three genera and are arranged in an established order.

Thus the first string is the proslambanomenos, which some people call prosmelodos. The second string is the hypate hypaton, and the third, the parhypate hypaton. Now the fourth string is called lichanos hypaton by everyone; but if it occurs in the diatonic genus, it is called "lichanos hypaton diatonos." If, on the other hand, it occurs in the chromatic genus, it is called "diatonos chromatice," or "lichanos hypaton chromatice." Finally it is called "lichanos hypaton enharmonios," or "diatonos hypaton enharmonios" in the enharmonic genus. The string called hypate meson occurs next, and then the parhypate meson. Then comes the lichanos meson, which is called simply "diatonos meson" in the diatonic genus, but in the chromatic genus, lichanos meson chromatice or diatonos meson chromatice, and in the enharmonic genus, diatonos meson enharmonios or lichanos meson enharmonics. The mese follows these, after which two tetrachords may follow, sometimes the synemmenon, sometimes the diezeugmenon. If the synemmenon occurs, the note placed after the mese is the trite synemmenon; then follows the lichanos synemmenon, which is diatonos synemmenon in the diatonic genus, diatonos synemmenon chromatice or lichanos synemmenon enharmonios in the enharmonic genus. The nete synemmenon occurs after this string. But if the synemmenon tetrachord is not placed after the mese string, but rather the diezeugmenon tetrachord, then the string after the mese is the paramese. Then come the trite diezeugmenon and lichanos diezeugmenon, the latter of which is diatonos diezeugmenon in the diatonic genus, diatonos diezeugmenon chromatice or lichanos diezeugmenon chromatice in the chromatic genus, and diatonos diezeugmenon enharmonios or lichanos diezeugmenon enharmonios in the enharmonic genus. This same note is also called paranete, with the addition of the word diatonos, chromatice, or enharmonios. Next follow the notes nete diezeugmenon, trite hyperboleon, and paranete hyperboleon. This paranete hyperboleon is naturally called diatonos hyperboleon in the diatonic genus, hyperboleon chromatice in the chromatic genus, and hyperboleon enharmonios in the enharmonic genus. Finally comes the highest note of all, the nete hyperboleon. Now this description of the notes in the three genera may be presented, in which one can observe the similarity and difference of names. If all these notes are counted, those with similar and different names, all together they make twenty-eight. This is clearly seen in the following description:

Diatonic

Proslambanomenos
Hypate hypaton
Parhypate hypaton
Lichanos hypaton diatonos
Hypate meson
Parhypate meson
Lichanos meson diatonos
Mese
Trite synemmenon
Paranete synemmenon diatonos
Nete synemmenon
Paramese
Trite diezeugmenon
Paranete diezeugmenon diatonos
Nete diezeugmenon
Trite hyperboleon
Paranete hyperboleon diatonos
Nete hyperboleon

Chromatic

Proslambanomenos
Hypate hypaton
Parhypate hypaton
Lichanos hypaton chromatice
Hypate meson
Parhypate meson
Lichanos meson chromatice
Mese
Trite synemmenon
Paranete synemmenon chromatice
Nete synemmenon
Paramese
Trite diezeugmenon
Paranete diezeugmenon chromatice
Nete diezeugmenon
Trite hyperboleon
Paranete hyperboleon chromatice
Nete hyperboleon

Enharmonic

Proslambanomenos
Hypate hypaton
Parhypate hypaton
Lichanos hypaton enharmonios
Hypate meson
Parhypate meson
Lichanos meson enharmonios
Mese
Trite synemmenon
Paranete synemmenon enharmonios
Nete synemmenon
Paramese
Trite diezeugmenon
Paranete diezeugmenon enharmonios
Nete diezeugmenon
Trite hyperboleon
Paranete hyperboleon enharmonios
Nete hyperboleon.[54]

XXIII. Which intervals occur in each genus.

Thus in this manner we make the appropriate division according to the genus through each tetrachord, so that we divide all five tetrachords of the diatonic genus into two tones and a semitone. In this genus the tone is called "non-composite" (*incompositus*), because it is a whole tone and no other interval is added to it. In this genus the tones are always whole tones in singular intervals.

In the chromatic genus the division is regarded as semitone, semitone, and a non-composite trihemitone. We

54. Cf. Nicomachus *Manual* xii (JanS. 264).

call this trihemitone "non-composite," because the interval is brought together as one. A semitone and a tone in the diatonic genus can be called a trihemitone, but not a non-composite trihemitone; for here it is made from two intervals.

The same thing occurs in the enharmonic genus; for its division consists of a diesis, a diesis, and a non-composite ditone. We call this ditone "non-composite " for the same reason, that is, because it is made of one interval.[55]

XXIV. What synaphe is.

But synaphe, which we can express in Latin as conjunctio (conjunction), occurs in tetrachords so positioned and arranged that one common middle boundary joins two tetrachords and makes them continuous, as in the following tetrachords:

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55. This chapter would appear to be a translation of Nicomachus Manual xii (JanS. 262).

[b] HYPATE HYPATON

[c] PARHYPATE HYPATON

[d] LICHANOS HYPATON

[e] HYPATE MESON

[f] PARHYPATE MESON

[g] LICHANOS MESON

[a] MESE

Thus here the hypate, parhypate, lichanos, and hypate meson form one tetrachord, and the hypate meson, parhypate meson, lichanos meson, and mese a second. The hypate meson must be counted in both tetrachords; and it is the highest sound of the upper tetrachord, and the lowest sound of the lower one. Thus one and the same string, the hypate meson, is the conjunction uniting the hypaton tetrachord and the meson tetrachord, as the above diagram illustrates. Thus _synaphe_, which we call conjunction, is the note in the middle of two tetrachords which is the highest pitch of the upper tetrachord and the lowest pitch of the lower one.

XXV. _What diazeuxis is._

Diazeuxis, which can be expressed as disjunction, occurs whenever two tetrachords are separated in the middle by a tone, as in the following two tetrachords:

[e]	HYPATE MESON
[f]	PARHYPATE MESON
[g]	LICHANOS MESON
[a]	MESE
[b]	PARAMESE
[c]	TRITE DIEZEUGMENON
[d]	PARANETE DIEZEUGMENON
[e]	NETE DIEZEUGMENON

Here it is apparent that there are two tetrachords, since there are eight notes. But diazeuxis, that is disjunction, occurs in this case between the mese and paramese, which are separated by a whole tone. These things will be explained more clearly, for every single thing should receive very careful explanation later in this treatise.[56] But the diligent student has ascertained five tetrachords, no more: the hypaton, the meson, the synemmenon, the diezeugmenon, and the hyperboleon.

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56. See Book IV. viii-ix, xii. pp. 246-254, 262-263.

XXVI. By what names Albinus called the strings.

However, Albinus has translated the names of the strings into Latin language, so that he called the hypaton "principal," the meson "middle," the synemmenon "conjunct," the diezeugmenon "disjunct," and the hyperboleon "high."[57] But let us not be lingering in another work.

XXVII. To what stars the various strings are compared.

At this time it seems appropriate to add that those strings from the hypate meson to the nete in the above tetrachords are a reflection of the order and differentiation in the heavenly spheres. For the hypate meson is attributed to Saturn, the parhypate to the orbit of Jupiter. The lichanos meson relates itself to Mars, and the mese to the sun. The trite synemmenon relates to Venus, whereas Mercury rules the paranete synemmenon. The nete reflects the orbit of the moon.[58]

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57. Concerning Albinus see above n. 38. Cf. Martianus Capella ix. "de tonis."

58. Cf. Nicomachus Fragment III (JanS. 271-72); Nicomachus Manual iii (JanS. 241-42); Pliny ii. 20. 84; Censorinus xiii. See also R. Bragard, "L'harmonie des

But Marcus Tullius gives a different order: for in sixth book of his De re publica he says the following:

> And nature is so borne that low sound emanates from its outermost part, whereas high sound emanates from its other part. Therefore the high celestial movement, that of the stars, whose revolution is faster, moves with a high and excited sound; whereas the movement of the moon and that of the very low celestial bodies move with a very low sound. Now the earth remains still as the ninth body, and it always holds to this first position.[59]

Therefore Tullius regards the earth as silent, that is, immobile. After this comes that which is nearest to silence, that is, the moon; and it gives the lowest sound. Thus the moon may be presented as the proslambanomenos, Mercury as the hypate hypaton, Venus as the parhypate hypaton, the sun as the lichanos hypaton, Mars as the hypate meson, Jupiter as the parhypate meson, Saturn as the lichanos meson, and the highest heaven as the mese.

Spheres selon Boèce," Speculum, IV (1929), pp. 206-13.

Boethius and	Nicomachus Frag. III	Nic. Manual	Pliny Zodiac	Censorinus Sphere
Hyp. mes.	Saturn	Saturn	Saturn	Saturn
Parh. "	Jupiter	Jupiter	Jupiter	Jupiter
Lich. "	Mars	Mars	Mars	Mars
Mese	Sun	Sun	Sun	Sun
Trite synem.	Venus	Mercury	Venus	Venus
Paran. "	Mercury	Venus	Mercury	Mercury
Nete "	Moon	Moon	Moon	Moon
			Earth	Earth

59. Cicero De Re Publica vi. 18.

Which of these tones are immobile, which are totally movable, and which are in part immobile and in part movable, these things will be explained in a more fitting place, that is, when I discuss the division of the monochord ruler.[60]

XXVIII. What the nature of consonance is.

Now although the sense of hearing discerns consonance, nevertheless reason is the final judge. For when two strings, one higher and one lower, are stretched and struck at the same time (simul), and they produce an intermingled sweet sound, and the two sounds agree together as if joined in one, then that is what we call consonance. On the other hand, when the strings are struck at the same time and each one desires to go its own way, and they do not impress the ear as a sound which is sweet, and as one sound made from two, then this is an occurrence of what we call dissonance.[61]

XXIX. When consonances are produced.

But in these comparisons of a high tone with a low, it

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60. See Book IV. xiii. pp. 263-267.

61. Cf. this more complete definition of consonance with that of Chapter VIII, p. 57, and with the theory of consonance in Chapter XXXI, pp. 98-99.

is necessary to ascertain the consonances which are commensurate with themselves, that is, those which can have a common denominator. In the multiple type of inequality, for example, the duple is that part which measures; it is that which is the difference between the two terms. For example, between 2 and 4, the double measures both terms, or between 2 and 6, which is a triple, the double measures both. In superparticular proportions, if there be a sesquialter proportion, such as 4:6, the double is again that which measures both, and indeed it is also the difference between them. Moreover, if there be a sesquitertian proportion, such as 8:6, again the double measures both terms.

This phenomenon does not occur in the other types of proportions which we discussed above. The superpartient is an example; for if we compare 5 to 3, the double, which is the difference between them, measures neither of them. If this double be compared to 3, it is smaller; and if the double itself be doubled, it is greater. Likewise when 2 is compared to 5, it is smaller; in fact is is surpassed by 3. And thus superpartient is the first type of inequality separated from the nature of consonance. Why? Because in those numbers which form consonances, more are similar than those just discussed. This is proved in the following manner: the duple is nothing other than a simple number twice; the triple nothing other than a simple number three times, and likewise the quadruple is nothing other than a simple unity four times. Sesquialter is the simple unity and a half of that unity; sesquitertian, a simple unity and a third of that unity. Now this much similarity is not easy to find in the other types of inequality.

XXX. How Plato said consonance is made.

Plato said that consonance was produced in the ear in the following way. He said it is necessary that a higher sound also be faster. Thus, since it has sped ahead of the low sound, it comes more quickly into the ear. Now it also hits the innermost part of the ear, and is thus bounced back by the renewed motion. But now it moves slower and not so fast as with its original impulse, and thus it is lower. Now this lower sound, returning, first comes together with the approaching low sound, and it mingles with it and, as Plato said, produces one consonance.[62]

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62. Plato *Timaeus* 80 A-B.

XXXI. <u>What Nicomachus held against Plato's theory</u>.

But Nicomachus did not agree with this theory, for consonance does not lie in a similar sound, but rather in a dissimilar sound coming into one and the same concord. Indeed if a low sound is mixed with a low sound, no consonance is produced; for similitude of sound does not produce consonance but rather dissimilitude. For although two dissimilar voices are at some distance as singular sounds, they become joined together as a mixture.

Nicomachus considered consonance to be made in the following manner.[63] One pulse alone does not emit a simple musical sound; but rather a string, once struck, hits the air again and again in order to produce a sound. But since these percussions occur very fast, so that one sound encompasses another, the space between the sounds is not sensed, and the sound impresses the ear as only one. If therefore the percussions of the low sounds are commensurate with the percussions of the high sounds as in the proportions we discussed above, then there is no doubt that

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63. This theory of consonance is not found in Nicomachus' extant works. Concerning Boethius' Nicomachus source see Chapter I of Commentary, pp. 338-365.

this commensuration will mix together and produce one consonance of sounds.

XXXII. Which consonances precede others in merit.

But these consonances which we have discussed ought to be judged by the reason as well as by the ear. The reason must contemplate which of them is more harmonious. For as the ear is affected by a sound, and the eye by a view, in the same way the mind (animus) is affected by numbers or continuous quantity.

Having supposed a line or a number, nothing is easier to contemplate, with either eye or the intellect, than its double. After this judgment of its double follows that of its half, and then its triple, and then a third part of it. Thus since the mental representation of the double is more easily accomplished, Nicomachus considered the diapason as the optimum consonance. After this followed the diapente, which is the half, and then the diapason and diapente, which is the triple. He ranked the other consonances in this same manner and form.[64] Ptolemy did not

64. This ranking of consonances is not found in Nicomachus' extant works; cf. Book II. xx. pp. 145-149, and Chapter I of Commentary pp. 338-365.

consider them in this manner; but I will explain all his thinking later.[65]

XXXIII. How the things thus far said ought to be accepted.

All these things must henceforth be very diligently explained; nevertheless up to now we have attempted to explain them cursorily and briefly. It is hoped that this discussion will meanwhile serve the mind of the reader as an orientation. The mind will descend to a deeper degree of knowledge in the subsequent discussion. Now the presentation thus far was in the manner of Pythagoras; for when the master Pythagoras said something, no one thereafter dared challenge his reasoning. The authority of the teacher was reason enough for them. Now this system was right so long as the student, strengthened by stronger doctrine, came to discover the rationale of these things for himself, even without a teacher. In this same way we commend to the faith of the reader what we have just set forth. He should consider that the diapason consists of the duple proportion, the diapente of the sesquialter, the diatessaron of the sesquitertian, the diapason and diapente of the triple, and

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65. See Book V. vii-xii. pp. 304-314.

the bisdiapason of the quadruple. Later we will explain very carefully the reason for these things as well as how and by what aural judgment the consonances should be drawn together. The coming books of this treatise will explain in great detail all the other things which were said above, such as that the sesquioctave proportion produces a tone, and this tone cannot be divided into equal parts, just as no superparticular proportion can be divided into equal parts. It will further explain that the diatessaron consists of two tones and a semitone, that there are two semitones, a major and a minor, that the diapente contains three tones and a minor semitone, while the diapason has five tones and two minor semitones, and these five tones and two semitones in no way equal six whole tones. I will demonstrate all these things later both by mathematical logic and aural judgment. And thus enough of this for now.

XXXIV. What a musician is.

Every art and discipline ought naturally to be considered of a more honorable character than a skill which is exercised with the hand and labor of a craftsman. For it is much better and nobler to know about what someone else is doing than to be doing that for which someone else is the authority. For the mere physical skill serves as a slave, while the reason governs all as a sovereign. And unless the hand acts according to the will of reason, the thing done is in vain. Thus how much nobler is the study of music as a rational science than as a laborious skill of manufacturing! It is nobler to the degree that the mind is nobler than the body. For he who is without reason spends his life in servitude. Indeed the reason reigns and leads to right action, for unless reason's commands are obeyed, the action, void of reason, will be senseless.

Thus we can see that rational speculation is not dependent upon an act of labor, whereas manual works are nothing unless they are determined by reason. The great splendor and merit of reason can be perceived in the fact that the so-called men of physical skill are named according to their instrument rather than according to the discipline. For the kitharist is named after the kithara, the aulos player from the tibia, and the others according to the names of their instruments. But that person is a musician, who, through careful rational contemplation, has gained the knowledge of making music, not through the slavery of labor, but through the sovereignty of reason.

Indeed this fact can be seen in the building of monuments and waging of wars, since they are given other names; for monuments are inscribed with the names of those with whose authority and reason they were ordained, and military triumphs are also similarly commemorated. But monuments and triumphs are not named or commemorated for the servitude and labor of those who carried these things to completion.

Thus there are three kinds of people who are considered in relation to the musical art. The first type performs on instruments, the second composes songs, and the third type judges the instrumental performances and composed songs.

But the type which buries itself in instruments is separated from the understanding of musical knowledge. Representatives of this type, for example kithara players and organists and other instrumentalists, devote their total effort to exhibiting their skill on instruments. Thus they act as slaves, as has been said; for they use no reason, but are totally lacking in thought.

The second type is that of the poets. But this type composes songs not so much by thought and reason as by a certain natural instinct. Thus this type is also separated from music.

The third type is that which has gained an ability of judging, whereby it can weigh rhythms and melodies and songs as a whole. Of course since this type is devoted totally to reason and thought, it can rightly be considered musical. And that man is a musician who has the faculty of judging the modes and rhythms, as well as the genera of songs and their mixtures, and the songs of the poets, and indeed all things which are to be explained subsequently; and this judgment is based on a thought and reason particularly suited to the art of music.

BOOK II

I. Introduction

The first book was an exposition of all things which we now propose to demonstrate very carefully. But before I begin to treat these things which should be taught thoroughly using their inherent logic, I will insert a few things; and, by means of the things inserted, the student's more enlightened mind should arrive at those things which ought to be said about the things which, up to now, have had to be accepted.

II. What Pythagoras established as "Philosophy."

Pythagoras was the first man to call the study of wisdom "philosophy." He held that philosophy was the knowledge and study of a thing which is considered true and real; and he considered things true and real which neither grew under strain nor shrunk under pressure, that is, those things which are not changed by any accidents. These things are indeed forms, magnitudes, qualities, relations, and other things, which, considered as such are immutable, but which may change and suffer many variations in conjunction with bodies. This occurs because of the relationship to a changeable thing.[1]

III. Concerning different types of quantities; and with which discipline each is considered.

According to Pythagoras all quantity is either continuous or discrete. But that which is continuous is called "magnitude," whereas that which is discrete is called "multitude." The property of these is different and almost contrary. For multitude progresses in such a way that it begins with a finite quantity and grows into an infinite quantity, and there is no limit to its increasing. It is limited with regard to the smallest thing, but unlimited with regard to the largest; and its origin is unity, of which there is nothing smaller. But magnitude, on the other hand, assumes a limited quantity as its measure, but it is infinitely divisible. For if there be a line one foot long, or any other length for that matter, it can be divided into two equal parts, and these two parts can be

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1. Cf. Nicomachus Introduction to Arithmetic i. 1; and De Institutione Arithmetica i. 1. (translated as preface to Principles of Music, pp. 23-30).

divided in half, and again this half divided into another half; and thus there will never by any limit to dividing this magnitude. Magnitude is thus limited insomuch as the largest thing is concerned, but it is infinite when one begins to divide it. But to the contrary, number is limited with regard to the smallest thing, but it begins to be infinite when one multiplies it. Thus although these things are infinite, nevertheless they are part of the study of philosophy as finite things; and philosophy discovers something limited in unlimited things whereby it can apply the ingenuity of its own system of thought.[2]

Now some magnitudes are immobile, such as squares, triangles, or circles, whereas others are mobile, such as the sphere of the universe and whatever turns in such fixed speeds. Moreover, some quantities are discrete in themselves, whereas others are discrete in relation to other quantities, such as a double, triple, or others which are born out of a comparison. Geometry speculates concerning that quantity discrete in itself, whereas music proves to

2. Cf. Nicomachus *Arithmetic* i. 2; and *De Institutione Arithmetic* i. 1.

hold the skill with those quantities related to something.[3]

IV. Concerning the differences of relative quantities.

We discussed sufficiently that quantity which is discrete in itself in The Principles of Arithmetic. There are three simple kinds of quantity which is related to something: the first type is multiple; the second, superparticular; and the third, superpartient. When the multiple is mixed with the superparticular or the superpartient, the other two types are produced: the multiple superparticular, and the multiple superpartient. The rule for all these types is as follows: if you want to compare unity with all numbers in natural order, a proportioned order of multiples should be formed. For 2:1 is the double, 3:1 is the triple, 4:1 the quadruple, and so on in the same manner as shown in the following illustration:

1	1	1	1	1	1
2	3	4	5	6	7

Now if you seek a superparticular proportion, then compare

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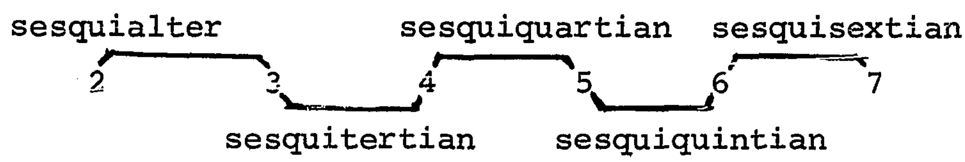
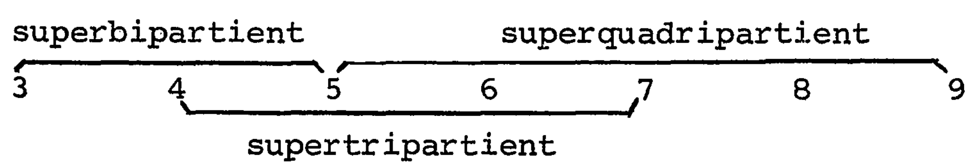
3. Cf. Nicomachus Arithmetic i. 3; and De Institutione Arithmetic i. 1.

the natural series with itself, of course with the unity removed; for example, 3:2 is the sesquialter, 4:3 the sesquitertian, 5:4 the sesquiquartian, and so on in this manner, as the following illustrates:



The superpartient proportions are found in this way: arrange a natural series, of course beginning from three. If you leave a one number interval, then you will see a superbipartient produced; if the interval is of two numbers, a supertripartient; if of three, a superquadripartient, and likewise in subsequent numbers



Keeping this same order, the diligent reader will work out the proportion put together from the multiple and superparticular or the multiple and superpartient. But all these things were discussed more fully in <u>The Principles of Arithmetic</u>.[4]

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4. <u>De Institutione Arithmetica</u> i. 22-31.

V. Why multiplicity is superior to the other types of inequality.

But now it should be considered that the multiple type of inequality seems far superior to the remaining two types. For an arrangement of numbers in natural order is coupled together in multiples of the unity which comes first.[5] But superparticular proportions are not made by comparison of a unity, but by comparison of those numbers which are set out after unity: for example, 3:2, 4:3, and so on in subsequent numbers.[6] The formation of superpartient proportions is quite backwards: for this type is not compared through continuous numbers, but rather through intermittent numbers. And the interruption of the series is never equal, for at one time it is interrupted by one number, at another by two, at another by three, at another by four, and thus the number of intermittent numbers increases to infinity.[7] Furthermore multiplicity begins

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5. Cf. Nicomachus *Arithmetic* i. 18-19; and *De Institutione Arithmetica* i. 23.

6. Cf. Nicomachus *Arithmetic* i. 19; and *De Institutione Arithmetica* i. 24.

7. Cf. Nicomachus *Arithmetic* i. 20; and *De Institutione Arithmetica* 1. 28.

with unity, superparticularity with two parts; but a superpartient proportion begins with three parts. But this should suffice for now.

In this place it is necessary to set out certain demonstrations which the Greeks call "axioms"; for only when we discuss every single thing by demonstration do we know all things as they are seen to be.

VI. What square numbers are; speculation concerning these.

A number is square which arises from multiplication of two equal factors, for example, 2x2, 3x3, 4x4, 5x5, 6x6, as the following diagram shows:

2	3	4	5	6	7	8	9	10
4	9	16	25	36	49	64	81	100

Thus the above arrangement of a natural series is the basis of the lower square numbers; for the square numbers are naturally continuous, and they follow one another in the following order: 4 9 16 25 36 49 64 81 100. If I subtract a smaller continuous square number from a larger continuous square number, that which remains will be that quantity which is the sum of the roots for these same squares. For example, if I subtract 4 from 9, 5 remains, which is the sum of 2 and 3, that is, the roots of the two squares. Likewise if I subtract 9 from that quantity which is numbered as 16, then 7 remains, of course the sum of 3 and 4 which are the roots of the just mentioned squares. The same thing occurs in the subsequent square numbers. But if the square numbers are not continuous, but one square number between two others is omitted, then half of the difference of these two will be that which is the sum of their roots. For example, if I subtract 4 from 16, 12 remains, the half of which is 6, which is the sum of the roots of the two squares; the roots of these squares are 2 and 4 which equal 6 when added. If two squares be skipped over, a third of their difference will be that which is the sum of their roots. For example, if I subtract 4 from 25, two squares having been omitted, then 21 remains; now the roots of these squares are 2 and 5, which equal 7, and 7 is a third part of 21. And thus the rule is as follows: if three squares be skipped, then a fourth part of the difference between the squares will be the sum of their roots. If four be skipped over, then a fifth, and thus the parts will be called one number larger than the number of squares skipped.[8]

VII. All inequality proceeds from equality, and the proof thereof.

As unity is the origin of numerical plurality, so equality is the origin of proportions. As was said in The Principles of Arithmetic, once we have assumed three numbers, we produce multiple proportions from equality.[9] We produce superparticular proportions, on the other hand, from transformed multiple proportions.[10] Likewise we make superpartient relations from transformed superparticular proportions.[11] For example, three unities are arranged one after another, or three doubles, or three triples, or any three equal terms, and thereunder is set down a row in which the first number is equal to the first number, the second is equal to the sum of the first and

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8. This axiom concerning square numbers has no direct relevancy to the following axioms or to the musical proofs which follow. Perhaps Boethius thought it would serve as a good orientation to the mathematical thought which follows.

9. De Institutione Arithmetica i. 32; Cf. Nicomachus Arithmetic i. 23. 6-8.

10. Ibid.; Cf. Nicomachus Arithmetic i. 23. 9.

11. Ibid.; Cf. Nicomachus Arithmetic i. 23. 10.

second, and the third equal to the first, twice the second, plus the third. Double numbers are thus made is a numerical progression, as this diagram shows:

1	1	1
1	2	4

Here the unity in the second row is equal to the unity of the first row. Likewise the double is equal to the first unity plus the second; and so the quadruplum is equal to the first unity, twice the second, and the third. The double proportion is thus 1:2:4. Now if you continue in this manner, a third row is made, and from the third a fourth, and from the fourth a fifth, and so on continues the production of these doubles.

$$1 + 2 + 4 + 1 = 8$$
$$1 + 2 + 4 + 8 + 1 = 16$$
$$1 + 2 + 4 + 8 + 16 + 1 = 32$$

Again having assumed three numbers, superparticular proportions are made as the following example illustrates. We now transform the above larger numbers and arrange them 4:2:1. A number is placed under the first number which is equal to it, that is 4; a second number is placed under the second number which is the sum of the first and the second, that is 6; a third number is now placed under the third number which equals the sum of the first, twice the second, and the third, that is 9. And having made this series we note that the proportion is a sesquialter.

4	2	1
4	6	9

If this process be performed with triple proportions, then the result will be sesquitertian proportions; and if with quadruple proportions, then sesquiquartian, and thus proportionality is born from multiplicity with similar names in any further part.

The superpartient proportion is put together from transformed superparticular: the sesquialter series is placed in reverse order, 9:6:4. Now a second row begins with a number equal to that of the first, that is 9; the second row's second number is the sum of the first and the second; and the third, the sum of the first, twice the second, and the third. Thus the diagram will be as follows:

9	6	4
9	15	25

Thus the superbipartient relation is produced from the transformed sesquialter. If someone approaches this system of thinking as a very diligent student, he will produce supertripartient proportions from transformed sesquitertians; and he will be amazed to discover that all superpartient proportions are produced from superparticularity with other similar names running parallel.

The multiple superparticular has to be created from non-transformed superparticular proportions, that is, superparticular proportions as they were brought forth from multiple proportions.[12] From non-transformed superpartient proportions, that is, from those just as they were produced from superparticular proportions, none other than multiple superpartient proportions are produced. But this is sufficient concerning these things; for we have discussed this comparison more diligently in _The Principles of Arithmetic_.[13]

VIII. _Rule for finding any continuous superparticular proportions_.

It often happens that someone discussing music seeks three or four or some other number of equal proportions in the superparticular type. But lest some error should

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12. _De Institutione Arithmetica_ i. 29; cf. Nicomachus _Arithmetic_ i. 23. 11.

13. _Ibid_. i. 31; cf. Nicomachus _Arithmetic_ i. 23. 12.

entangle the process in difficulty through ignorance or sheer chance, we should produce any number of equal proportions from multiplicity with the following rule: any single multiple proportion, itself computed from a unity, leads to as many superparticular relations, of course in a part contrary to its own designation, as the multiple itself departs from unity.[14] Thus a duple proportion precedes a sesquialter, a triple a sesquitertian, a quadruple a sesquiquartian, as the following description of double numbers shows:

1	2	4	8	16
	3	6	12	24
		9	18	36
			27	54
				81

Thus in the above diagram the first double multiple proportion has a 3 in proportion to it, which is thus able to make a sesquialter proportion. There is naturally nothing related to the 3 which could make a sesquialter proportion, for it has no half. Now the 4 is the second duple, which precedes two sesquialter proportions, those of 6 and 9, the

14. De Institutione Arithmetica ii. 2; Nicomachus Arithmetic ii. 3. 1.

latter of which has no half, and for that reason nothing is related to it by a sesquialter proportion. The pattern is the same in subsequent numbers.

Triple proportions create sesquitertian proportions in the same manner as this similar diagram in triple proportion illustrates:

1	3	9	27	81
	4	12	36	108
		16	48	144
			64	192
				256

Thus we see sesquitertian proportions born in this arrangement. Here the first triple precedes one sesquitertian, the second two, the third three, and each last number is always naturally closed to division by 3.

If you should form a quadruple row, you will discover sesquiquartian proportions in the same way; or if a quintuple row, then sesquiquintian proportions, and so on in subsequent numbers. And thus multiple proportions of a singular denomination precede as many superparticular proportions as they themselves have been multiplied from unity in a given place. Thus we give one such disposition in the quadruple proportion, so that in it, as well as in the others, the diligent reader may further sharpen his mind.

1	4	16	64	256
	5	20	80	320
		25	100	400
			125	500
				625

Thus it seems that this system of reason should be of use whenever someone wants to seek out four or five or even more sesquialter or sesquitertian or any other proportions; for here no error is likely to occur. But one should not try to fit such proportions to a certain first number which cannot precede or have behind it as many numbers as have been placed before it; but rather one sets our multiple proportions in order and figures how many superparticular proportions are required. Then he works from that multiple which departs from unity that number of times. Now if someone perhaps seeks three sesquialter proportions in the above diagram, then he should not begin his search at 4; for this number, since it is the second duple, precedes only two proportions, and no one can fit a third to it. But rather he should try to add halves from the 8; for this number, since it is the third duple, produces the three sesquialter proportions which are sought. And one should continue in subsequent numbers in this same manner.

There is another way of forming proportions in this manner. The smallest proportions in these comparisons are called the "roots" of the proportions. For if one arranges a natural series of numbers, multiplied from a unity, for example, 2 3 4 5 6 7, then the smallest proportions are the sesquialter 3:2, the sesquitertian 4:3, the sesquiquartian 5:4, and so on (to infinity) in proportion to the number of proportions that surpass the unity. But one might propose that two sesquialter proportions be produced in continuous relation. Thus I take the root sesquialter proportion and arrange it 2 and 3. Then I multiply the 2 x 2, which makes 4, and likewise the 3 x 2, which makes 6; but we multiply the three again by itself, which makes 9. These numbers are placed in this order:

2 3
4 6 9

We thus find the two proposed sesquialter proportions: 6:4 and 9:6. Now someone might propose that we find three such proportions. Thus I take these same numbers which I set forth in the above investigation as two sesquialter relations and use these same sesquialter proportions. I multiply the 4 x 2 which makes 8, likewise the 6 x 2, which makes 12, and also the 9 x 2 which makes 18; I multiply the 9 again by 3, which makes 27. These numbers are arranged as follows:

		2		3		
	4		6		9	
8		12		18		27

And this system will be valid for the other proportions: if you want to extend the sesquitertian root, for example, then you place the roots of the sesquitertian proportion, that is, 4 and 3, in relation to each other; then you multiply these as was done above. Similarly you take the sesquiquartian roots of this proportion and extend them by the same multiplication. Just how useful these considerations are to us will be shown in the following chapters.

IX. Concerning the proportion of numbers which are reduced by other numbers.

If two numbers are completely measured by their own difference, the numbers resulting from this measurement are in the same proportion as those numbers which were measured by their difference. Let the numbers 50 and 55 be assumed. These numbers are related according to the sesquidecimal proportion; and their difference is 5, which is of course a tenth part of 50. Thus 5 measures 50 ten times, and 55 eleven times. Thus the actual difference, that is, 5, measures the numbers 50 and 55 according to the numbers 10 and 11, and 10 and 11 are related by a sesquidecimal proportion. Therefore those numbers measured by their own difference are in the same proportion as those numbers into which their own difference measured them.

But if some numerical difference measures certain numbers--the difference being that of the numbers--in such a way that (1) a numerical plurality is left over in the measurement, and (2) this excess is the same in both numbers, and (3) the measurement of the difference is smaller than the numerical plurality, then, if that which remains after the measurement be subtracted from the original numbers, the resulting numbers will be a larger proportion than that of the numbers when their own difference measured them. Let the numbers 53 and 58 be considered. Let the number 5, which is their difference, measure them; it measures the number 53 up to 50 ten times and leaves three, whereas it measures the number 58 up to 55 eleven times and again leaves 3. Now if the three be subtracted from these numbers, 50 and 55 will remain. Let these numbers be arranged in this manner

53	58
50	55

Now here it is demonstrated that the 50 and 55 form a larger proportion than 53 and 58. For a larger proportion is always found in smaller numbers. But I will demonstrate this a little later.[15]

But if the measurement by a difference surpasses the magnitude of the numbers, and both numbers are surpassed by the same plurality, then, if the amount by which the measuring difference surpassed be added to the original numbers, the resulting numbers will be a smaller proportion than the proportion of those numbers when their own difference measured them. Let the numbers 48 and 53 be assumed. Their difference is 5, and if it measures the number 48 ten times, it will make 50 and the number 50 surpasses the number 48 by 2. Likewise, if it measures the number 53 eleven times it will make 55, which again surpasses the number 53 by the same 2. Let this 2 be added

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15. See last paragraph of this chapter.

to both numbers and let them be arranged in the following manner:

48	53
50	55

Thus 50:55--achieved by the addition of the 2 by which the measuring difference surpassed the first numbers--are related by a smaller proportion than 48:53, which were measured by the same difference of 5.

Indeed larger and smaller proportions are recognized in this manner: a half is larger than a third, a third is larger than a fourth, a fourth is larger than a fifth, and so on. Thus it follows that a sesquialter proportion is larger than a sesquitertian, and a sesquitertian surpasses a sesquiquartian, and so on in subsequent proportions. From whence it follows that a proportion of superparticular numbers is always seen to be larger in smaller numbers. This is apparent in a natural series of numbers: we arrange the series 1, 2, 3, 4. Thus the 2 forms a duple with the unity, and the 3 a sesquialter with the 2, and the 4 a sesquitertian with the 3. Here the larger numbers are 3 and 4, the smaller 3, 2, and unity. Thus the smaller proportion is found in the larger numbers, the larger in the smaller numbers. Hence it is apparent that whenever an equal plurality is added to two numbers which contain a superparticular proportion, the proportion is larger before the addition of the equal plurality than after the equal plurality has been added to the numbers.

X. What results are produced from the multiplication of multiple and superparticular proportions.

It seems that something which will be demonstrated shortly should be anticipated in this place.[16] If a multiple interval be multiplied by 2, the product of this multiplication is a multiple proportion. If the product of this multiplication should not be a multiple proportion, that which was multiplied by 2 was not a multiple.

Likewise if a superparticular be multiplied by 2, the product is neither a superparticular nor a multiple. Whenever that which is born from such a multiplication is neither multiple nor superparticular, then that which was

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16. See Book IV. ii. pp. 216-218.

multiplied by 2 is either superparticular or another type, but indeed not a multiple.[17]

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17. The following gloss by Gerbertus Scholasticus accompanies this chapter in Brussels 5444-6, f. 58v. *Que in decimo capitulo secundi libri musicae institutionis ratio superparticulari ut breviter facta est, eadem latius et per exempla tractantur in secundo capitulo libri quarti, nos quoque ipsa verba ponemus et de eisdem ita protenus dicemus. Si superparticularis proportio binario multiplicetur, id quod fit neque superparticularis esse neque multiplex. Binario dicitur multiplicari proportio quando eadem duplicatur ut qualis est prima, talis sit et secunda, id est, quemadmodum habet se primus terminus ad secundum, sic secundus se habeat ad tertium. Sit superparticularis proportio iiii:vi. Haec quoniam una est, binario multiplicetur; Bis enim unum, duo fiunt. Oportet igitur ut sunt iiii:vi, sic esse 6 ad alium quemlibet numerum. Hic sit viiii. Dico quoniam viiii:iiii neque multiplex est neque superparticularis. Quod si id quod ex tali multiplicatione nascetur neque multiplex neque superparticulare, tunc illud quod binario multiplicatum est vel superparticularis vel alterius generis, non vero multiplicis. Quod ex priore multiplicatione natum est duples sesquiquartus est, ut sunt viiii:iiii, id est neque multiplex neque superparticularis, sed multiplex-superparticularis. Et quod binario multiplicatum est multiplicis generis non est, sed vel superparticularis vel alterius. At hic neque multiplicis neque alterius, sed definite superparticularis. Est enim multiplicata proportio sesquialter.* "Since the rationale of the superparticular proportion was explained rather briefly in Chapter X of the second book of *The Principles of Music*, and the same thing is discussed later using examples in Chapter II of the fourth book, we also will set forth these same words and discuss them forthwith thusly. If a superparticular proportion be multiplied by two, the product is neither superparticular nor multiple. A proportion is said to be multiplied by two when it is duplicated in such a way that a second proportion is the same as the first; that is, just as the first term is related to the second, so is the second related to a third. Let the superparticular proportion 4:6 be assumed. Since

XI. Which superparticular proportions produce which multiple proportions.

To this we should add that the first two superparticular proportions make the first multiple. Thus if a sesquialter and a sesquitertian be added they will create a duple. Take the numbers 2, 3, and 4, for example: 3:2 is a sesquialter, 4:3 a sesquitertian, and 4:2 a duple. Likewise the first multiple added to the first superparticular creates the second multiple. Take the numbers 2, 4, and 6, for example: 4:2 forms the duple, that is, the first multiple; 6:4 is the sesquialter, which is the first superparticular; 6:2 forms the triple, that is, the

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this is one, let it be multiplied by two, for two times one equals two. Therefore as 4 is to 6, so 6 must be to some other number. This number should be 9. Thus I say that 9:4 is neither multiple nor superparticular, for if the product of such a multiplication is neither multiple nor superparticular, then that which was multiplied by two is either of the superparticular or of some other type, but not of the multiple type. The product of the above multiplication is a duple-sesquiquartian, since it is 9:4; it is neither multiple nor superparticular but multiple superparticular. And that which was multiplied by two is not of the multiple type, but either of the superparticular or another type. But this case was neither of the multiple nor of another type, but definitely of the superparticular type. For a sesquialter proportion was multiplied."

second multiple. When you add the triple to the sesquitertian, the quadruple is produced; if you add the quadruple to the sesquiquartian the quintuple is produced; and in this manner, that is, by joining proportions of the multiple and superparticular types, multiple proportions are infinitely produced.

XII. Concerning the arithmetic, geometric, and harmonic mean.

Since we have discussed the things which had to be said concerning proportions, now we should discuss means. A proportion is the mutual relation of two terms; moreover, I call terms a numerical whole. Proportionality is the putting together of equal proportions. Proportionality consists of at least three terms; for when a first term related to a second term retains the same proportion as the second term related to the third, then that is called "proportionality." Moreover, among the three terms the second one is the mean. Thus a triple partition is formed with these mean terms joining together the proportion. For either (1) the difference of the first term and the mean term is equal to that of the mean and the last term--but here the proportion is not equal, for example, in the numbers 1:2:3, wherein one is the difference between 1 and 2 as well as 2 and 3, but the proportion is not equal, 2:1 forms a duple whereas 3:2 is a sesquialter--or (2) the proportion is equal between both terms, but their difference is not equal, as in the numbers 1:2:4; here 2:1 is a duple, as is 4:2, but 2 forms the difference of 4 and 2, whereas 1 forms the difference of 1 and 2. There is also a third type of mean term, which consists in neither the same proportion nor the same difference; but in this type the largest term is related to the smallest term in the same proportion as the difference of the larger terms is related to the difference of the smaller, as in the numbers 3:4:6. Now 6:3 is a duple; 2 stands between 6 and 4, whereas 1 stands between 4 and 3. But 2 related to 1 again forms the duple. Therefore the largest term is related to the smallest just as the difference of the larger terms is to that of the smaller. The mean term in which the differences are equal is called "arithmetic"; the one in which the proportions are equal is called "geometric"; and the one we described third is called "harmonic."[18] Of this we submit the following diagram:

Arithmetic	Geometric	Harmonic
1:2:3	1:2:4	3:4:6

We know that there are also other mean terms of proportions which we discussed in The Principles of Arithmetic.[19] But these are the ones necessary for the present discussion. But among these means the one for geometry is normally and actually called "proportionality;" for it is totally constructed according to equal proportions. Nevertheless we use the word proportionality somewhat indiscriminately and call all the other types proportionality as well.

XIII. Concerning continuous and disjunct means.

But relating to these matters, some proportionality is disjunct, some is continuous. Continuous proportionality is that as discussed above; for one and the same mean number is added to a larger number, and at the same time placed in

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18. These means, essential to Pythagorean musical speculation, probably date back to the time of Pythagoras himself. Their invention and definition have been attributed to Hippasus, Philolaus, and Archytas; see Diels Fragmente der Vorsokratiker (ed. Kranz), I. 18. 15, p. 110 (Hippasus), I. 44. A24, p. 404-5 (Philolaus), I. 47. B2, pp. 435-36 (Archytas).

19. De Institutione Arithmetica ii. 61-62; cf. Nicomachus Arithmetic i. 28. 3-11.

front of a smaller number. On the other hand, when there are two mean terms, we call the proportionality "disjunct," as in a geometric series of the following type: 1:2, 3:6. For here the 2 is related to the 1, and the 6 to the 3; and this is called a disjunct proportionality. Hence it can be recognized that continuous proportionality is found in at least three terms, whereas disjunct proportionality is found in four terms. However, it is possible for continuous proportionality to be in four or more terms if they be similar to the following: 1:2:4:8:16. But here there are not two proportions, but more, and there is always one less proportion than the number of terms set down.

XIV. Why the means set forth above are named as they are.

Thus the first of these types of means is called "arithmetic," for the difference between the term is equal according to number. The second one is called "geometric," because it has the quality of equal proportions. The last one is called "harmonic," because it is so composed that an equality of proportions is set down in the difference of the terms as well as the terms themselves. But a more diligent exposition of these things took place in *The Principles of Arithmetic*,[20] whereas now we merely review it quickly so that we only recall it to mind.

XV. *How the above discussed means come forth from equality*.

But now we should discuss how these proportionalities proceed from equality. We have already discussed how unity prevails in number, and likewise equality prevails in proportions; and just as unity is the source number, thus equality is the first principle of proportions. Hence the arithmetic mean is born in the following manner: once three equal terms are assumed, there are these two ways of producing proportionality. The first number is placed equal to the first, the second equal to the first and the second, and the third equal to the first, second, and the third. This is shown in this example: assume three unities, and thus place under the first one one equal to the first unity, that is one; the second is the sum of the first and the second, that is 2; and the third is the sum of the first,

20. *Ibid*. ii. 42-50.

second, and third, that is 3. The result will be as follows:

1	1	1
1	2	3

Likewise if there be three twos set out in equality, then place the first term equal to the first, that is 2, the second equal to the first and the second, that is 4, and the third equal to the first, the second, and the third, that is 6. The result is as follows:

2	2	2
2	4	6

And likewise with 3:

3	3	3
3	6	9

But in relation to these things, it should be considered that if unity was established as the first principle of equality, unity will also be the differences of numbers, and these numbers allow nothing to come between themselves. If a double be the foundation of equality, then 2 is the difference, and one such number is allowed to come between the terms. But if the foundation be 3, then the difference is the same, with two numbers so constituted naturally coming between the numbers and so on in this manner.

There is another way of creating arithmetic proportionality. Again three equal terms are set down and another first term is established by adding the first and the second; a second term is formed by adding the first and twice the second, and a third by adding the first, twice the second, and the third. Thus if there are three unities, the first should be equal to the first and the second, that is 2; the second equal to the first and twice the second, that is 3; and the third equal to the first, twice the second, and the third, that is 4.

1	1	1
2	3	4

Thus here unity is the difference of the terms. For there is one unity between 2 and unity, and between 3 and 2. Furthermore no natural number is allowed to come between them. Immediately following the unity a 2 is set down, and naturally after the 2, a 3.

The same occurs again in the case of a 2, for the three twos are set in order and the first term is equal to the first and the second, that is 4; the second is equal to the first and twice the second, that is 6; and the third is equal to the first, twice the second, and the third, that is 8.

2	2	2
4	6	8

Here again 2 is the basis of the difference of the terms with one natural number being allowed to come between them; for 5 is naturally found between 4 and 6, and 7 between 6 and 8.

But if 3 be the foundation of the equality, the difference will be 3, with one less than this number always being allowed to come between. This same pattern is observed in the numbers 4 and 5. And the diligent reader, using these same rules, will discover for himself the things concerning which we now remain silent for the sake of brevity.

We showed how a geometric proportionality can be discovered from equality when we demonstrated that all inequality flows from equality. Nevertheless, unless it should cause squeamishness, this should now be repeated briefly. One having taken three terms, a first one is placed equal to the first, a second equal to the first and the second, and a third equal to the first, twice the second, and the third. And this same thing follows continuously. And thus geometric proportionality takes its foundation from equality. But we discussed the properties of these proportions extremely diligently in The Principles of Arithmetic.[21] If the reader instructed in these things should read this, then he will not be disturbed by any doubt or error.

The harmonic mean, which we should now discuss a little further, is produced in the following manner: if we desire to produce duple proportions, we assume three equal terms and place a first term equal to the first plus twice the second, a second one equal to twice the first and twice the second, and a third equal to the first, twice the second and three times the third. Thus let three unities be assumed.

1 1 1

The first is made equal to the first and twice the second, that is 3; the second is equal to twice the first and twice the second, that is 4; and the third is equal to the first,

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21. De Institutione Arithmetica ii. 44 ; see also Book II. vii. pp. 113-116.

twice the second and three times the third, that is 6. And if the equality consists of a 2 or a 3, the same rule applies for computing the mean terms, with the terms and their differences being spaced according to the duple proportion. This is shown in the following diagram:

1	1	1	2	2	2	3	3	3
3	4	6	6	8	12	9	12	18

If it should ever be required that the outside terms be related by a triple proportion, again assume three equal terms, and make a first one from the first and the second, a second from the first and twice the second, and a third from the first, twice the second, and three times the third, as the following description shows:

1	1	1	2	2	2	3	3	3
2	3	6	4	6	12	6	9	18

XVI. Concerning the harmonic mean; a much fuller investigation thereof.

But since we are engaging in a discussion of harmony, I do not believe we should tacitly pass over things which can be discussed more thoroughly. Thus harmonic proportionality should be set out and the differences of the terms be placed between them according to the order of terms in this diagram:

differences
1 2
3 4 6
terms

And thus do you not see that 4:3 produces a diatessaron consonance, 6:4 harmonizes in a diapente, 6:3 mixes a diapason consonance, and their very differences bring forth the same consonance again? For 2:1 is a duple, established in the diapason consonance. Whenever the outer numbers are multiplied by each other, and the middle term is increased by itself times itself, then these numbers compared with each other will hold the relation and sound of a tone. For 3 x 6 is 18, 4 x 4 is 16, and 18 transcends the minor 16 according to an eighth part. Again if the smaller term be multiplied by itself, it will make 9; if the larger number be multiplied by itself, it will make 36; and these numbers compared with each other hold the quadruple proportion, that is, the bisdiapason consonance. When we consider this more thoroughly, then everything will come to be from the multiplication of either the different terms or the terms times themselves. For the smaller term multiplied by the middle one will make 12; likewise the smaller multiplied by the larger will make 18; and the middle term treated the same equals 16; finally the 6, which is the largest, if multiplied by itself, will make 36. Thus these numbers are arranged in order:

36   24   18   16   12   9

Thus the resoundings of 24:18 and 12:9 are the diatessaron consonance, whereas 18:12, 24:16, and 36:24 are the diapente. However, 36:12 is the triple, which is the diapason and diapente, whereas 36:9 is the quadruple, which is the bis-diapason. The *apogdous*, which is the tone, is held in the relation of 18:16.[22]

XVII. *How the above discussed means are in turn located between two terms.*

Two terms are usually presented and arranged in such a way that we sometimes place between them an arithmetic mean, sometimes a geometric mean, and sometimes a harmonic mean. We also discussed this in *The Principles of*

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22. Boethius should have also noted that in this series of numbers 24:9 holds the proportion of the diapason and diatessaron, an interval considered consonant by Ptolemy (see Book V. ix.). See discussion of harmonic proportionality in Commentary, Chapter III, pp. 413-414.

Arithmetic,[23] but nevertheless we should also briefly explicate this same thing here.

If the arithmetic mean be sought, then the difference of the given terms must be found and divided, then added to the smaller term. Thus let the terms 10 and, on the other side, 40 be assumed and their mean be sought according to arithmetic proportionality. I first consider the difference of the two, which is 30; I divide this in half, which makes 15; I add this to the smaller term, that is 10, which makes 25. If this mean be placed between 10 and 40, it produces arithmetic proportionality in this manner: 10:25:40.

Again we can place a geometric mean between these same numbers. I multiply the extremes by their own numerosity, that is 10 x 40, which makes 400; I take the square root (*tetragonale latus*) of this, which makes 20, for 20 x 20 equals 400. If I place this mean of 20 between 10 and 40, it makes a geometric middle as we see in the following description: 10:20:40.

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23. *De Institutione Arithmetica* ii. 1; cf. Nicomachus *Arithmetic* ii. 27.

If we seek the harmonic mean, we add the extremes with themselves, whereby 10 plus 40 equals 50; we multiply the difference of these numbers, which is 30, by the smallest term, that is 10, and 10 x 30 is 300; this we divide by 50, which makes 6, which added to the smaller term, makes 16. If we place this mean number between 10 and 40, we extricate harmonic proportionality: 10:16:40.

XVIII. Concerning the merit or manner of consonance according to Nicomachus.

But enough concerning these things. Now it seems that we should discuss how the Pythagoreans proved that musical consonances were found in these proportions we have discussed. Ptolemy does not seem to agree with them concerning this matter, but we will discuss this a little later.[24]

That consonance whose character is more easily grasped by the sense ought to be considered the first and pleasant consonance. For just as every single thing is in itself, so it is perceived by the sense. Therefore

24. See Book V. vii-ix. pp. 304-309.

if that consonance which consists in duplicity is easier to know than the others, then there is no doubt that the diapason is the first consonance of all and that it excells the others in merit, since it precedes the others in being known. The remaining consonances, according to the Pythagoreans, necessarily stand in the order given by multiplications of multiple and diminutions of superparticular proportions. For it was demonstrated that, according to the ancients, multiple inequality transcends superparticular proportions in merit. Therefore let a natural numerical series be set out from 1 to 4: 1, 2, 3, 4. Thus the 2 related to the 1 makes a duple proportion, and resounds the diapason consonance, which is the highest and most knowable because of its simplicity. If the 3 be compared with 1, it produces the diapason and diapente consonance. The 4 compared to 1 holds the quadruple proportion, naturally producing the bisdiapason consonance. When the 3 is compared with 2, it produces a diapente consonance, whereas the 4 to 3 produces a diatessaron. And now one comparison remains: if we compare 4 to 2 it ends in the duple proportion which the 2 compared to the 1 had held. Thus sounds are at their greatest distance in the bisdiapason, since the interval here is formed according to the quadruple proportion. The smallest interval of consonance is that which occurs when the higher sound transcends the lower by a third part of the lower (4:3). And thus the sizes of the consonances are established, and they can neither be extended beyond the quadruple nor compressed to less than a third part. And according to Nicomachus[25] this is the order of consonances: first, the diapason; second, the diapason and diapente; third, the bisdiapason; fourth, the diapente; and fifth, the diatessaron.

XIX. Concerning the order of consonances according to Eubulides and Hippasus.

But Eubulides[26] and Hippasus[27] held another order of

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25. This ordering of consonances is not found in the extant works of Nicomachus; cf. Chapter XX of this Book, pp. 147-148, n. 28.

26. No works or fragments of the early Pythagorean Eubulides survive. In fact, this is probably the only specific theory which can be definitely attributed to him; see Diels Vorsokratiker (ed. Kranz), I. 14. 8, p. 99, l. 27:

27. Hippasus (fifth century B.C.) belongs to the very early Pythagorean school. No complete works or fragments survive, but through later Greek writers he is known to have worked in the area of musical mathematics (cf. p. 130, n. 18); see Diels Vorsokratiker (ed. Kranz), I. 18, pp. 107-110.

consonances; for they say the multiplication of multiple corresponds to the diminution of superparticular in a fixed order. Accordingly there can not be a duple unless there be a half, nor can there be a triple without a third. Therefore if there be a duple from which a diapason is produced, then there is a half from which a sesquialter proportion is made in the opposite (superparticular) division, bringing about a diapente. When these, the diapason and the diapente, are mixed together, a triple is made, containing the consonance of the same name. But again a third part is produced in the opposite division, from which the diatessaron consonance is born. Furthermore, by uniting the triple and the sesquitertian, the quadruple proportion is produced. Thus it follows that from the diapason and diapente, that is, the one consonance of this name, plus the diatessaron, one consonance is produced: this consonance consists in the quadruple proportion and takes the name of bisdiapason. Thus this is the order according to these theorists: first, diapason; second, diapente; third, diapason and diapente; fourth, diatessaron; and fifth, bisdiapason.

XX. Which consonances are placed opposite which consonances according to Nicomachus.

But Nicomachus does not agree with this positioning of the consonances against each other. He holds rather that, as unity was the first principle of increasing and diminishing in arithmetic, so the diapason consonance is the first principle of the other consonances; and from the diapason the other consonances can be set down in opposite division.

A thing will be easier to know if it is first discerned in numbers. Thus a unity is established and two parts flow from it, one in the multiple category, another in the other as this formula shows:

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            1
        1/2   2
     1/3         3
  1/4               4
1/5                   5
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And in this manner the scheme progresses to infinity. For 2 is the duple of unity, whereas its opposite part reveals the half of this unity. Three is the triple, and its opposite part a third; four the quadruple, and its opposite part a fourth. Thus the first principle of increasing and diminishing is simple unity. Thus we transform the consonances in the same manner. Naturally the diapason, that is the duple, will have the highest position of first principle, and the remaining consonances are arranged in opposite division in this manner: the sesquialter opposite the triple, and the sesquitertian opposite the quadruple, because they will be appraised in this order of augmentation. For the same number is the first sesquialter which is the first triple, of course in relation to the first unity. For three is the first triple if compared to unity, and it is likewise the first sesquialter if compared to two. Moreover the same three forms a triple with the difference of twc and three, which numbers prove to be related according to the sesquialter. Therefore since the sesquialter is rightly placed opposite the triple, the diapente consonance should by all reason be placed opposite the diapente and diapason.

Again the quadruple holds the division opposite the sesquitertian. For that number which is first quadruple is also discovered to be the first sesquitertian, as the following demonstrates: four is naturally the first quadruple compared with unity, the first sesquitertian if compared with three; moreover with the difference of itself and three, it again forms a quadruple. Whence it follows that the sesquitertian proportion, which is the diatessaron, is located in the position opposite the quadruple, which is the bisdiapason.

Now since the duple has no proportion opposite it, and is itself not the sesquialter of any other; and since there exists no number with which two, the first duple, can be joined as a superparticular proportion, thus it is beyond any form of opposite proportion. And therefore Nicomachus held the diapason to be the first principle of consonances in the manner of the following:

Diapason

Diapason and Diapente — Diapente

Bisdiapason — Diatessaron

But Nicomachus held that, although the whole scheme is related as above, nevertheless the multiple consonances precede in sweetness, and the superparticular proportions follow thereafter, just as we described a little earlier.[28]

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28. The reference to "earlier" refers to Chapter XVIII of this book. But like the contents of that chapter, the theories presented here are not to be found in the extant writings of Nicomachus. However, these theories follow quite logically from Nicomachus' Arithmetic. In Book I. 18. 1, and 19. 1, Nicomachus ranks the multiple type of inequality

Therefore, since consonance is the fitting mixture of two voices, whereas sound is the event of an inflected voice produced at a single pitch (intentio), and since sound is the smallest particle of modulation, whereas all sound consists in pulse, and all pulse is from motion, and since some motions are equal and others unequal, and of unequal motions some are more unequal, others less, others between: from all this it follows that equality of sound is born from equality, whereas inequalities of sound stem from inequality corresponding to the size of interval. Thus the clear, first, and simple proportions come into being, and they are naturally the consonances of the multiple and superparticular types: duple, triple, quadruple, sesquialter and sesquitertian. Dissonance is born from those inequalities which occur in the other types of proportions, or those sounds whose intervals are

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the highest in order and by nature and the superparticular type second to it. In Book I. 23. 7, discussing the generation of inequalities from unity, the theory presented at the beginning of this chapter is clearly implicit. Given the Pythagorean doctrine of the primacy of number, plus these theories of proportions and their origins, this order of consonances attributed to Nicomachus by Boethius is entirely consistent. Cf. Chapter I of Commentary, pp. 338-365, concerning Boethius' Nicomachus source.

situated variously or vaguely, or at a considerable distance; and from these sounds no concord is produced.

XXI. What must be learned in advance so that the diapason can be proved to lie in the multiple type of inequality.

Thus having distinguished this matter, we will demonstrate that the diapason consonance, which is the most excellent of all, is found in the multiple type of inequality and in the duple proportion. First we must demonstrate how the diapason consonance can be known in the multiple type of inequality. Thus we must set out a few things in advance, and by knowing these things the demonstration will be made easier.

If someone subtracts a continuous superparticular proportion from any superparticular proportion, the one subtracted naturally being smaller, then that which remains is less than the mean of the proportion which was subtracted. For example, consider the sesquialter and sesquitertian. Since the sesquialter is larger, we subtract the sesquitertian from the sesquialter, and there remains a sesquioctave. A sesquioctave proportion doubled does not make a whole sesquitertian proportion, but it is smaller by that distance which is found in a semitone. Since a doubled sesquioctave proportion does not equal a whole sesquitertian, a simple sesquioctave is not a true mean of the sesquitertian proportion. Likewise when you subtract a sesquiquartian from a sesquitertian, that which remains does not equal half of the sesquiquartian. This principle applies to all the following superparticular proportions.[29]

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29. The following gloss by Gerbertus Scholasticus is found on f. 63^v of Brussels 5444-6, placed adjacent to this chapter. Constantino suo Gerberti Scolastici: Capitulo xxi secundi libri musicae institutionis sic positum a Boetio est: si continuam ei superparticularem quis auferat, usque non est sesquitertiae proportionis plena medietas. Sit propositus unus et idem numerus ad quem aptetur sesquialter ac sesquitertia proportio. Hic sit vi. ad quem viiii. sesqualter est, viii. vero sesquitertius. Qui disponantur hoc modo vi:viii:viiii. Et quoniam hae duae proportionis continuae superparticulares sunt in tribus terminis constitutae, aufero primum terminum ac quem viii. est sesquitertius, viiii. sesquialter, remanent viii. et viiii., qui sesquioctavi sunt. Sed sesquioctava proportio non est minoris proportionis medietas, id est sesquitertiae, quoniam duplicata non efficit eam sed minor est. Duplicemus igitur sesquioctavam proportionem, et sint tres numeri ita dispositi, qui a proportione viii. et viiii. non recedant. Fiantque octies viii. et octies viiii. et novies viiii., id est lxiiii:lxxii:lxxxi. Dico quoniam primus ad secundum et secundus ad tertiam sesquioctavam custodiunt habitudinem, sed tertius ad primum minus est quam sesquitertius, non est ergo sesquioctava

XXII. Demonstration by contradiction (per impossibile) that the diapason is in the multiple type of inequality.

We now return to the diapason consonance. If the

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medietas sesquitertii. Et in omnibus superparticularibus continuis hoc in commune speculandum est. Quoniam si minor a maiore subtrahbitur, id quod relinquitur minus est medietate subtracte proportionis, quoniam duplicatum non ei coequatur. Quod monstrat subiecta descriptio. "In Chapter XXI of the second book of The Principles of Music the following is set out by Boethius: if someone subtract a continuous superparticular from any superparticular, the remainder is not as much as a complete half of a sesquitertian proportion. Let one and the same number be set forth, to which is fitted a sesquialter and a sesquitertian proportion. This number should be 6, to which 9 is the sesquialter, whereas 8 is the sesquitertian. These should be disposed in this manner: 6:8:9. And since these two proportions are continuous superparticulars, founded in three terms, I subtract the first term to which 8 is the sesquitertian, and 9 the sesquialter; 8 and 9 remain, which are sesquioctaves. But a sesquioctave proportion is not half of the smaller proportion, that is, the sesquitertian, since, if it is doubled, it does not make that proportion, but is less. Thus let us double the sesquioctave proportion, and it should thus be placed in three numbers which will not depart from the proportion of 8 and 9. And 8 should be multiplied by eight, and 8 by nine, and 9 by nine: that equals 64:72:81. I say that since the first to the second and the second to the third hold the sesquioctave relation, but the third to the first is less than a sesquitertian, therefore the sesquioctave is less than half of a sesquitertian. And all speculation concerning all continuous superparticular proportions must follow this same line of reasoning; for if a smaller one be subtracted from a larger one, the remainder is less than the mean of the subtracted proportion, since, when it is doubled, it does not become equal to it, as this attached discussion proves.

diapason is not in the multiple type of inequality, then it should be in the superparticular type. Thus let the diapason be a superparticular proportion. Now subtract the next consonance, that is the diapente, from it, and a diatessaron remains. Therefore twice the diatessaron is less than one diapente and the same diatessaron is not as much as a half of the diapente consonance. But all of this is impossible; for it has been clearly demonstrated that twice a diatessaron transcends a diapente by a tone and a semitone. Therefore a diapason cannot possibly be placed in the superparticular type of inequality.[30]

XXIII. Demonstration that the diapente, diatessaron, and tone are in the superparticular type.

Now it remains for us to demonstrate that the diapente, diatessaron, and tone should be placed in the superparticular type of inequality. Although we demonstrated this indirectly in the first proof when we showed the diapason should not be placed in the superparticular type,

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30. These proofs found in the last part of this book concerning various intervals are closely related to those found in *Sectio Canonis*, Propositions ix-xiii (JanS. 157-163).

nevertheless we will consider this thoroughly and singularly here.

Now if it be said that these proportions should not be placed in the superparticular type, it should be conceded that they are placed in the multiple type. Just why they cannot be placed in the superpartient or the other mixed types according to my opinion was explained earlier.[31] Thus let them be placed in the multiple type, if that can be done. Since the diatessaron is smaller and the diapente larger, the former should fit into the multiple duple proportion and the latter into the triple; for it is probable that the diatessaron consonance is next in order to the diapente consonance. Thus if the diatessaron be placed in the duple, then the diapente is placed in the next proportion, that is, the triple. But the tone, since it is located after the diatessaron in musical relationships, should certainly be placed in a proportion smaller than the duple. However, this cannot be found in the multiple type of inequality. Thus it follows that the tone lies in the superparticular type. So let the first

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31. See Book I. v.-vi. pp. 53-56.

superparticular proportion, that is, the sesquialter, be that of the tone. Now if we subtract duple from triple, that which remains is a sesquialter. If the diatessaron indeed is the duple, and the diapente the triple, and I subtract a diatessaron from a diapente a tone remains, then there can be no doubt that the tone ought to be placed in the sesquialter proportion. But two sesquialter proportions surpass one duple, exactly how, anyone instructed from The Principles of Arithmetic can ascertain for himself.[32] Thus these two tones are more than a diatessaron, which is inconsistent; for a diatessaron transcends two tones by the space of a semitone. Thus it cannot follow that the diapente and diatessaron are not located in the superparticular type of inequality.

But if someone suggest that the tone also be placed in the multiple type, since the tone is smaller than the diatessaron, and the diatessaron is smaller than the diapente, then the diapente should be placed in the quadruple

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32. This passage refers not so much to a specific passage in Boethius' arithmetical work as to the general skill in arithmetic one would have acquired from studying that work:

Two sesquialter proportions = 4:6:9
One duple proportion = 4 : 8
∴ Two sesquialter surpass one duple

proportion, the diatessaron in the triple, and the tone in the duple. But the diapente consists of a tone and a diatessaron; thus by this logic the quadruple should consist of the duple plus the triple, which cannot possibly follow. Again let the diapente be placed in the quadruple and the diatessaron in the triple. If we subtract a triple from a quadruple, a sesquitertian remains. By this logic a tone must correspond to the sesquitertian proportion. But three sesquitertian proportions are less than one triple. Therefore three tones will for no reason fill up a diatessaron, which is most certainly false. For two tones and a semitone complete a diatessaron consonance. Thus these arguments demonstrate that the diatessaron consonance is not a multiple.

But I say that the diapente also cannot be placed in the multiple type. For if it be placed there, since the diatessaron is the next smaller consonance, the diapente could not be located in the smallest multiple proportion, that is, the duple; for this would be the place where the diatessaron should be able to fit. But the diatessaron consonance is not in the multiple type, thus the diapente cannot be fitted to any multiple proportion larger than the duple, which is the smallest of this type. Thus let the smallest multiple, the duple, be the diapente. Now the diatessaron, which is the smaller, cannot be fitted in the multiple type, for there is none smaller than the duple. Thus let the diatessaron be sesquialter, the tone sesquitertian; for it is located in a continuous proportion. But two sesquitertian proportions are more than one sesquialter. Thus two tones surpass one diatessaron consonance, which will occur for no reason. Thus these arguments prove that the diapente, diatessaron, and tone cannot be found in the multiple type of inequality and are thus by all right placed in the superparticular type.

XXIV. Demonstration that the diapente and diatessaron are in the largest superparticular proportions.

We should also now necessarily add that, since the diapente and diatessaron are superparticular proportions, they are found in the largest superparticular proportions. Moreover the sesquialter and the sesquitertian are these largest proportions. This is proved in the following manner. If the diapente and diatessaron consonances are found in proportions smaller than the sesquialter and sesquitertian, then there is no doubt that, just as no other superparticular proportions besides the sesquialter and sesquitertian join together to produce one duple, so the diapente and diatessaron together would in no case encompass a diapason. For since the diapason was proved to be in the duple proportion, and the duple proportion is composed of a sesquialter and a sesquitertian, then there is no doubt that if we place the whole diapason in duple, then the diapente and diatessaron must be located in the sesquialter and sesquitertian proportions. For if they do not consist of the sesquialter and sesquitertian proportions, they could not join together to make a diapason consonance which consists of a duple; for the other superparticular proportions will in no case join together in a diapason.

XXV. That the diapente lies in the sesquialter proportion, the diatessaron in the sesquitertian, and the tone in the sesquioctave.

Thus I assert that the diapente consists of the sesquialter proportion, and the diatessaron of the sesquitertian. For since the sesquialter proportion is the larger of these two proportions, and the sesquitertian is the smaller, and since the diapente is the larger consonance, whereas the diatessaron is the smaller, then it is clear that the larger proportion ought to be fitted to the larger consonance while the smaller proportion to the smaller consonance. Therefore the diapente is to be found in the sesquialter proportion, the diatessaron in the sesquitertian. But if we subtract a sesquitertian from a sesquialter, a sesquioctave proportion is left; from whence it follows that the tone ought to be set down in the sesquioctave proportion.

XXVI. That the diapente and diapason is in the triple, and the bisdiapason in the quadruple.

But since it was demonstrated that the diapason is the duple, the diapente the sesquialter, whereas these proportions joined together create the triple proportion, then it is quite clear that the diapason and diapente is found in the triple proportion. But if someone adds a sesquitertian relation to the triple, he makes a quadruple. Thus if a diatessaron consonance be added to a diapente and diapason consonance, the interval of sound will be the quadruple, which we demonstrated above to be the bisdiapason.

XXVII. The diatessaron and diapason is not a consonance according to the Pythagoreans.

From these arguments the observant reader should recognize that some consonances placed over others have produced certain other consonances. For a diapente and diatessaron joined together create a diapason, as has been said. Moreover, if a diapente consonance be added to this, that is, the diapason, the consonance named for both of these is made, that is, the diapason and diapente. If a diatessaron be added to this one, a bisdiapason is made, which holds the quadruple proportion.

Suppose then that we join a diatessaron and a diapason: will these according to the Pythagoreans produce another consonance? Not at all. For this combination falls into the superpartient type of inequality, and it preserves neither the order of multiplicity nor the simplicity of superparticularity. Just let this matter be considered in numbers, whereby we can test it more easily. Let there be a 3, to which 6 is the duple, consisting of the diapason proportion. Let a sesquitertian, which we have said to be the diatessaron, be placed in relation to the 6, namely 8; for 8:6 holds the sesquitertian proportion. The

8 compared to the 3 contains the latter twice; but it is not a multiple, for it contains also other parts besides. For 8 surpasses 2 x 3 by 2 unities, which are two third parts of the term which we placed as the first and minimum term. Thus let these numbers be set out: 3:6:8. Here also something falls between these two consonances next to each other. In fact, it is neither a true duple, whereby it should produce a diapason consonance, nor a triple, whereby it should produce a diapason and a diapente consonance. If a tone be added to it, it would presently make a triple proportion. For since a diapason and a diapente joined together make a triple, and a diatessaron and a tone combine to make a diapente, if a diatessaron consonance be added to a diapason, the result will not be consonant, for between the duple and the triple no natural multiple proportion can be found. If I add a tone to this, it makes a diapason, a diatessaron, and a tone, which will be the same as the diapason and diapente. For a diatessaron and a tone make a diapente. Thus let the diapason be 3:6, the diatessaron 6:8, the tone 8:9, the diapente 6:9, the diapason and diapente, 3:9. Thus this row will be a triple proportion:

3 : 6 : 8 : 9

But although Nicomachus said much about this matter,[33] nevertheless we have presented here most briefly the same things which the Pythagoreans affirm; and thereby we have argued and proved the same consequence: if a diatessaron consonance be added to a diapason, their combination cannot possibly form a consonance. We will discuss later what Ptolemy thought concerning this matter,[34] but this suffices for now. It is time to consider the semitone.

XXVIII. Concerning the semitone; in what smallest numbers it is found.

It appears that semitones are so named, not because they are a real half tone, but because they are not whole tones; and these intervals which we now call "semitones," were called either "limma" or "diesis" by the ancients.

When two sesquioctave proportions, which are tones, are subtracted from a sesquitertian proportion, which is

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33. Nothing about this matter, namely that of the diapason and diatessaron, is found in the extant works of Nicomachus; See Chapter I of Commentary, pp. 338-365.

34. See Book V. ix-x. pp. 307-310.

a diatessaron, that interval which we call a semitone remains. Thus let us seek two tones arranged in continuous order. But since these consist of the sesquioctave proportion as was discussed above, and since we cannot add two continuous sesquioctave proportions unless that multiple be found from which these proportions can be derived, let us assume a first unity and its first octuple, that is 8. From this 8 I will be able to derive one sesquioctave proportion. But since we seek two, we multiply this 8 by 8, and from it is extended the number 64. Thus this will be the second octuple from which we can build two sesquioctave proportions. Now 8, which is an eighth part of 64, is added to 64, which makes 72. Then if an eighth part of 72, which is 9, be added to it, it makes 81. And these two continuous tones are presented as their first possible description in numbers: 64:72:81. Thus now we seek the sesquitertian of the unity 64. But it so happens that 64 has no third part. If all these numbers be multiplied by 3, the third part is forthwith achieved and all numbers remain in the same proportion as they were before the multiplicator 3 was applied to them. Thus three times 64 makes 192. One third of that, that is 64, added to it makes 256. Thus this will be the sesquitertian proportion holding the diatessaron consonance. Now let us add to the 192 in appropriate order the two sesquioctave proportions contained in their two numbers. Thus 3 x 72 equals 216; again 3 x 81 equals 243. Now these two terms are placed between the two written above in this manner:

192 : 216 : 243 : 256

Thus in this arrangement the first number to the second and the second to the third constitute identical tones. Therefore that interval which remains from 243 to 256 constitutes the smallest form of a semitone.

XXIX. Demonstration that 243:256 is not a half tone.

Thus I now demonstrate that the interval 243:256 is not the true dimension of a half tone. For the difference of 243 and 256 is contained in only 13 unities, and this 13 is less than an eighteenth part of the smaller number (243), but more than a nineteenth part. For if you multiply 18 x 13, you produce 234, which is in no way equal to 243; whereas if you multiply 19 x 13, it goes beyond a semitone, since every semitone, if it is to be an exact half of a tone, must fall between a sixteenth and a seventeenth part. Moreover we will demonstrate this later.[35]

Now it becomes clear that such an interval as the semitone, if doubled, cannot complete the space of one whole tone. Let us arrange, according to the rule described above, two continuous proportions related in the same proportion as the numbers 243:256 are. Thus we multiply 256 times itself and arrive at the largest number 65536. Likewise 243 increased according to its own numerosity leads to the smallest term, 59049. Again 256 is multiplied by 243, and thus the product will be 62208. This number is placed as the mean in this manner:

65536 : 62208 : 59049

Therefore 256:243 are in the same proportion as 62208: 59049. The same is true of 65536:62208. But the largest of these terms, namely 65536, related to the smallest, namely 59049, does not produce one whole tone. But if the proportion of the first to the second, which is equal to the proportion of the second to the third, is proved to be a whole semitone, then two halves joined together should necessarily make one tone. But since the proportion

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35. See Book III. i. pp. 174-176.

of the extreme terms is not a sesquioctave, it is made manifest that these two intervals do not represent halves of tones. For whatever is half of something, if it be doubled, makes that of which it is said to be half. But if it cannot fill that, then it is less than a half part. Moreover it is proved that 65536 compared to 59049 does not make a sesquioctave proportion if an eighth part of 59049 be taken according to the rules given in arithmetic. Now since this eighth part does not consist of a whole number, we leave the same to the computation of the more diligent reader. Thus it is evident that the proportion which is found in 256:243 is not a whole half of a tone. Therefore that which is called a semitone is in truth less than a half part of a tone.

XXX. Concerning the larger part of the tone: in what smallest numbers it consists.

The other part of the tone, which is larger, is called "apotome" by the Greeks, but can be called "decisio" (remainder) by us. For it follows by nature that when something is divided and not divided into equal parts, then just as much as the smaller part is smaller than a half, by just that margin does the larger part surpass the half. Therefore the apotome surpasses an exact half tone by exactly the same margin that the semitone is less than an exact half tone. And since we taught that a semitone is first of all found in 256:243, now we should prove in what smallest numbers that which is called apotome is found. If 243 could be divided into eight equal parts, since this would produce a sesquioctave comparison to 243, then the relation of 256 compared to the larger sesquioctave of the smaller number would necessarily reveal the apotome. But since an eighth part is obviously missing in this number, both numbers are multiplied by eight. Thus 8 x 243 equals 1944. Now if one eighth part of this number, which is 243, be added to it, it will make 2187. Likewise 256 is multiplied by 8, and this makes 2048. This number is placed as the mean of the above derived terms:

1944 : 2048 : 2187

Therefore the third term to the first retains the proportion of the tone, whereas the second to the first holds the proportion of a minor semitone; the third to the second holds the apotome. Thus it is seen that the apotome proportion consists in these prime numbers, since the interval of a semitone is contained in the prime numbers 256:243. Therefore 1944 and 2048 are in the same proportion as 243 and 256, since each of these were multiplied by eight. For if one number multiplies any other two numbers, those numbers which are born from this multiplication will be in the same proportion as those first numbers which were multiplied by one number.

XXXI. Of what proportions the diapente and diapason consist; and that the diapason does not consist of six tones.

Since we have discussed the diatessaron consonance at some length, now we should briefly and with clear numbers discuss the diapason and the diapente.

The diapente consists of three tones and a semitone, that is, a diatessaron and a tone. Thus let the numbers of the above description be set forth: 192:216:243:256. In this disposition the first term to the second and the second to the third hold the proportions of tones; but the third to the fourth holds that of a minor semitone, all of which was proved above.[36] Thus if an eighth part of 256

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36. See Chapter XXVIII of this book.

(32) be added to 256, it will make 288, which, compared to 192, will make the interval of the sesquialter proportion. Therefore there are three tones if the first be related to the second, the second to the third, and the fifth to the fourth. The comparison of the third to the fourth holds the semitone.

But if a diatessaron is made of two tones and a minor semitone, and a diapente of three tones and a minor semitone, then it would seem that the conjunction of a diatessaron and a diapente makes one diapason: there will be five tones and two minor semitones, the latter of which do not seem to complete one tone. Therefore the diapason is not a consonance consisting of six tones as Aristoxenus thought.[37] This also becomes quite clear if a numerical arrangement is made. Thus let six tones be set out in order, of course consisting of sesquioctave proportions.

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37. Aristoxenus *The Elements of Harmony* ii. 56-58.

diatessaron	=	2-1/2 tones
diapente	=	3-1/2 tones
diatessaron + diapente	=	diapason
∴ diapason	=	6 tones

Six sesquioctave proportions are produced from the sixth octuple. Thus let six octuples be set out in this manner:

1 8 64 512 4069 32768 262144

Now six tones in the sesquioctave proportion are figured from this highest number in this manner: first of all compute an eighth part of the terms, and then add the eighth part to the term from which the eighth part was taken. Thus let the arrangement be as follows:

sesquioctave proportions	eighth parts
262144	32768
294912	36864
331776	41472
373248	46656
419904	52488
472392	59049
531441	

The logic of this arrangement is this: the continuous row, which is called a boundary, holds the octuple numbers; thus sesquioctave proportions are derived from 6 octuples. The place in which we have written the eighth parts presents the eighth parts of the same numbers to which they are added. Now when this addition takes place, the following number is created; for example, the first number is 262144, an eighth part of which is 32768; if these be added together they make the next number, 294912, and so on the principle is carried to the other numbers.

Therefore if the largest number, 531441, were the duple proportion to the first number, 262144, then the diapason would by right seem to consist of six tones. However, if we compute the duple of the first and smallest number, it will be less than that number which is the largest. For the duple of 262144 is 524288, which holds a diapason consonance with the first number. This number is smaller than that number which held the sixth tone, that is 531441. Therefore the diapason consonance is smaller than six whole tones, and that interval by which six tones surpass a diapason I call a "comma." The smallest numbers in which the comma is found are 524288:531441.

But I will discuss later what Aristoxenus, who gave all judgment to the ears, thought concerning these matters.[38] Now I should draw the series of this book to a close in order to avoid distate.

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38. See Book III. iii. pp.178-180; and Book V. xiv. pp. 315-317.

BOOK III

I. _Demonstration against Aristoxenus that a superparticular proportion cannot be divided in half, and for this reason, neither can the tone._

In the last book we demonstrated that the diatessaron consonance consists of two tones and a semitone, and the diapente consonance consists of three tones and a semitone; but these semitones, considered and investigated in themselves, cannot fill the space of one whole tone. Thus the diapason consonance by no means consists of six tones.

But the musician Aristoxenus trusted merely in aural judgment,[1] and thus he did not conclude like Pythagoras, that semitones were smaller than a half tone. On the other hand, since they were called "semi" tones, he held that they were halves of tones.[2] Thus again we should argue and demonstrate that no superparticular proportion can be divided in exactly half by any conceivable number. For no mean number will be able to come between two numbers containing a superparticular

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1. Aristoxenus _Harmonics_ ii. 33-34.
2. _Ibid._ II. 46, 57.

proportion in such a way that the smallest number is related to the mean in the same proportion that the mean is related to the largest number, that is, as in a geometric proportion. This holds true whether the numbers are principal numbers, the difference of which is unity, or if they are larger numbers. But a mean number located between two such numbers can either produce equal differences, whereby there is equality according to an arithmetic mean, or it will produce a harmonic mean, or even some other type of mean which we mentioned in The Principles of Arithmetic.[3] Once this is proved, the position holding that a sesquioctave proportion--that is a tone--can be divided in half will not be able to stand at all; for all sesquioctave proportions belong to the superparticular type of inequality.

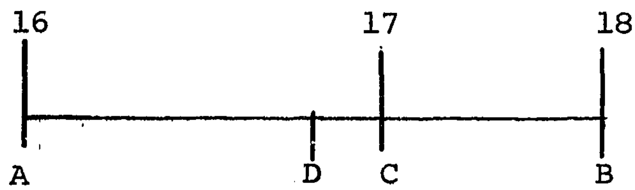
This will be better demonstrated by induction. For if, in testing the matter through single superparticular proportions, no instance occurs in which the interposed mean term divides the superparticular into equal proportions, then there is no doubt that a superparticular proportion cannot

3. De Institutione Arithmetica ii. 51-52.

be divided into equal proportions. But if it sounds consonant to the ears when some slight sound is compared to some other sound standing at the interval of two tones and a whole half tone, this does not prove that it is consonant by nature; for just as a sense cannot grasp all those things which are extremely small, so the ear cannot distinguish this little difference which proceeds beyond consonance. However, this will come to be perceived by the ear if such particles continue to be added through these same errors; for a thing which cannot be sensuously discerned as an extremely small quantity, can be clearly perceived when composed and joined together, for it then begins to be larger.

Thus from which proportion should we begin? Perhaps we will make the matter brief if we begin from that which is in question: that is, whether the tone can be divided into equal parts or not. Thus here and now the tone ought to be thoroughly discussed, and we should demonstrate exactly how it cannot be equally divided. If someone should transfer this demonstration to the other superparticular proportions, then it will similarly be demonstrated that a superparticular proportion cannot be divided into any equal or whole number.

The first numbers containing a tone are 8 and 9. But since these follow one another in natural order so that no mean number lies between them, I multiply them by two, which is the smallest number by which I can multiply them. This produces 16 and 18; and one number by nature falls between these two, that is 17. Thus 18:16 is a tone. But 18 compared to 17 contains its total plus one seventeenth part of it. In that 1/17 part is naturally smaller than 1/16 part, the proportion contained in the numbers 17:16 is smaller than that of 18:17. This is illustrated as follows: let 16 be A, 17 C, and 18 B. A true half of a tone in no way falls between C and B, for the proportion C:B is smaller than C:A. Thus a true half ought to be placed in relation to the larger part. Let D represent the half. Since the proportion D:B, which represents the half tone, is larger than the proportion C:B, which represents the smaller part of a tone, and since the proportion A:C, which is the larger part of the tone, is larger than the proportion A:D, which is again the half tone, and moreover, since the proportion A:C is a sesquisixteenth and the proportion C:B is a sesquiseventeenth: there is no doubt that the true half falls between the sesquisixteenth and the sesquiseventeenth. But this half will not be found in a whole number.



Indeed since the number 17 compared to the number 16 holds a sesquisixteenth proportion, let us seek a sixteenth part of the number 17: that will be a unity and 1/16 part of the unity (1 1/16). If we join this to the same number 17, it will produce 18 1/16. Thus if 16 1/16 be compared to the number 16, it is seen to exceed the interval of a true tone, since only the number 18 holds the sesquioctave proportion to 16. Thus it follows that since the sesquisixteenth proportion twice in succession transcends a tone, it may not represent a true half of a tone; for if something taken twice surpasses something, it is seen not to be a half of that which it surpasses. Therefore the sesquisixteenth will not be a half of a tone, since the sesquisixteenth itself is larger than a true half of a tone.

But since the sesquiseventeenth proportion is the next proportion following the sesquisixteenth, let us see if it will not make a tone if multiplied by two. The term

18 contains a sesquiseventeenth part of the number 17: thus if we compare another number in this same proportion to 18, it will be 19 1/17. But if we compare a number to 17 which stands in the sesquioctave proportion to it, the number will be 19 1/8. An eighth is larger than a seventeenth, and thus the proportion of the numbers 19 1/8:17 is larger than that of 19 1/17:17, the latter of which is of course two sesquiseventeenth proportions. Therefore two sesquiseventeenth proportions are seen not to complete one tone, and a sesquiseventeenth is not half of a tone. For if it is doubled it does not fill up the space of a whole, and thus it is not a half. A half doubled is always equal to that of which it is half.

II. <u>That half of a tone does not remain when two tones are subtracted from a sesquitertian proportion</u>.

If we consider those numbers which remain when two tones have been subtracted from a sesquitertian proportion, we are able to see whether the proportion which remains after the two tones have been subtracted should be judged as the space of a whole half tone. If this proves true, it also proves that a diatessaron consonance is composed of two tones and a half tone. Above there was the first term, 192, to which the term 256 was the sesquitertian proportion.[4] Furthermore, in relation to the first term (192), 216 formed a whole tone, and 243 held another tone in relation to 216. Thus the proportion which remains from a diatessaron proportion after two tones is that which consists of 243 and 256. If this proves to be a half of a tone, then it cannot be doubted that a diatessaron consists of two tones and a half tone. Since it was demonstrated that half of a tone is found between a sesquisixteenth and a sesquiseventeenth proportion, this proportion (256:243) ought to measure out this relationship. But lest this matter be prolonged any more, I take an eighteenth part of 243: that is 8 1/2. If I add this to the same number (243), it will make 256 1/2. Thus it appears that the proportion 256:243 is smaller than a sesquieighteenth proportion. But if a half tone is less than a sesquisixteenth proportion, but more than a sesquiseventeenth, whereas a sesquieighteenth is less than a sesquiseventeenth, and the proportion 256:243, the remainder

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4. See Book I. xxii., Book II. xxviii.

when two tones are subtracted from a diatessaron, is smaller than a sesquieighteenth, then there is no doubt that the proportion of these two numbers is very greatly smaller than half a tone.

III. Demonstrations against Aristoxenus that the diatessaron consonance does not consist of two tones and a half tone, nor the diapason of six tones.

But if, as Aristoxenus said, the diatessaron consonance is composed of two tones and a half tone,[5] then two diatessaron consonances necessarily make five tones, and a diapente plus a diatessaron, just as they make one diapason, should also be equal to six tones in continuous proportion. A little while ago we set forth six tones,[6] the smallest of which was the number 262144, and the highest number in relation to this first, 531441, was located in the sixth tone; the number 472392 held the fifth tone. Let these numbers be set forth in the following manner:

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5. Aristoxenus *Harmonics* ii. 56-67.
6. See Book II. xxxi.

6 tones

262144 472392 531441

5 tones

First we should consider the smaller numbers, that is, five tones. If the diatessaron consists of two tones and a semitone (that is, a half tone), and two diatessarons consist of five tones, when I have added a diatessaron to 262144 and subtracted another diatessaron from 472392, the same number should result. This is done in this manner: I compute a diatessaron interval, that is a sesquitertian, from the number 262144 by addition, which results in the number 349525 1/3; again I compute a sesquitertian proportion from the number 472392 by subtraction, which results in the number 354294. Thus we set forth these proportions in the following manner, letting A represent the first, B the second, C the third, and D the fourth.

A	262144
B	349525 1/3
C	354294
D	472392

Since the term A stands at the interval of five tones in relation to term D, and since the diatessaron consists of two tones and a half tone, as Aristoxenus thought, and since one diatessaron is found between A and B, while another is found between C and D, then it is necessary that terms B and C not be different, but one and the same number. Only then could five tones really appear to be produced from two diatessarons. But since there is indeed a difference of 4768 2/3, it is proved that the diatessaron is in no way composed of two tones and a half tone.

IV. That six tones exceed a diapason consonance by a comma, and in what minimum number the comma is found.

But let us try to express the difference of the above A and B in a whole number. Since a third part, which is half of 2/3, makes a full unity, if I would add half of the total difference, which is 2384 1/3, to this difference, it would make the whole number 7153. This number a short while ago held the proportion of the comma.[7] For a comma is that interval by which six tones surpass a diapason consonance, and the smallest number which contains the comma is 7153. Thus just as we added the exact half of the

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7. See Book II. xxxi.

difference in order to make 7153, likewise we can add exact halves of the terms A, B, C, and D, and it will be the same proportion as above in all instances. Moreover, the same difference will be produced between five tones and two diatessarons as that which is the difference between six tones and a diapason consonance, that is, of course, 7153. Whence it is concluded that five tones and six tones surpass respectively two diatessarons and a diapason by only a comma, the interval just found in the number 7153. The following diagram demonstrates this:

5 tones
diatessarons

A	B	C	D
262144	349525 1/3	354294	472392
	halves of the above numbers		
131072	174762 2/3	177147	236196
	first numbers plus halves		
393216	524288	531441	708588

difference of the
middle terms

7153

262144:531441	six tones
262144:524288	diapason
7153	difference

V. How Philolaus divided the tone.

The Pythagorean Philolaus tried to divide the tone in another manner.[8] He considered the first principle of the tone to be that number which made the first cube of the first odd number; for this number was considered most honorable among the Pythagoreans. Since 3 is the first odd number, if you multiply 3 x 3, and then the product of this by 3, the number 27 is produced. Twenty-seven stands at the interval of a tone when compared to 24, and the same difference of 3 remains. For 3 is an eighth part of 24, and when added to 24, it produces the first cube of 3, that is, 27. Philolaus made two parts from this number: the first was larger than a half--and he called it the apotome--and the second was smaller than a half--and he called it the diesis (but it could also be called the minor semitone). He held that the diesis first of all was found in the number 13, since this was discerned to be the difference of 256 and 243. Moreover, this same number 13

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8. This Chapter and Chapter XIII of this same Book are two fragments attributed to this late fifth century (B.C.) Pythagorean which are found in no other works. Another important musical fragment of Philolaus is found in Nicomachus Manual ix (JanS. 252-53); See Diels Vorsokratiker (ed. Kranz), I. 44, pp. 398-415.

is the sum of 9, 3, and unity; unity represents a point, 3 the first unequal [odd] line, the 9 the first odd square number. Thus since he placed the diesis in the number 13 for these reasons, and this number represented the semitone, he considered the other part which is contained in 14 the apotome. Since unity is the difference between 13 and 14, he held that unity ought to be considered the representative space of the comma. Thus he placed the tone in the number 27; for between the number 216 and 243, which stand at the interval of a tone, 27 is the difference.

VI. That a tone consists of two semitones and a comma.

From all this it is easily seen that a tone consists of two minor semitones and a comma. For if a whole tone is made of an apotome and a semitone, whereas a semitone differs from an apotome by a comma, then an apotome is nothing other than a minor semitone plus a comma. Thus if someone subtracts two minor semitones from a tone, a comma will remain.

VII. Demonstration that the difference between a tone and two semitones is a comma.

This conclusion can also be proved in this manner: if a diapason is contained in five tones and two minor semitones, and six tones surpass a diapason consonance by one comma, then there is no doubt that when five tones are subtracted from both intervals at the same time, the remainder will be two minor semitones in the case of the diapason, and one tone in the case of six tones. Moreover the remaining tone surpasses the two remaining semitones by a comma. For if a comma were added to the same two semitones, they would equal a tone. Therefore one tone consists of two minor semitones and a comma, the latter of which is found to be equal to the prime number 7153.

VIII. Concerning intervals smaller than a semitone.

Philolaus included these intervals and intervals smaller than these in the following definitions.[9] The diesis, he said, is the space by which a sesquitertian proportion is larger than two tones. A comma is the space by which a sesquioctave proportion is larger than two diesis, that is, larger than two minor semitones. A schisma is half of a comma, a diaschisma is half of a diesis, that is, of a minor semitone. From these definitions the following is

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9. See p. 182 , n. 8.

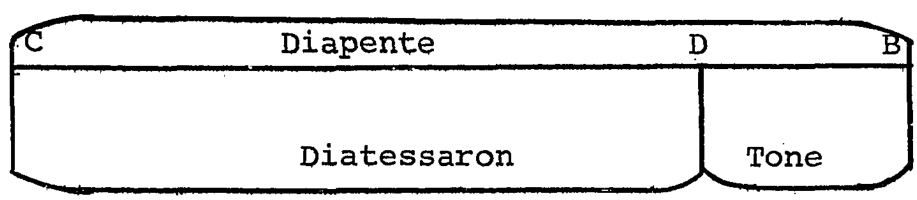
inferred; since a tone is principally divided into a minor semitone and an apotome, it is also divided into two minor semitones and a comma; from this it follows that it may be divided into four diaschisma and a comma. Thus a semitone, that is, a whole half of a tone, consists of two diaschisma, which equal the minor semitone, plus a schisma, which equals half of a comma. For since the total tone is joined together from two minor semitones and a comma, if someone wants to divide it equally, he should make two minor semitones and a comma. But one minor semitone is divided into two diaschisma, and half of a comma is a schisma. Therefore one can rightly say that a true half tone can be divided into two diaschisma and a schisma. Moreover it follows that a whole half tone seems to be separated from a minor semitone by one schisma. The apotome, on the other hand, is separated from a minor semitone by two schisma; for the difference between them is a comma, and two schisma equal one comma.

IX. How to adduce the parts of a tone by means of consonances.

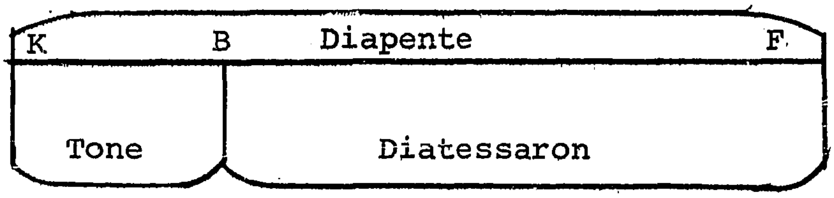
But enough has been said concerning this matter.

It seems that we should now discuss how we can, on the one hand, extend the spaces governed by musical consonance, and, on the other hand, how we can turn back or subtract from these same spaces. This matter should be considered linearly, and let the lines which we will draw represent musical space. But let reason through itself demonstrate the matter to itself.

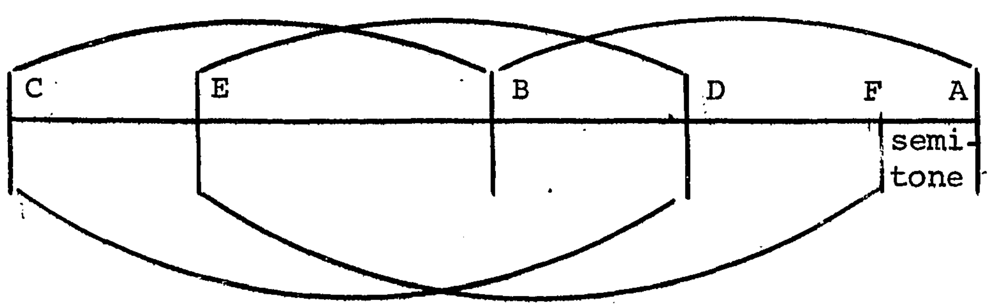
Let the problem be to adduce the interval of a tone through musical consonance, both to a lower and to a higher pitch. Assume the sound B; from B, I rise to another sound which stands at the interval of a diapente from B, and let this sound be called C. From here I turn back the space of a diatessaron consonance, and this sound is D. Since a tone is the difference between a diatessaron and diapente, a tone has been ascertained in the space DB.



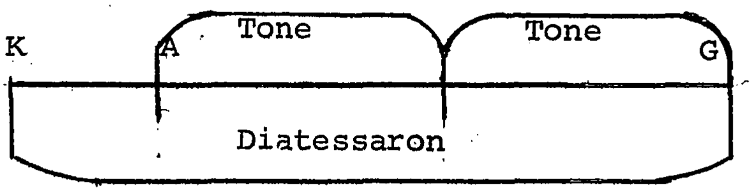
And thus we will sing a tone in relation to a lower pitch: I rise a diatessaron from B to F, and from F, I turn back a diapente to K. Thus KB will be a tone. The diligent reader will notice that the tone made by DB is to a higher pitch, whereas KB is to a lower.



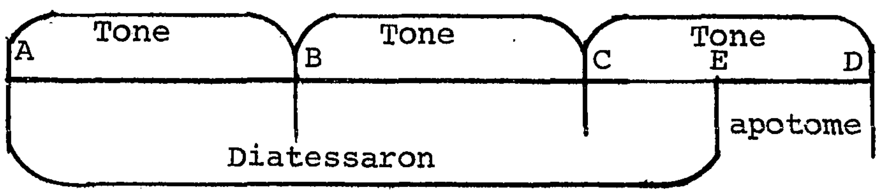
Let the problem be to adduce the smaller part of a tone, both to a higher and lower pitch, by means of consonance. The minor part of a tone is that interval by which a diatessaron transcends two tones. Thus let there be sound A; I rise from here a diatessaron consonance to B; again I rise a diatessaron consonance from B to C, and from C, I turn back a diapente to D. Thus BD is a tone. I again rise a diatessaron consonance from D to E, and drop down a diapente again from E to F. Thus DF is a tone. Since BD and DF are tones, and BA was a diatessaron, FA will be the smaller part of a tone, which is called a semitone.



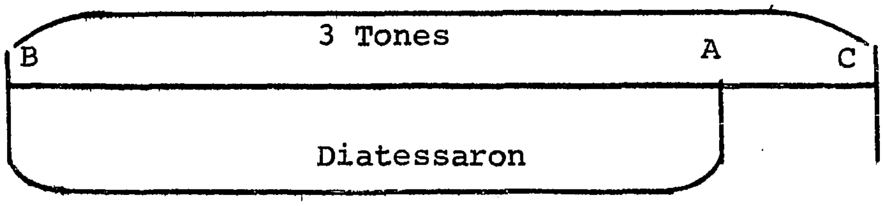
The same to a lower pitch in this manner: let there be the sound A. By means of consonance I rise the space of two tones to G, and then I turn back a diatessaron from G to K. Therefore KA will be that minor part of a semitone which was required.



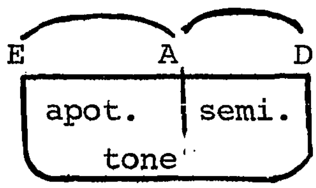
If we subtract a diatessaron from three tones, an apotome will be left. Thus let there be three tones: AB, BC, and CD. From these we subtract the diatessaron AE. Thus EC will be a minor semitone, but ED is an apotome.



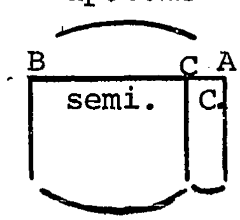
If it be of advantage, we might adduce the apotome, first to a higher pitch, in this manner: I rise three tones which are represented by AB, and from B, I turn back a diatessaron consonance to C, and CA, which remains, is an apotome.



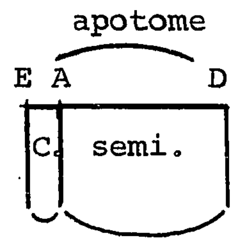
But if we want to make the same interval to a lower pitch, it should be done in this manner: let the sound A be assumed; I rise a minor semitone which is AD, and turn back a tone from D, which is DE. Thus AE will be the apotome we seek.



Let the problem be to adduce a comma to a higher pitch. Let the sound A be assumed. I rise an apotome AB, and then turn back a minor semitone BC. Since a semitone is smaller than an apotome by the space of a comma, CA will be the comma.



And this same thing to a lower pitch in this manner: from A, I rise to a minor semitone, that is AD, and from D, I turn back an apotome, that is DE. Thus EA will be the comma.



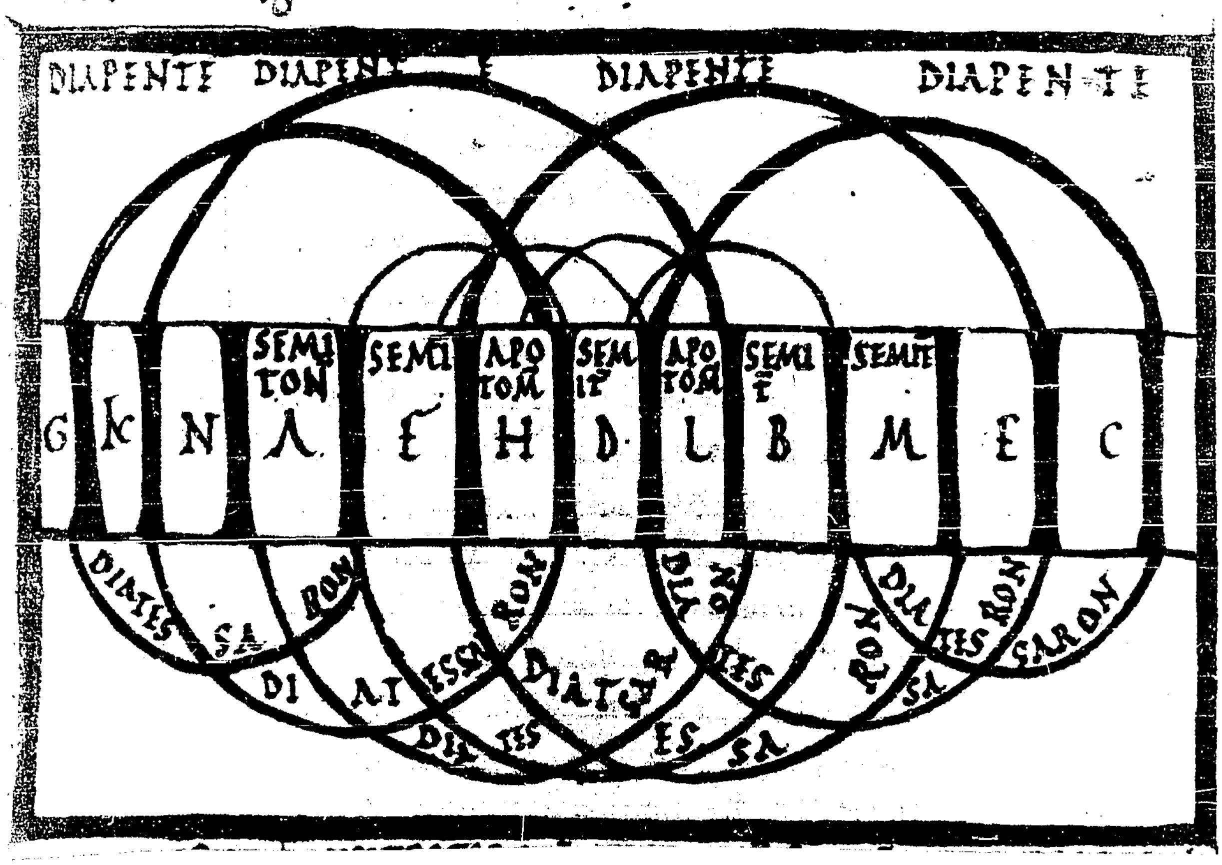
X. Rule for adducing the semitone.

It is necessary that all these consonances be recognized by both the mind and the ears. For unless these things we have brought together by knowledge and reason be of some use and very obvious application, they are indeed collected in vain. But since this matter of the semitone, which we have now addressed in The Principles of Music, is one of the most advanced problems in music, it may not be decided directly by the ears, but all the same by the reason. Thus we will give one example of how to find this seemingly more difficult interval. Moreover, the example will show how one can find this interval in either direction, both to a higher and to a lower pitch.

Let there be a diatessaron: AB. It is required that we subtract a minor semitone in relation to this consonance AB toward an upper pitch and a lower pitch. Thus I extend a diatessaron BC. In turn I come back a diapente CD. Thus BD will be a tone; for a diapente consonance surpasses a diatessaron consonance by a tone, and the space DC transcends CB by the space BD. I extend another diatessaron DE, and turn back a diapente EF. Thus DF is a tone. But BD was also a tone. Therefore AF, the interval which remains when the two tones FD and BD have been subtracted from the diatessaron, AB, is a minor semitone.

Again I turn back a diatessaron AG and extend the diapente GH. Thus AH will be a tone. But AF was a semitone. Therefore FH will be an apotome. Again I turn back a diatessaron HK and extend the diapente KL. Thus HL is a tone. HA was also a tone. Thus LB is a minor semitone. But DB was a tone; thus LD will be an apotome.

Again I extend back a diatessaron FM, and thus BM is a semitone. I turn back a diatessaron LN, and NA is a semitone. Thus by means of consonance two semitones have been computed in relation to the diatessaron AB: BM at the top and NA at the bottom. Moreover the total MN is less than a diapente, for it consists of five semitones and two apotome, that is, from two tones and three minor semitones. Since two semitones cannot make a whole tone, but the space of a comma is left, the total space MN is less than the space of a diapente consonance by one comma. The diligent reader recognizes this quite easily.



(Cologne, Stadt Archiv, W. 331 p. 126)

But since we discussed the comma only to a small extent, we should not neglect showing in what proportion the comma itself is contained. For the comma is the ultimate interval heard which can really be perceived. Thus we must discuss the problem of how many commas the major and minor semitones singularly consist, and also of how many commas the tone is constructed. Thus let us now begin discussing in this direction.

XI. Archytas' demonstration that a superparticular cannot be equally divided, and the refutation of this same demonstration.

A superparticular proportion cannot be divided exactly in half by the insertion of a number placed in the middle. This will be demonstrated quite firmly later.[10] The demonstration which Archytas gives concerning this matter is very unsatisfactory. He demonstrates it in the following manner:

Assume, he says, the superparticular proportion A:B. I take the lowest common denominator of this proportion, that is C:DE. Since these smaller numbers C:DE are in the same proportion, they are superparticular. The

10. Book IV. ii. pp. 215-216.

number DE transcends the number C by one of its parts. Let this part be D. I say "part" because D will not be a number but a unity. For if D is a number and a part of DE, then it measures the number DE, whence it follows that it also measures C. Therefore the number D would measure both numbers, C and DE, which is impossible. For when minimum numbers are the common denominators of any proportion they are in the same proportion as the other numbers, and their difference can only be unity. Thus D is unity, and DE transcends C by unity. Therefore there is no mean to fit into this proportion which will divide it equally. Whence it follows that no mean number which divides the proportion equally can be located between those numbers which hold the same proportion as these.[11]

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11. This argument attributed to Archytas (c. 430-365 B.C.) is found only in this work by Boethius. It is viewed by mathematical historians as a most significant fragment of Euclidean mathematical thought (Euclid's Elements, trans. with introduction and commentary by Sir Thomas L. Heath [2nd ed., Cambridge: 1908] vol. II, p. 295). The proof is almost identical with that presented in Sectio Canonis, Proposition III (See Book IV. ii. pp. 215-216), a work definitely of the mature Euclidean school if not by Euclid himself.

In proof of Archytas, A:B equals 4:6, whereas C:DE equals 2:3.

Porphryr In Harmonica Ptolemaei Commentarius 92. cites

Thus according to the reasoning of Archytas, no mean term falls in a superparticular proportion which divides the proportion equally, because the minimum terms in the same proportion are separated only by a unity, just as though the smallest terms in multiple proportion did not also share the same difference of unity, while we see that there are more multiple proportions besides these [differentiated by unity], which are found in roots between which the mean term can fit dividing the same proportion equally.[12] But he who has been well versed in arithmetic will easily recognize this. It should only be inserted that, according to Archytas, this occurs only in superparticular proportions;

Archytas as author of a περὶ μουσικῆς. Fragments attributed to Archytas concerning his theory of sound, proportions, and harmony are found in Diels *Vorsokratiker* (ed. Kranz), I. 47. B1-3, pp. 431-438.

12. Boethius' objection, although somewhat awkwardly stated, seems to apply to Archytas' statement that "when minimum numbers are the common denominators of any proportion, they are in the same proportion as the other numbers *and their difference can only be unity*." For example, 4:1 is the common denominator of 16:4, and thus it is in the same proportions; but 2 forms the geometric mean of 4:1, equally dividing the proportion. Therefore, this statement of Archytas is not universally valid. The proof of this premise which Boethius accepts (Book IV. ii. pp. 215-216 from *Sectio Canonis*, Proposition III) is, with exception of this false generalization, practically an identical argument.

but this should not be universally asserted. Now we should turn to the following.

XII. In what numerical proportion the comma is found, and that it is in that proportion which is larger than 75:74 and smaller than 74:73.

First I say that those numbers containing the comma stand in a proportion larger than 75:74 and smaller than 74:73. This will be demonstrated thusly: first of all one should remember that six tones transcend a diapason by a comma. Thus assume that A is 262144, and B, 524288; the last number contains a diapason consonance in relation to A, being of course its double. Let term C stand at a distance of six tones from A, that is 531441. These numbers are collected from the second book where we discussed the placement of tones.[13] Therefore, between B and C the proportion of a comma is found. If I subtract the number B from C, the number D remains, consisting of 7153 unities. Number D is less than a seventy-third part of the number B, and likewise larger than a seventy-fourth

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13. Book II. xxxi. pp. 167-170.

part of it. For if I should multiply D (7153) by 73, I would arrive at the number 522169; if I multiply D by 74, I produce the number F, 529322. Indeed E, achieved by multiplying by 73, is less than the number B, whereas F, achieved by multiplying by 74, is larger than B. Thus it has rightly been said that D is smaller than a seventh-third part of B, but larger than a seventy-fourth part of B. Thus the proportion C:B is larger than 75:74, but smaller than 74:73. For in the former (75:74) unity is a seventy-fourth part of the smaller number, whereas in the latter (74:73) it is a seventy-third part.

A	262144				
B	524288			E	522169
		D	7153		
C	531441			F	529322

This same matter is explained in another manner which has already been anticipated.[14] If the actual difference of two numbers be equally added to their proportion, then the proportion that will be contained between the two numbers made from the addition will be smaller than the proportion before the addition took place. Consider, for

14. Book II. ix. pp. 123-124.

example, 4 and 6; if their difference of 2 be added to each, 6 and 8 are produced. But whereas 6:4 contained a sesquialter proportion, 8:6 contains a sesquitertian, and a sesquitertian is, of course, smaller than a sesquialter.

Let the number C measure the smaller number A 75 times; if I should multiply the number C by 75, I produce the number D, 536475. Thus the number D surpasses A by the number E, that is 5043. Again let the number C measure the number B 74 times, and if these are multiplied the number F is produced, 529322. Now F is larger than B by the same number E, that is, 5043. Therefore D transcends A by E, and moreover F surpasses B by this same E. Thus if we add E to A we make D; if we add the same to B, we make F. The number D is produced by C x 75, whereas C x 74 creates the number F. Thus D and F related as a proportion hold the same relationship as 75:74. But D and F are A and B plus one E added to each. Thus the proportion contained between A and B is necessarily larger than that between D and F. For D and F are made by the addition of an E to the numbers A and B. But the proportion D:F is the same proportion as 75:74. Therefore the proportion A:B is larger than 75:74. A and B contain the comma; therefore the proportion of a comma is larger than 75:74.

A	531441	B	524288	C	7153
D	536475	F	529322		
	E 5043				

Since we have shown that the proportion of the comma is larger than 75:74, now it must be shown that the same proportion is smaller than 74:73. This will be demonstrated in the following manner: first of all that which we discussed concerning measurement by differences in the second book must be remembered.[15] If we subtract from any proportion the difference of the numbers, that which remains will hold a larger proportion than those numbers before their difference was subtracted. Let the proportion 8:6 be assumed; I subtract their own difference from them, which makes 6:4. Whereas a sesquitertian was contained in the first numbers, a sesquialter is contained in the latter. The sesquialter proportion is naturally larger than the sesquitertian.

Thus let us assume again the same A and B of the above discussion, as well as their difference C. I multiply C x 74, which makes the number F, 529322. This

15. Book II. ix. pp. 122-123.

number compared to A is surpassed by the number G, 2119. Again C is multiplied by 73, which makes K, 522169. This number compared to B is surpassed by the same G, 2119. Thus F and K are made by subtracting G from A and B. Thus A and B retain a proportion which is larger than F and K. But F and K are the same proportion as 74:73, for they were made by multiplying these numbers by C. Therefore the proportion A:B, containing the comma, is smaller than 74:73. Moreover it was just demonstrated that this same proportion was larger than 75:74. Thus that which was necessary to prove has been demonstrated: the numbers containing the comma hold between themselves a proportion larger than 75:74, but smaller than 74:73.

A	531441	B	524288	C	7153
F	529322	K	522169		
	G 2119				

XIII. That the minor semitone is larger than 20:19, but smaller than 19 1/2:18 1/2.

If this type of speculation be applied to the minor semitone, we will also quite easily ascertain its proportion as 256:243. Thus let A represent 256 and B 243, and let C represent their difference of 13. I say that A:B holds a proportion smaller than 19 1/2:18 1/2. Let C divide A 19 1/2 times, and C x 19 1/2 produces the number D 253 1/2. Now C transcends D by 2 1/2, and we let this difference of 2 1/2 be represented by F. Again let the same difference C divide the number B 18 1/2 times, and C x 18 1/2 makes the number E, 240 1/2. Thus when E is compared to B, it is transcended by the same F, namely 2 1/2. Thus D and E are smaller than A and B by the same difference F. So F subtracted from A and B leaves D and F. Therefore A:B is a larger proportion than D:E. But the proportion D:E is the same as 19 1/2:18 1/2. Thus the necessary proof has been given: A:B is a smaller proportion than 19 1/2:18 1/2.

A 256	B 243	C 13
D 253 1/2	E 240 1/2	
	F 2 1/2	

Nevertheless the proportion 256:243 seems to be larger than 20:19. Thus let the A, B, and C of the above discussion be once more assumed. Let the difference C measure the term A 20 times, and 20 x C makes term D, 260. This number will transcend A by 4, and 4 can be represented by F. Again let C measure B 19 times, and C x 19 gives us term E, 247. E transcends term B by the same F. Thus D and E transcend A and B respectively by the same difference of F. If I added F to A and B, I would produce D and E. Therefore the proportion A:B is larger than D:E; but D:E is nothing more than 20:19 multiplied by C. Thus A:B, the proportion of the semitone, is a larger proportion than 20:19.

A	256	B	243	C	13
D	260	E	247		
	F 4				

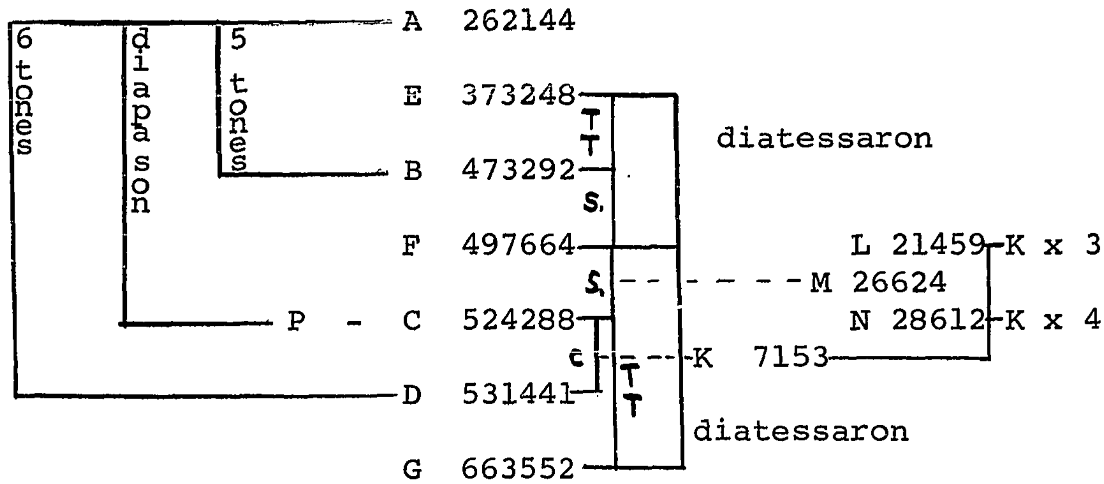
Thus it has been demonstrated that the minor semitone is a larger proportion than 20:19, and likewise smaller than 19 1/2:18 1/2.

Now we should compare the semitone to the comma; for just as the comma is the last interval to be heard, so is it the last proportion.

XIV. <u>That a minor semitone is larger than three commas, but smaller than four</u>.

We now propose to demonstrate that the minor semitone is larger than three commas, but smaller than four. You should be able to recognize this quite easily in the following manner: let those three numbers be set forth which contain the proportion of the diapason and also of six tones. Thus A should be 262144. This term is extended to five continuous tones, and this should be term B, 472392. Term C will represent the diapason consonance, that is the number 524288. And A is extended six tones to term D, and this will be the number 531441. Thus when these terms have been so set forth and arranged, it is clear that the comma is contained between C and D, and that their difference is 7153; let K represent this difference. Figuring back now two tones from B, term E is produced, that is, the number 373248. Moreover, I now extend a diatessaron from E, and produce F, 497664. Thus since there are two tones between E and B, and a diatessaron between E and F, a minor semitone is found between B and F. For when two tones are subtracted from a diatessaron, the remainder is a minor semitone, which as already discussed, lies in the numbers 256:243. If you multiply these same numbers by 1944, then you unfold the numbers B and F. And these numbers contain the same proportion as the original numbers, because they were both multiplied by the same number (1944). Once again I extend a diatessaron from term F up to term G, and G represents the number 663552. Moreover I figure back two tones from this G to term P, which is 524388. This P presents the same sound as term C, since through this reasoning it comes to be equal with it. For AD is a diapason consonance consisting of five tones and two minor semitones, and it is separated from six tones by a comma. Term P is thus at a distance of five tones and two minor semitones from A in the following manner: five tones are located in the space from A to B; a minor semitone is found between B and F, and F to P again holds a minor semitone. Thus the space from A up to P has produced five tones and two semitones. Thus the terms P and C should be signified by the same number. Since there is a minor semitone between F and C, we should see what their difference is so that we might compare it with the comma. The difference of these numbers is 26624, which will be term M. Thus K is the difference of the comma, and M that of the minor semitone. If we should multiply K x 3, the product would be 21459, and let this number be L. If you want to multiply K x 4, the product is 28612, and let this be N. Therefore M is indeed larger than L, but at the same time smaller than N. But N is made from the comma multiplied by 4, while

L is the comma multiplied by 3; M of course holds the difference of the minor semitone. Therefore it is rightly said that the minor semitone is smaller than four commas, but larger than three.



XV. <u>That the apotome is larger than four commas, but smaller than five, and that the tone is larger than seven commas, but smaller than eight.</u>

Following this same line of reasoning, we can discover how many commas are contained in a major semitone, which we discussed above under the name of apotome.[16]

Let term A be 262144, and term B, standing five tones from A, 472392. Let D, standing six tones from A, be 531441. Thus between B and D there is a tone. B stands

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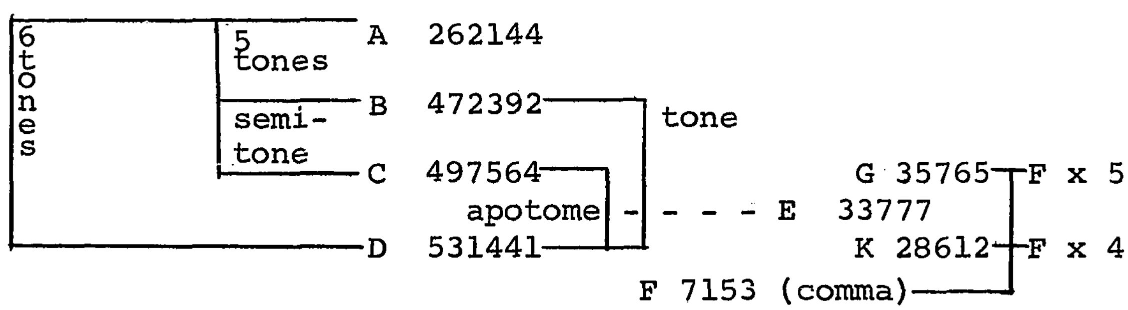
16. Book II. xxx. pp. 165-167.

at the distance of a tone from C, represented by the number 497664. But let C stand at the distance of a minor semitone from B, and thus C should be 497564. Therefore there remains an apotome proportion between C and D; for since BD is a tone, if I subtract the minor semitone BC from it, there remains CD, which we have above said to be an apotome. Between D and C there is a difference of 33777; let term E represent this number. But the difference of the comma was 7153, so let this be term F. If I multiply F, the comma, by five, I produce 35765, and let this be G; if on the other hand, I multiply F by four, I produce the number K, which is 28612. Term G is larger than E, but smaller than K. But G equals the comma multiplied by five, K the comma multiplied by four. Moreover E is the difference of the apotome. Therefore it has rightly been said that the apotome is less than five commas, but larger than four.

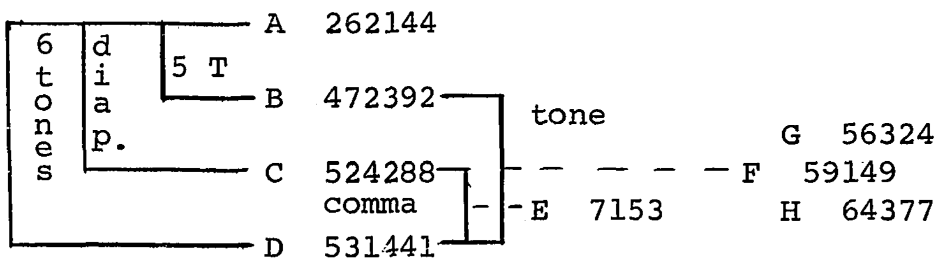


From this it is also proved that the tone is larger than eight commas, but smaller than nine. For if the minor semitone is larger than three, but less than four commas, and the apotome is larger than four but less than five, then the minor semitone joined to the major semitone, that is the apotome, will be larger than eight commas but smaller than nine. Now a minor semitone plus an apotome makes a tone. Therefore a tone is larger than eight commas, but smaller than nine.

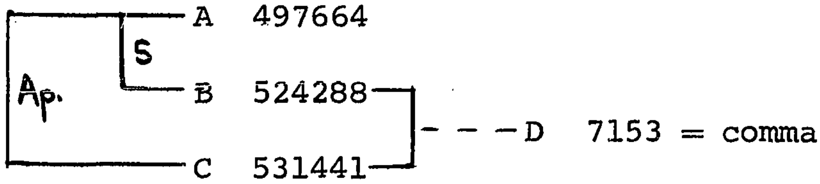
XVI. <u>Proof of what was just said through numbers</u>.

But although the relationship of the tone to commas was just demonstrated by one type of reasoning, nevertheless we should not now, like lazy ones, tire from demonstrating by means of the tone itself that it holds this same relationship to commas. Thus let A be 262144, and B, standing at the distance of five tones, 472392. C is in the number 524288, standing at the distance of a diapason consonance from A. D is 531441, differing from A by the interval of six tones. Therefore the distance between D and C is a comma, of course the difference between six tones and a diapason consonance. Let this difference, 7153, be represented by E. Moreover D is separated from B by a whole tone, that is, the difference between five and six tones;

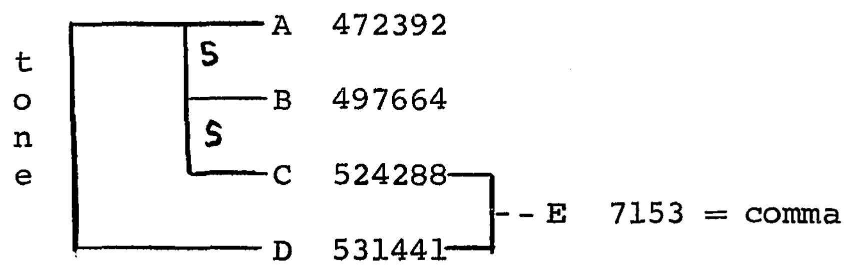
let F signify this difference: 59049. But H surpasses the number F, whereas term G is surpassed by F; now F represents the difference of a tone, H the comma multiplied by nine, and G the comma multiplied by eight. Therefore it has been demonstrated that the tone is less than nine commas, but more than eight.



Although it may have been proved in these premises that the distance between a major and minor semitone is a comma, nevertheless I will likewise demonstrate this in itself through fitting numbers in the following demonstration. Let the number A be 497664, and let the number B, standing at a minor semitone from A, be 524288, as was just described above. Finally let term C stand at the distance of an apotome from A, located in the number 531441. Since AB is a minor semitone, and AC a major semitone, we should thus seek the difference between C and B. It is 7153, and let this be D. But this number was just demonstrated to hold the comma. Therefore the comma forms the difference between a major and minor semitone.



Again I propose to demonstrate that the tone is exactly one comma larger than two minor semitones. Let A be 472392, and let a tone be extended from A, and let this be B, 497664, and likewise extend a semitone from B, and let this be C, 524288. Since AD is a tone, while AC is two minor semitones, let us see what constitutes the difference between D and C. Let this be term E, 7153. Thus it is demonstrated that the tone is a comma larger than two minor semitones.



But since everything which we propose to prove by appropriate argument has been demonstrated, we must now go to that which remains to be discussed in The Principles of

Music, namely the division of the monochord. Since this object requires a more extended treatment, we have decided that the discussion should be transferred to the next book.

BOOK IV

I. <u>That the differences in sound consist in quantity</u>.

Although we have explained in the discussions of the last book everything which had to be demonstrated, nevertheless, in order to refresh our memory, it would do no harm to present certain things again briefly with some difference in the treatment. In this way we should come to the division of the ruler with these things reinforced in our minds, and thus our entire attention can be given to the monochord.

<u>If</u>[1] <u>all things were still, no sound would strike our ears; for with all motion stopped, there would be no striking together of objects. Thus a certain striking is necessary for the existence of sound. But if there be a stroke, a motion must necessarily have preceded it.</u>

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1. This brief recapitulation of the previous books which Boethius presents in the first two chapters of this book is a somewhat expanded translation of the introduction and first nine propositions of the Euclidian <u>Sectio Canonis</u> (JanS. 148-158). The more or less directly translated passages will be underlined, while the additions will not. This concluding paragraph of Book IV, Chapter I, presents the introductory passage of the treatise (JanS. 148-149).

Therefore, if there be sound, there must necessarily be motion. But all motion partakes at one time in speed and at another in slowness. If the motion be slow at the moment of impact, a low sound is made. For as slow motion is close to motionlessness, so is a low sound near to silence. On the other hand, a fast motion produces a high sound. Moreover a low sound rises toward the middle range with any increase in motion, while a high sound descends toward the middle range with any decrease in motion. Whence it follows that all sound is seen to consist of certain parts.[2] *But every conjunction of parts is put together through a certain proportion. Therefore the conjunction of sounds is organized and regulated through proportions. Proportions are primarily considered in numbers. A simple numerical proportion is found either in multiple, superparticular, or superpartient relationships. Consonant or dissonant sounds are clearly heard according to multiple or superparticular proportions. Consonant*

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2. That is, parts or quantities of motion. Boethius' translation of πυκνος and αραιος as "fast" and "slow" (*velox* and *tardus*) rather than "dense" and "sparse" (*densus* or *frequens* and *rarus*) somewhat obscures the logic of this passage.

sounds are those that sound sweet and intermingled when struck at the same time, while dissonant sounds are those which do not sound sweet and intermingled when struck at the same time.[3] Having thus pre-arranged these things, we should discuss proportions a little.

II. Diverse speculations concerning intervals.

If a multiple interval is multiplied by two, the resulting interval of this multiplication will be a multiple. Let the multiple interval BC be assumed, and B is the multiple of C; moreover, as C is related to B, so let B be related to D. Thus since B is the multiple of C, C divides the term B either two or three or some other number of times. But as C is related to B, so is B related to D, and thus term B divides D. Therefore, term C also divides D. Thus D is a multiple of C,[4] and the interval DC is made by bringing together and uniting the interval BC twice with itself, or multiplying it by

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3. "Simul pulsae" is added to these definitions by Boethius or his source. See Commentary, Chapter IV, pp. 440-443 concerning Boethius' concept of consonance.

4. Sectio Canonis, Proposition 1 (JanS. 150).

two. The same is also proved in numbers. Let B and C be a duple, such as 2:1. As C is to B, let B be to D; thus D will be 4. Thus since B:C is the multiple 2:1, the multiple D:C is the multiple 4:1. For 4:1 is a quadruple interval, or its half--the interval BC--multiplied by two.

D	B	C
4	2	1

If an interval multiplied by two produces a multiple, then the multiplied interval will have been a multiple. Let the interval BC be assumed, and let B:C equal D:B. Moreover, let D:C be a multiple. I say that B:C is a multiple, for since D:C is a multiple, C divides D. Now it was shown that if numbers were in proportion and the first was naturally related to the last, if the first term divides the last, it likewise divides the middle.[5] *Thus C divides B, and B is a multiple of C.*[6] If this likewise be considered in numbers, C should be unity, while D should be twice the proportion B:C, that is 4, and a multiple of C; for it is the quadruple.

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5. Cf. Euclid *Elements* viii. 7.
6. *Sectio Canonis*, Proposition 2 (JanS. 151).

Thus since this quadruple is produced from twice the proportion B:C, B:C will be its half. Therefore the proportion B:C is a duple proportion. But a duple proportion is multiple. Thus the proportion B:C is multiple.

D	B	C
4	2	1

In the case of superparticular intervals, neither one nor more middle numbers will be able to come proportionally between the numbers of the proportions. Let the superparticular proportion B:C be assumed, and let DF and G be the smallest numbers in the same proportion. Since DF and G are the smallest numbers in this proportion, they are also the first numbers in this proportion. Thus unity alone measures them. Thus if G be subtracted from DF, D remains, and it is the common denominator of both numbers. This will therefore be unity. For this reason no number which is less than FD or more than G will fall between DF and G. Unity alone is between these numbers. Now in superparticular proportions, the same quantity proportionally comes between larger numbers in a certain proportion as that which comes between the smallest numbers in the same proportion.[7] But nothing can come between FD and G, the minimum terms of this proportion, and thus nothing can proportionally come between B and C.[8] In order to consider this in numbers, assume any superparticular proportion, for example the sesquialter, and let it be the numbers 10 and 15. The minimum numbers in this proportion are 2 and 3. I subtract 2 from 3, and unity is left; and it measures both numbers. Thus there will be no number between 2 and 3 which is larger than 2 and smaller than 3, unless unity be divided, which is inconsistent. Therefore, no number will come between 10 and 15 which holds the same proportion to 10 as 15 does to this number placed between them.

DF	G
3	2
B	C
15	10

If an interval which is not a multiple be multiplied by 2, the result is neither superparticular nor multiple. Let the non-multiple interval B:C be assumed, and let B be made related to a D in the same proportion as C is to B.

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7. Cf. Euclid *Elements* viii. 8.
8. *Sectio Canonis*, Proposition 3 (JanS. 152-53).

I assert that D is neither a multiple nor a superparticular of C. But first, if it can be done, let D be a multiple of C. Since it is known that if an interval be multiplied by 2 and it creates a multiple interval, that which was multiplied by 2 must be a multiple interval,[9] *then BC will be multiple. But this was not thusly set forth; so D will not be a multiple of C. Indeed neither is it a superparticular; for no middle term can proportionally come between the numbers of a superparticular proportion.*[10] *The term B falls proportionally between terms D and C, for as C is to B, so B is to D. Thus it will be impossible for D to be either a multiple or superparticular of C,*[11] and this is what was necessary to prove. In numbers, let the non-multiple interval be 6:4, and as 4 is related to 6, so let 6 be placed in relation to some other number. This other number will thus be 9, which is neither a multiple nor a superparticular of 4.

D	B	C
9	6	4

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9. See above proof (p. 213) from *Sec. Can.*, Prop. 1.
10. See above proof (p. 215) from *Sec. Can.*, Prop. 3.
11. *Sectio Canonis*, Proposition 4 (JanS. 153-54).

If an interval be multiplied by two and the product of this multiplication is not a multiple, then the multiplied interval itself is not a multiple. Let the interval BC be assumed, and as C is related to B, so let B be related to D, and D should not be a multiple of C. I assert that B will not be a multiple of C; for if it is, D must also be a multiple of C.[12] *But this is not the case. Therefore B will not be a multiple of C.*[13]

The duple interval is combined from the largest two superparticular proportions, the sesquialter and sesquitertian. Thus let A be a sesquialter of B, and B a sesquitertian of C. I say that A is a duple of C. Since A is a sesquialter of B, A contains the total B plus one of its halves. Therefore two A's are equal to three B's. Again, since B is a sesquitertian of C, B contains C plus a third part of it. Thus three B's equal four C's. However, three B's were equal to two A's. Therefore two A's are equal to four C's. Therefore one A equals two C's. And thus A is the duple of C.[14] In numbers let the sesquialter be 12:8, and

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12. See above proof (p.213) from *Sec. Can.*, Prop. 1.
13. *Sectio Canonis*, Proposition 5 (JanS. 154).
14. *Sectio Canonis*, Proposition 6, second demonstration (JanS. 155).

the sesquitertian 8:6. Therefore 12:6 forms the duple proportion.

A	B	C
12	8	6

The triple interval is born from a duple plus a sesquialter proportion. For let A be the duple of B, and B the sesquialter of C. I say that A is the triple of C. For since A is the duple of B, A is equal to two B's. Moreover, since B is the sesquialter of C, B contains the whole of C plus its half part. Thus two B's are equal to three C's. But two B's were equal to one A. Thus one A is equal to three C's. Therefore C is the triple of one A.[15] In numbers let the duple be 6:3, and sesquialter 3:2; thus 6:2 forms the triple proportion.

A	B	C
6	3	2

If a sesquitertian interval be subtracted from a sesquialter, the remainder will be a sesquioctave interval. Thus let A be the sesquialter of B, and C the sesquitertian of B. I affirm that A is the sesquioctave of C. For since A:B is a sesquialter, A contains B plus its half. Thus 8

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15. Ibid., Proposition 7 (JanS. 156).

A's are equal to 12 B's. Moreover, since C:B is a sesquitertian, C contains B plus one-third of it. Thus 9 C's are equal to 12 B's. However, 12 B's were equal to 8 A's. Thus 8 A's are equal to 9 C's. Therefore A is equal to C plus an eighth part of it [C]. Therefore A is the sesquioctave of C.[16] In numbers let the sesquialter interval be 9:6, and the sesquitertian 8:6. Therefore 9:8 is the sesquioctave proportion.

A	B	C
9	8	6

Six sesquioctave proportions are more than one duple interval. Let the number A be assumed, to which C in turn is a sesquioctave, to which D is a sesquioctave, to which F is a sesquioctave, to which G is a sesquioctave, to which finally K is a sesquioctave. But let this matter be proved in an arithmetical manner, and let A, B, C, D, F, G, and K be numbers. Thus let A equal 262144, to which is related

the sesquioctave B, 294912

the sesquioctave of which is C, 331776

the sesquioctave of which is D, 371776

16. Ibid., Proposition 8 (JanS. 156-57).

the sesquioctave of which is F, 419904

the sesquioctave of which is G, 472392

the sesquioctave of which is K, 531441.

Thus K, 531441, is larger than the duple of A (2x262144),[17] and therefore six sesquioctave proportions are larger than one duple interval.

III. Naming of the notes through Greek and Latin letters.

Since we now have to divide a string by means of a ruler in accordance with the consonances already discussed, and since this partition will produce the necessary sounds in the three genera of song, there remains for us now to explain the musical notes, so that, when we indicate the divided line with these same marks, the name of every single one will be very easily called to mind.

For the sake of an abbreviated form of writing, the ancient musicians did not always consider the complete name of a note necessary, and they figured out certain signs by which the names of the notes were denoted. Moreover, they distributed these signs throughout the genera and modes. By means of this brevity they reached the point that if a

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17. *Ibid.*, Proposition 9 (JanS. 167-170); cf. Book II. xxxi. pp. 167-170.

musician wanted to write down some melody over a verse composed in the rhythm of a meter, he wrote these signs for the respective sounds. Thus they found that in this remarkable way not only the words of a song written out with letters but also the melody itself signified by these signs would be preserved in the memory and for posterity.

But for the present we will take the Lydian mode from all these modes and present its signs or notes through the three genera. We will do the same for the remaining modes at another time. But surely if I describe this arrangement of the notes using Greek letters, the reader should not be disturbed by the novelty. For the total arrangement of the notes is organized around Greek letters, and sometimes they are altered letters, whereas other times they are turned in various positions. But we guard against changing anything handed down from the authority of antiquity. Thus the notes or signs given first and above in the arrangement will be those for music with words, whereas the second and lower notes are for instruments.[18]

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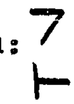
18. Boethius' source for this exposition of Greek notation is closely related to the Isagoge Musica of Alypius (JanS. 367-406). This fact is evidenced more by Boethius' textual description of the notes than by the letters

The proslambanomenos, which can be called *adquisitus*, is an imcomplete zeta and a lying tau: 

The hypate hypaton, which is the *principalis principilium*, is a backward gamma and a normal gamma: 

The parhypate hypaton, that is the *subprincipalis principalium*, is an incomplete beta and an upside-down gamma: 

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themselves. The medieval scribes copying this work have passed down so many minor inconsistencies with regard to the various Greek signs that it would be practically impossible to arrive at Boethius' original rendering of these letters. Nevertheless there remains a surprising similarity between notational signs in MSS of this work and those of the Alypius treatise. It is also significant to note that the Lydian mode, presented here as an introduction to Greek notation, is the first mode presented by Alypius. This text, however, here integrates the descriptions of the chromatic and enharmonic genera with that of the diatonic genus, whereas Alypius treats each separately. Concerning probable source of this theory, cf. Commentary, Chapter I, pp. 357-359. The Latin names for the notes were probably taken from Albinus (cf. Book I, xxvi).

Insofar as the following descriptions and notational signs are consistent with those of Alypius, no further note seems necessary; however, each inconsistency between Boethius and Alypius will be indicated by an additional note.

The enharmonic hypaton, or *principalium enharmonios*, is an upside-down alpha and a backward gamma with a little line on the back[19]: ∀ ⅂

The chromatic hypaton, or *principalium chromatica*, is an upside-down alpha with a line and a backward gamma with two lines[20]: ∀ ⅂

The diatonic hypaton, or *principalium extenta*, is a Greek phi and a digamma: Φ Ϝ

The hypate meson, or the *principalium mediarum*, is a sigma and a sigma: C C

The parhypate meson, or *subprincipalis mediarum*, is a rho and a lying down sigma: P ∪

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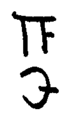
19. Alypius describes the instrumental enharmonic hypaton as an "upside-down digamma" rather than a "backward gamma with a little line"; Ⅎ rather than ⅂ .

20. The Alypius treatise places the line differently on the upside-down gamma: ∀ rather than ∀ ; moreover, Alypius describes the instrumental chromatic hypaton as an "upside-down digamma with a line" rather than a "backward gamma with two lines"; Ⅎ rather than ⅂ .

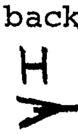
The enharmonic meson, or *mediarum enharmonios*, is a Greek pi and a backward sigma: 

The chromatic meson, or *mediarum chromatica*, is a Greek pi with a line and a backwards sigma with a line through the middle[21]: 

The diatonic meson, or *mediarum extenta*, is a Greek mu and an imcomplete Greek pi: 

The mese, or *media*, is an iota and a tilted lambda: 

The trite synemmenon, or *tertia coniunctarum*, is a theta and a reclining lambda: 

The enharmonic synemmenon, or *coniunctarum enharmonios*, is a Greek eta and a backward lambda leaning over with a line through the middle[22]: 

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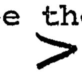
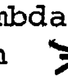
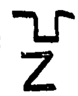
21. The Alypius treatise places the line differently on the vocal pi:  rather than  .

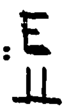
22. Alypius does not describe the leaning lambda as having a line through the middle:  rather than  .

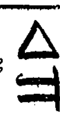
The chromatic synemmenon, or *coniunctarum chromatica*, is a Greek eta with a line and a backward lambda with a line[23]: 

The diatonic synemmenon, or *coniunctarum extenta*, is a gamma and nu: 

The nete synemmenon, or *ultima coniunctarum*, is an upside-down square  and a zeta: 

The paramese, or *submedia*, is a zeta and a Greek pi on its side: 

The trite diezeugmenon, or *tertia divisarum*, is a square eta and an upside-down pi: 

The enharmonic diezeugmenon, or *divisarum enharmonios*, is a delta and a Greek pi lying on its side backward: 

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23. The descriptions of the chromatic synemmenon read similarly in Boethius and Alypius; however, the lines are placed differently in the Alypius treatise:  and  rather than  and .

The chromatic diezeugmenon, or <u>divisarum chromatica</u>, is a delta with a line and a Greek pi lying on its side backward with an angular line[24]: 

The diatonic diezeugmenon, or <u>divisarum diatonos</u>, is a square upside-down ω and a zeta: 

The nete diezeugmenon, or <u>ultima divisarum</u>, is a tilted phi and an upside-down shortened nu[25]: 

The trite hyperboleon, or <u>tertia excellentium</u>, is a downward y leaning toward the right and half of an alpha leaning upward toward the left[26]: 

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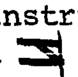
24. Alypius (JanS. 384 describes the vocal chromatic diezeugmenon as a delta with a dot rather than with a line:  rather than  ; moreover, the line on the instrumental pi appears slightly different:  rather than  .

25. The symbols for this note are the same in Alypius and Boethius, but Alypius (JanS. 369) describes the instrumental note as a shortened eta poorly made (ἦτα ἀμελητικὸν καθειλκυσμένον), which also describes the form  .

26. Some confusion regarding the diatonic trite hyperboleon appears in both Boethius and Alypius. The symbol for the vocal trite hyperboleon (λ) is the same in both treatises, whereas the descriptions are in conflict; for Alypius (JanS. 369) describes it as merely a downward tilted epsilon (ν κάτω νεῦον), whereas Boethius adds the phrase "leaning toward the right" (y <u>deorsum respiciens dextrum</u>). Based on the symbol itself, it appears that

The enharmonic hyperboleon, or <u>excellentium enharmonios</u>, is an inverted tau and half of an alpha leaning toward the right[27]: ⊥ Y

The chromatic hyperboleon, or <u>excellentium chromatica</u>, is an inverted tau with a line and half of an alpha leaning to the right with a line behind[28]: ⊥ Y

Boethius' additional phrase is a mistake.

The descriptions of the instrumental trite hyperboleon, on the other hand, are word for word the same (ἡμίαλφα ἀριστερὸν ἄνω νεῦον and semialpha sinistrum sursum respiciens), but the symbol given by Alypius is ⅄ rather than Y. It would seem that Y fits the words ἄνω and sursum better than ⅄. Moreover, Alypius' description of the trite hyperboleon in the chromatic genus (JanS. 384), the exact same pitch as this one in the diatonic genus, agrees word for word with this description, but the symbols given are ⅄ and Y, the same as for the note under consideration here. Thus it would seem that Y is a more consistent symbol for this note than the symbol given by Alypius in the diatonic genus. Cf. JanS. Prolegomena, p. 366.

27. The Alypius and Boethius descriptions agree concerning the vocal enharmonic hyperboleon. However, Boethius' symbol for the instrumental notation of this note is Y rather than the ⅄ of Alypius. Moreover the word ἄνω (upward) appears in Alypius' description of the note (JanS. 399), which is more consistent with Boethius' Y than his own ⅄.

28. Alypius (JanS. 384) describes these symbols as having dots rather than lines: ⊥' and Y' rather than ⊥ and Y. Note that the form of this half alpha agrees with the half alpha form used by Boethius for the preceding note.

The diatonic hyperboleon, or *excellentium extenta*, is a Greek mu with a dot and an incomplete pi with a dot: 

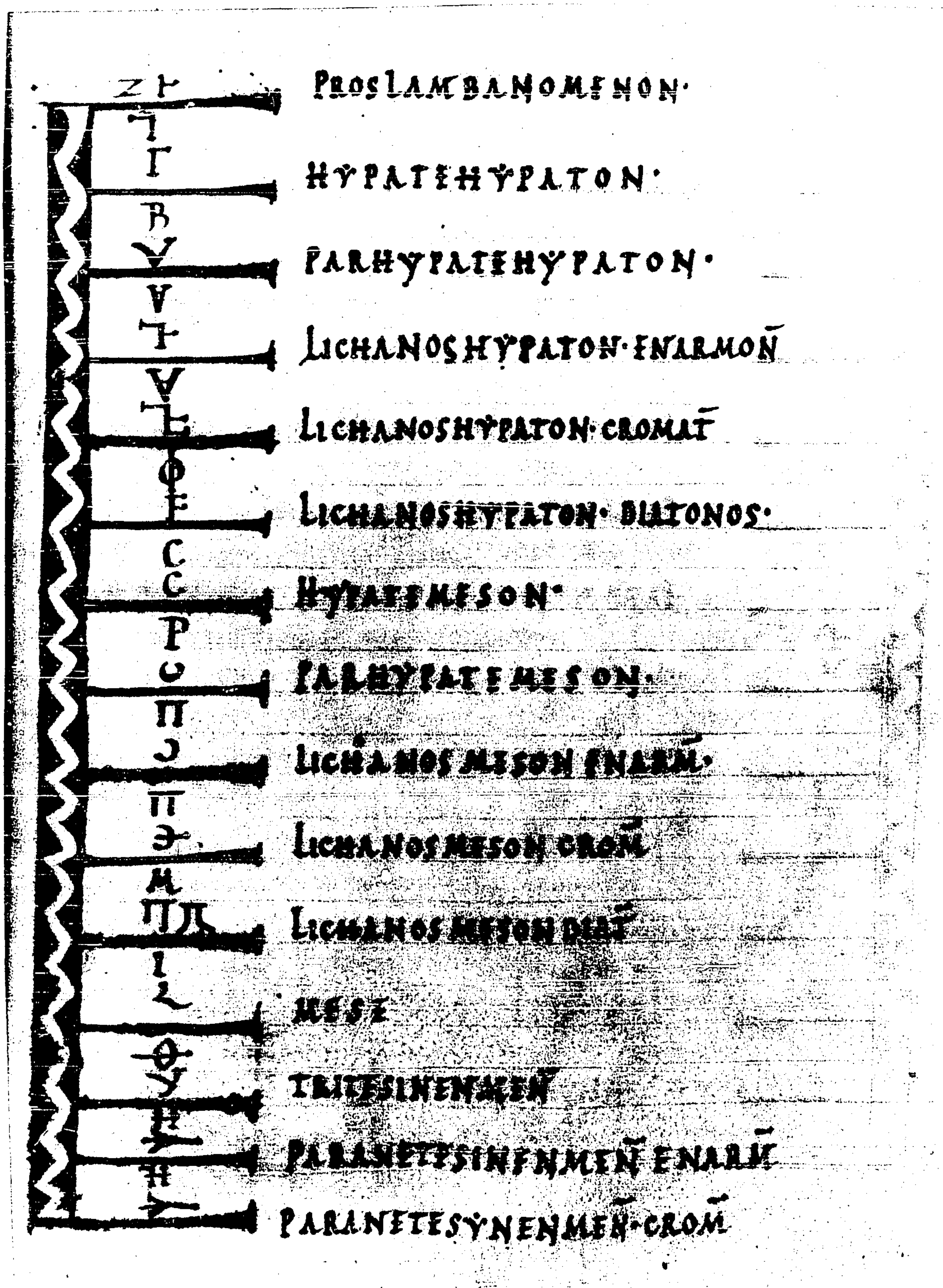
The nete hyperboleon is an iota with a dot and a tilted lambda with a dot: 

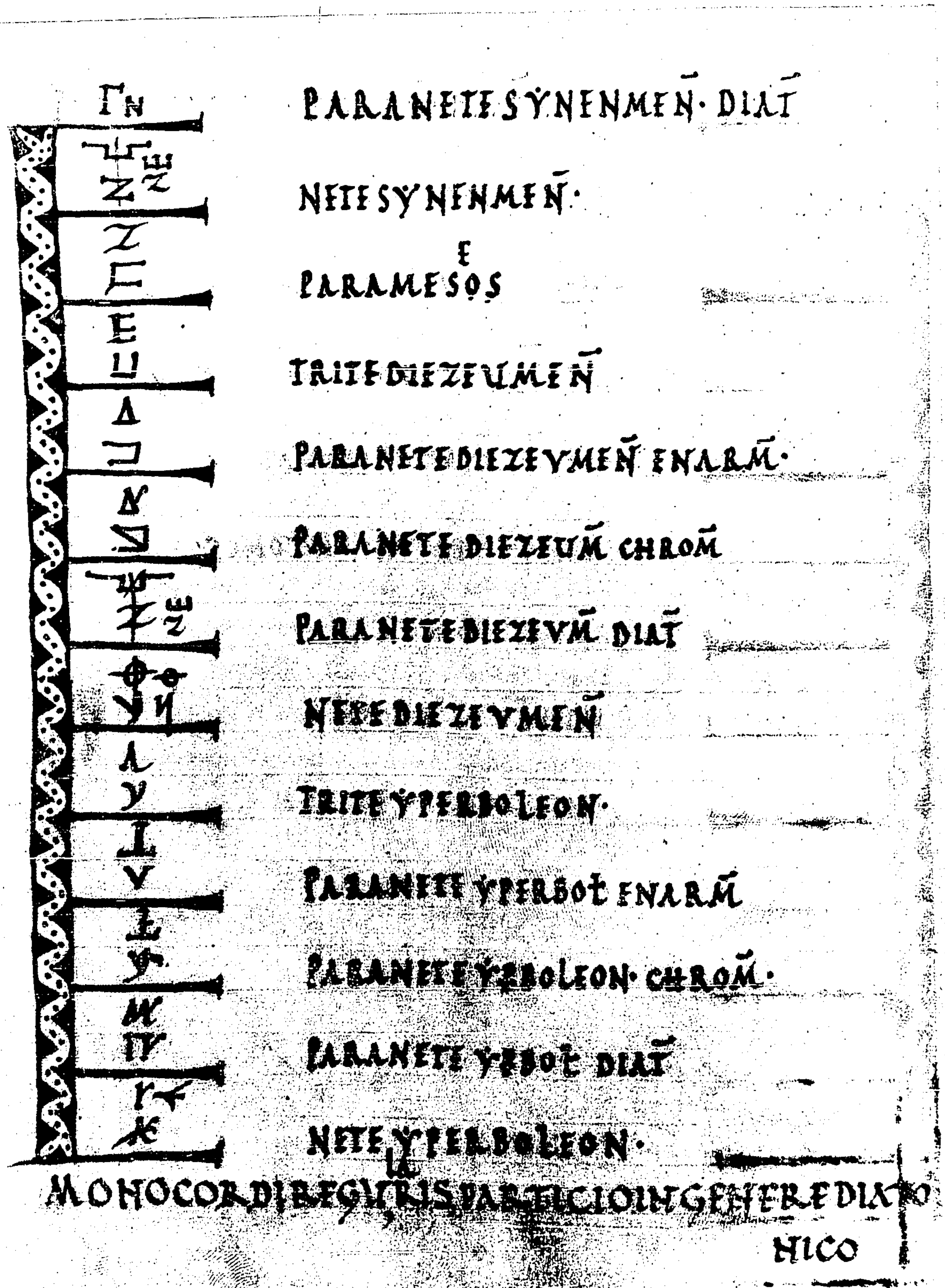
IV. *Arrangement of the musical notes according to their appropriate sounds in the three genera.*

(See plates from Cologne, Stadtarchiv, W. 331, 147-148, on the following two pages).

V. *Division of the monochord ruler into the three genera.*

But now it is time to approach the division of the monochord ruler. Yet we should first make it clear that whether the division we are about to describe is considered in the measurement of the string or in numbers and their proportions, a longer length of string and a larger plurality of numbers make lower sounds. But if the string be shortened and the plurality in the numbers be lessened, it is necessary that higher sounds be produced. From this comparison we see that a higher or lower sound is found


PROSLAMBANOMENON·
HYPATEHYPATON·
PARHYPATEHYPATON·
LICHANOSHYPATON·ENARMON
LICHANOSHYPATON·CROMAT
LICHANOSHYPATON· DIATONOS·
HYPATEMESON·
PARHYPATEMESON·
LICHANOS MESON ENARM·
LICHANOS MESON CROM
LICHANOS MESON DIAT
MESE
TRITESINENMEN
PARANETESINENMEN ENARM
PARANETESYNENMEN·CROM


PARANETE SYNENMEN· DIAT
NETE SYNENMEN·
PARAMESOS
TRITE DIEZEUMEN
PARANETE DIEZEYMEN ENARM·
PARANETE DIEZEUM CHROM
PARANETE DIEZEVM DIAT
NETE DIEZEVMEN
TRITE YPERBOLEON·
PARANETE YPERBOL ENARM
PARANETE YPERBOLEON· CHROM·
PARANETE YPERBOL DIAT
NETE YPERBOLEON·
MONOCORDI REGULIS PARTICIO IN GENERE DIATO
NICO

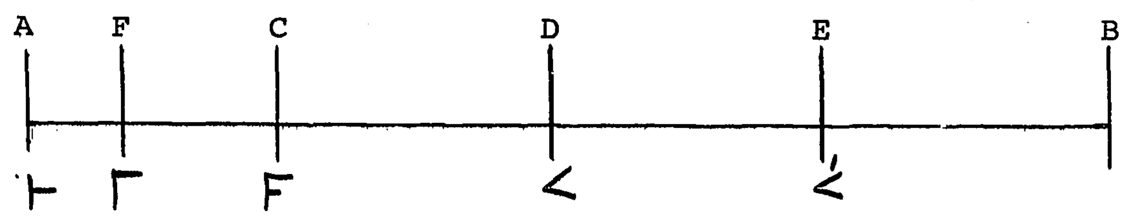
respectively according to whether it is designated by a smaller number and a shorter string or by a larger number and a longer string.

But let the reader not be perturbed because we have been signifying the spaces toward the higher notes with larger numbers whereas those toward the lower notes have been signified with lower numbers; for increasing and intensifying does bring about a higher sound, whereas remitting and releasing does create a lower sound. But in this context we were discussing merely the spaces of the proportions, and this had nothing to do with a property of highness or lowness; thus we ascended in pitch with ever larger numbers and we descended with ever smaller numbers. But here where we are measuring the spaces of strings and the sounds themselves, it is necessary to follow the nature of things. Thus larger numbers are given to the longer lengths of string which hold low sounds, and smaller numbers are given to the shorter lengths which hold high sounds.

Let a string AB be strung under tension, and let there be a ruler equal to this string which is divided according to the proposed partitions. Thus when the ruler is placed next to the string, the same divisions we have designated on it will also be designated on the string. So in effect we are dividing the string itself rather than partitioning a ruler. Thus AB is divided into four parts with three points: C, D, and E. Therefore the total AB will be the duple of DB and AD, and AD and DB will each be duples of AC, CD, DE, and EB. Since AB is the lowest, it is the proslambanomenos, whereas DB is the mese; for it is half of the total length. And just as AB has the duple, or twice the space of DB, so DB is the duple or twice as high a pitch as AB; for, as just discussed, the order of space and pitch is always reversed. Inasmuch as a note is a higher pitch, it will be proportionally smaller in space. Thus EB will be the nete hyperboleon, for since EB is half the quantity of DB, it is higher than DB by a duple proportion. Moreover, since EB is a fourth part of the space AB, it will be higher than AB by a quadruple proportion. Thus as has been discussed, the nete hyperboleon will be higher than the mese by a duple proportion, whereas the mese is higher than the proslambanomenos by a duple proportion. The hyperboleon is higher than the proslambanomenos by a quadruple proportion. Thus the proslambanomenos to the mese will resound a diapason consonance, as will the mese to the nete hyperboleon. But the proslambanomenos to the nete hyperboleon resounds a bisdiapason consonance.

Again since AC, CD, DE, and EB are equal parts, AB equals four of these parts, but CB equals only three; therefore AB is the sesquitertian of CB. Moreover, since CB is made of three equal parts, while EB is only one of these, CB is separated from EB by a triple proportion. CB will thus be the diatonic lichanos hypaton, and the proslambanomenos will resound a diatessaron consonance with the diatonic lichanos hypaton. On the other hand, the diatonic lichanos hypaton will resound a diapente consonance with the mese. This same note will resound a diapason and diapente with the nete hyperboleon.

If I subtract a ninth part from AB, the ninth part being AF, FB will be the other eight parts. Thus FB will be the hypate hypaton, to which AB, the proslambanomenos, holds a sesquioctave proportion, or in music, a tone.



The signs written beneath the line in this description are from the table in which we presented the signs for the notes. It would be too long to write out their names.

If we likewise divide AB into three parts, AG will be one third part, and GB will be two of these same third parts. Therefore the proslambanomenos AB will resound a diapente consonance, consisting of the sesquialter proportion, in relation to GB, which is the hypate meson. But CB to GB will be a sesquioctave proportion and will hold a tone; for the interval between the diatonic lichanos hypaton--CB--and the hypate meson--GB--is a tone. Moreover, the proslambanomenos AB to the diatonic lichanos hypaton holds a diatessaron consonance, whereas the proslambanomenos AB to the hypate meson GB holds a diapente consonance. Likewise the diatonic lichanos hypaton to the mese, that is CB:DB, holds a diapente consonance, whereas the hypate meson to the mese, that is GB:DB, holds a diatessaron consonance. Therefore the diatonic lichanos hypaton CB stands at the distance of a tone from the hypate meson GB.

If I take a fourth part of CB, it will make CK. Thus CB to CK holds a sesquitertian proportion. But KB stands at the distance of a sesquioctave from DB. Thus

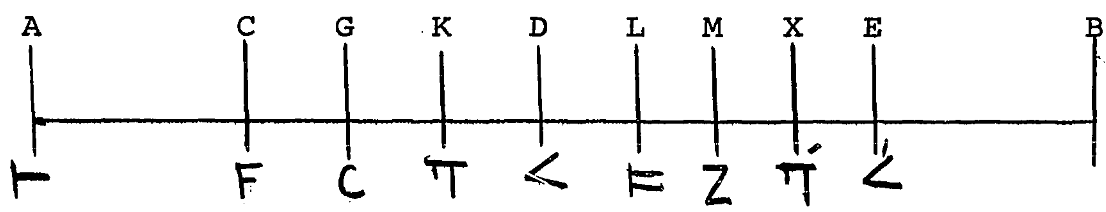
KB will be the diatonic meson, and CB, the diatonic lichanos hypaton, to KB, the diatonic meson, will contain a diatessaron consonance.

Moreover, if I take a ninth part of DB, DL will be made, and LB will thus be the paramese.

If I take a fourth part of DB, it will be DM, and MB will be the nete synemmenon.

If I take a third part of DB, it will be DN, and thus NB will be the nete diezeugmenon.

If KB should be divided into two equal parts, it will be KX, and XB will be the paranete hyperboleon.



VI. <u>The division of the nete hyperboleon monochord in the three genera</u>.

This description of the diatonic genus was given in the principle and more simple mode called Lydian. We should not at this time discuss the other modes. But so that the description may run at the same time through the three genera, and so that the exact numerical plurality may be given in all cases, even to the proportions of the tones and diesis, the numbers which will allow all this to be fulfilled have been thought out. Thus the largest number is given to the proslambanomenos, and this number should be 9216, while the smallest number should be 2304. Thus we proceed from the lowest number and we label all the notes not only with their names, but with letters as well. But since the partition must be made in all three genera, and the number of notes exceeds the number of letters, when we have come to Z and run short of letters, we will use double letters to describe the remaining notes, for example AA, BB, and CC.

Thus let the first and largest number holding the place of the proslambanomenos be 9216, and let the whole string be extended from this note, signified by A, up to the note signified by LL.[29]

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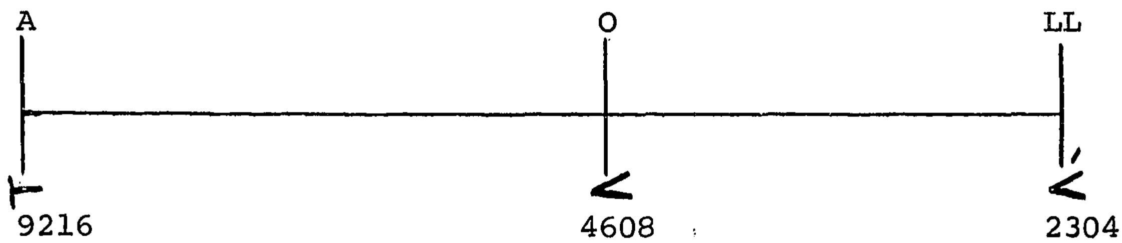
29. The text is here a little ambiguous; for the string will in fact have to extend beyond, not merely up to, the note signified by LL if the pitch LL is to have any vibrating length. Since LL becomes the nete hyperboleon (<'), it is equivalent to the E of the immediately preceding diagram (p. 236). Therefore, just as in the preceding

I divide this A, the proslambanomenos 9216, in half at O, so that the whole A is the duple of O. Likewise let O be the duple of LL. A will thus be the proslambanomenos, O, the mese, and LL, the nete hyperboleon. A will have the number 9216, O half of that, 4608, so that the mese may harmonize with the proslambanomenos according to the diapason consonance. LL will be half of the mese, thus 2304, so that the proslambanomenos with the nete hyperboleon will render a bisdiapason consonance according to a quadruple proportion.



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diagram, exactly one-fourth of the string must continue past LL if these divisions are to have any validity in sound. The text, for the sake of clarity, should have begun this section by dividing the total string A:MM into four equal parts using three points: E (the future diatonic lichanos hypaton), O, and LL. Thus O (the mese) to MM will be twice the length of LL (the nete hyperboleon) to MM, forming a duple proportion and sounding a diapason. Thus he could have proceeded with the numbers in the next paragraph, giving A 9216, O 4608, and LL 2304. Thus MM would equal the origin of numerosity (or some such term) from which numerical plurality takes its beginning. For a proportional division of the monochord ruler according to the following chapters see Appendix.

If I should take an eighth part of 2304, that is 288, and add it to the same, it will give me 2592, and this will be KK. KK is thus the paranete hyperboleon, standing at the distance of a tone from the nete hyperboleon. If I likewise take an eighth part of KK, that is 2592, thus 324, and I add this to KK, the number 2916 will be made, and I have produced FF, the diatonic trite hyperboleon in the diatonic genus. The number 2916 stands at the distance of a tone from the diatonic paranete hyperboleon, but two tones from the nete hyperboleon. This same FF will be the chromatic trite hyperboleon in the chromatic genus but the enharmonic paranete hyperboleon in the enharmonic genus. One will recognize why this is evident after we have described the first three tetrachords of the three genera beginning from the nete hyperboleon.

Since if I subtract two sesquioctave proportions from a sesquitertian it will leave me a semitone, I now take a third of LL, the nete hyperboleon, and this is 768. I add this to LL, and thus make 3072. This number is the nete diezeugmenon, DD, which stands a minor semitone away from the trite hyperboleon. For since the nete diezeugmenon holds a diatessaron consonance in relation to the nete hyperboleon, whereas the distance between the nete hyperboleon and the diatonic hyperboleon is two tones, that which stands between the nete diezeugmenon and the trite hyperboleon must be a minor semitone.

Thus we have explained the division of the hyperboleon tetrachord in the diatonic genus, and now the tetrachords of the enharmonic and chromatic genera are added as follows: since the paranete hyperboleon stands at the distance of a tone from the nete hyperboleon in the diatonic genus, but at the distance of three semitones in the chromatic genus or two tones in the enharmonic genus, if we would take half of the distance between the paranete hyperboleon and the nete hyperboleon, and then add this to the paranete hyperboleon of the diatonic genus, we would have the number standing at the distance of three semitones from the nete hyperboleon. This number will be the paranete hyperboleon in the chromatic genus. Thus I subtract the nete hyperboleon, 2304, from the paranete hyperboleon, 2592, and I get a remainder of 288; I divide this and the result is 144; I add this to 2592, and I have made HH, 2736. This number will be the chromatic paranete hyperboleon.

Since the trite hyperboleon, either diatonic or chromatic, is two tones from the nete hyperboleon, and the paranete hyperboleon in the enharmonic genus is also two tones from the nete hyperboleon, the same number will be the paranete hyperboleon in the enharmonic genus that is the trite hyperboleon in the diatonic and chromatic genera. But since the trite hyperboleon of the diatonic and chromatic genera stands a minor semitone from the nete diezeugmenon, but the enharmonic tetrachord consists of two whole tones and a diesis plus a diesis, each of which are half of a minor semitone, I take this distance which lies between the nete diezeugmenon and enharmonic paranete hyperboleon; since the nete diezeugmenon is 3072 and the enharmonic paranete hyperboleon is 2916, the distance between them will be 156. I take half of this number, which is 78, and I add this to 2916, and this produces 2994. This number will be EE, the enharmonic trite hyperboleon.

Thus the hyperboleon tetrachord has been described according to the three genera, the outline of which we have added below:

Diatonic	Chromatic	Enharmonic
2304 LL	2304 LL	2304 LL
$\overset{O}{T}$ 2592 KK	$\overset{S}{T}\overset{S}{T}\overset{S}{T}$ 2736 HH	$\overset{O}{T}\overset{O}{T}$
$\overset{O}{T}$ 2916 FF	$\overset{S}{T}$ 2916 FF	2916 FF
$\overset{S}{T}$ 3072 DD	$\overset{S}{T}$ 3072 DD	ẟ 2994 EE ẟ 3072 DD

VII. <u>The rationale of the description presented above</u>.

Thus by such reasoning three tetrachords have been described; for every tetrachord sounds a diatessaron consonance, and the nete hyperboleon and nete diezeugmenon hold a diatessaron consonance in all three genera, diatonic, chromatic, and enharmonic. A diatessaron consists of two tones and a minor semitone. In the above description the division took place in this manner. In the diatonic genus, the first genus, the paranete hyperboleon, 2592, holds the distance of a tone in relation to the nete hyperboleon, 2304. We signify this with the sign $\overset{O}{T}$. Likewise the diatonic trite hyperboleon, 2916, holds the difference of a tone in relation to the paranete hyperboleon, 2592, and this distance is again signified by $\overset{O}{T}$. But the nete diezeugmenon to the trite hyperboleon 3072:2916, is a semitone, which we signify by $\overset{S}{T}$. The total space between the nete diezeugmenon and the nete hyperboleon is two tones and a semitone.

But in the chromatic genus the two tones and semitone are divided in this manner. The second genus was described as follows: the chromatic paranete hyperboleon, 2736, compared to the nete hyperboleon, 2304, holds the same space as the diatonic paranete hyperboleon to the nete hyperboleon, that is a tone or two semitones (a major and a minor semitone), plus half again of this space between the diatonic paranete hyperboleon and the nete hyperboleon. In this way a half tone was made; but it is not a whole half-tone because a tone cannot be divided into two equal parts. This was demonstrated most thoroughly above. We signify this space of three semitones, that is a tone and a semitone, in this manner: $\overset{SSS}{TTT}$. Again the chromatic paranete hyperboleon to the trite hyperboleon retains the part of a tone that is a semitone. This is the space remaining from the two tones which were contained between the diatonic trite hyperboleon and the nete hyperboleon. Likewise four semitones subtracted from a whole tetrachord leave the space of a semitone. A semitone is thus contained between the nete diezeugmenon and the trite hyperboleon. Therefore this tetrachord also consists of two tones and a semitone, but it is divided into a space of three semitones and two spaces, each of which consists of one semitone.[30] These three spaces are contained between four notes.

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30. Boethius here departs from his mathmatical logic which has been perfectly consistent up to this point. The inconsistency arises in his computation of the chromatic paranete hyperboleon. This genus consists of two separate semitones and a combined space of three semitones, presented by Boethius in the numbers 3072:2916: 2736:2304. The first semitone (3072:2916) is perfectly consistent with the proven proportion of the semitone, 256:243 (Book III, i-ii, I, xvii). However the next semitone (2916:2736) was computed by subtracting 2304 from 2592, the space of a tone, dividing the remainder in half, and adding this half to 2592. Thus Boethius arrived at the number 2736. But by dividing this tone in half, two parts are left, the first of which falls into the proportion 2736: 2592. If one reduced this proportion to its smallest numbers, the result is the proportion 19:18. But Boethius does not actually use this semitone (2736:2592) as such, rather it becomes the third part of the interval consisting of three semitones (2736:2304). However, the remainder of the tone after this proportion has been subtracted becomes one of the semitones in the chromatic tetrachord. If the two semitones of this division be compared, 2916:2736:2592, it becomes evident that 2736:2592 is the smaller; for its difference is 144 whereas the difference between 2916 and 2726 is 180. Thus Boethius' minor semitone in this computation equals the proportion 19:18. But Book III, xiii (pp. 200-201) proved that a minor semitone was smaller than 19-1/2:18-1/2, whereas the proportion 19:18 is larger than 19-1/2:18-1/2. Therefore 2916:2736, the remainder (the apotome) after the proportion 19:18 is subtracted from a tone,

It is quite easy to understand this tetrachord in the enharmonic genus. From the nete hyperboleon 2304, to the enharmonic paranete hyperboleon, 2916, there is the space of two tones which we have signified by TT. Thus there remains a semitone in this tetrachord consisting of two tones and a semitone, and the semitone is contained between the nete diezeugmenon and the enharmonic hyperboleon. We have divided this into two diesis with the insertion of the enharmonic trite hyperboleon.[31] We signify this space of a diesis with the sign  .

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must be smaller than the remainder when 256:243 is subtracted from a tone. Moreover, if this remainder (2916:2736) be added to 256:243 (found in the numbers 3072:2916), the sum will be smaller than a 9:8 tone. This becomes evident when 3072, the nete diezeugmenon, is compared to 2736, the chromatic paranete hyperboleon; for these two notes are supposedly two semitones apart, but their proportion is less than 9:8. The sesquioctave relation of 2736 is 3078, and 2736:3072 is a smaller proportion than 2736:3078. Thus it seems that here Boethius definitely contradicts himself. Boethius uses the same method of computing the second semitone of the chromatic genus in the following tetrachords, and thus they all share this same inconsistency. (For further proof of this note see Commentary, Chapter III, pp. 416-418).

31. Up to this point the diesis has not been considered in numbers, but rather has been considered merely half of a minor semitone (Bk. I, xxi). However, like the tone, a semitone (256:243) cannot be divided into two equal parts. Moreover, no interval can be subtracted from another to compute a diesis as two tones can be subtracted from a

Thus the hyperboleon tetrachord has been described for us. Having completed this we should move on to the diezeugmenon tetrachord. But the present tetrachord should not be forgotten, for it can serve as an example in the division of the other tetrachords.

VIII. The division of the nete diezeugmenon notes in three genera.

If I take half of the nete diezeugmenon (3072), it will be 1536; and if I add this to the first number, it will make the number 4608, which is the mese. We have designated this note with the letter O.

If I take a third part of this nete diezeugmenon, DD, (3072), it will be 1024; and this added to DD will make

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diatessaron to compute a semitone. Consequently Boethius here merely takes the space of a semitone (3072:2916; 3072 - 2916 = 156), divides the difference in half (78), and adds this half to the lowest number of the semitone. Thus he arrives at the numbers containing a divided semitone: 3072:2994:2916. If these proportions are reduced to their lowest numbers, the result is 512:499:486. The difference between these numbers is the same (13), and thus they are an arithmeticly proportional series. In arithmetic proportionality the difference is the same but the proportions are different (Bk. II, xii). Thus there is a major diesis (499:486) and a minor diesis (512:499) just as there is a major and a minor semitone. Boethius uses this same method of computing the diesis of the following tetrachords, and thus they all share this arithmetic proportionality. (See Commentary, Chapter III, pp. 418-419).

4096, which will be called the paramese and signified with the letter X. Since the comparison of the nete diezeugmenon to the mese consists of a sesquialter proportion, 3027:4608, it will resound a diapente consonance. Moreover the same nete diezeugmenon related to the paramese, 3072:4096, will resound a diatessaron consonance, for this relation consists of a sesquitertian proportion.

If I take an eighth part of the nete diezeugmenon (3072), thus 384, and add this to the same, it will make 3456. This number will be the diatonic paranete diezeugmenon, notated by the letters CC. Thus CC stands a tone away from the nete diezeugmenon. If I likewise take an eighth part of the number 3456, thus 432, and add it to the same, it will make 3888. This number will be the diatonic trite diezeugmenon, Y. Since the nete diezeugmenon held a sesquitertian proportion in relation to the paramese, whereas the interval between the diatonic diezeugmenon and the nete diezeugmenon consists of two tones, a minor semitone will be contained between the trite diezeugmenon and paramese. Thus the diatonic genus has been completed in this tetrachord and pentachord in such a way that the tetrachord from the nete diezeugmenon to the paramese is a diatessaron consonance, whereas the pentachord from the nete diezeugmenon to the mese is a diapente consonance.

The enharmonic and chromatic genera are interwoven as follows. I take the difference of the diatonic nete and paranete diezeugmenon; 3456 - 3072 equals 384. This I divide and 192 is made. If I take this and add it to the diatonic paranete diezeugmenon (3456), it will make 3648. This will be the chromatic paranete diezeugmenon, denoted by the two letters BB. Moreover, where there was before a diatonic whole tone between the paranete diezeugmenon and the trite diezeugmenon (3888), now there is the chromatic semitone left over from that tone which was divided between the diatonic paranete diezeugmenon and the diatonic trite diezeugmenon. There is still another semitone left between the chromatic trite diezeugmenon and the paramese. This semitone is the remainder from the diatessaron consonance between the nete diezeugmenon and the paramese when two tones have been subtracted; these two tones were contained between the nete diezeugmenon and the chromatic trite diezeugmenon.

But the same note that is the diatonic trite diezeugmenon in the diatonic genus is called the chromatic trite diezeugmenon in the chromatic genus. The note stands at the distance of two tones from the nete diezeugmenon and is notated by AA. No note is placed between the nete diezeugmenon and the enharmonic paranete diezeugmenon, and thus the latter is named "para nete." But the semitone which is left between the enharmonic paranete diezeugmenon and the paramese, that is, between AA and X, is divided according to the following reasoning so that it makes two diesis: I take the difference of the enharmonic paranete diezeugminon and the paramese: 4096 - 3888 equals 208. I divide this in half and make 104. I add this to 3888 and make 3992. This will be the enharmonic trite diezeugmenon, and it is written with the letter Z.

I have added below a diagram of this tetrachord in the three genera, and I have also inserted the hyperboleon tetrachord discussed above. Thus there is one diagram for both tetrachords, and the combined form of the total disposition might begin to be seen.

	Diatonic	Chromatic	Enharmonic
	2304 LL	2304 LL	2304 LL
Hyperboleon	T 2592 KK	SSS TTT 2736 HH	TT
	T 2916 FF	S T 2916 FF	2916 FF
	S T 3072 DD	S T 3072 DD	◊ 2994 EE ◊ 3072 DD
Diezeugmenon	T 3456 CC	SSS TTT 3648 BB	TT
	T 3888 Y	S T 3888 AA	3888 AA
	S T 4096 X	S T 4096 X	◊ 3992 Z ◊ 4096 X
	T 4608 O	T 4608 O	T 4608 O

IX. The nete synemmenon monochord in the three genera.

The above diagram of the three genera shows how two tetrachords are arranged so that they are conjunct with each other but disjunct from the mese. Now we have to approach the tetrachord called synemmenon, because it is conjunct to the mese.

Since we have already established that there is a diapente consonance between the nete diezeugmenon and the mese, and moreover, a diapente consists of three tones in this pentachord: one between the nete diezeugmenon and the diatonic paranete diezeugmenon, a second between the diatonic paranete diezeugmenon and the diatonic trite diezeugmenon, and a third between the paramese and the mese. There remains a semitone between the diatonic trite diezeugmenon and the paramese. Since the tetrachord between the nete diezeugmenon and the paramese is separated from the mese by one tone (that between the paramese and the mese), if we should subtract one tone from the pentachord between the nete diezeugmenon and the mese, indeed the tone between the nete diezeugmenon and the diatonic paranete diezeugmenon, then we would be able to join this other tetrachord to the mese. Thus the synemmonon, or conjunct, tetrachord should be made in this manner: since the diatonic paranete diezeugmenon, CC, is the number 3456, a third part of this added to the same equals the mese. Thus this number annotated by CC in the diezeugmenon tetrachord is separated from the nete diezeugmenon by a tone in the diatonic genus, and it is called the diatonic paranete diezeugmenon. But in the synemmenon or conjunct tetrachord this number will make the nete synemmenon in all three genera, and it is annotated by the letter V. If an eighth part of this number be taken, thus

432, and in turn added to the same, it will make 3888, which is the diatonic paranete synemmenon signified with the letter T. Let an eighth part of this be taken and added to the same, and it will make 4374, which is the diatonic trite synemmenon, that is Q. But since the nete synemmenon to the mese (3456:4608) holds a sesquitertian proportion or a diatessaron, whereas the distance between the trite synemmenon and the nete synemmenon (4374:3456) is one of the two tones, the proportion of a semitone is thus left between the trite synemmenon and the mese. This tetrachord is thus connected with the mese, and for this reason it is called continuous, or conjunct. Thus the proportion of the diatonic genus has been completed.

The division of the chromatic genus is as follows: I take the difference between the nete synemmenon and the diatonic paranete synemmenon: 3888 - 3456 equals 432. To produce a semitone I divide this, making 216, and add this to 3888. The three semitone interval will thus be produced and it will be in the number 4104, which will be the chromatic paranete synemmenon signified by the letter S. There is now a semitone between the chromatic paranete synemmenon and trite synemmenon where there was a tone in the diatonic genus. Another semitone is found between the chromatic synemmenon and the mese.

But since there are two tones between the diatonic or chromatic trite synemmenon and the nete synemmenon, the same note that is the chromatic or diatonic trite synemmenon in the chromatic or diatonic genus is the enharmonic paranete synemmenon in the enharmonic genus. The number for this is 4374, and let it be the letter R. From R to the mese is a semitone; so I divide this into two diesis in the following manner: I take the difference of the enharmonic paranete synemmenon and the mese: 4608 - 4374 equals 234. I divide this and make 117. This I add to the enharmonic paranete synemmenon (4374) and thus make the number 4491. This number is signified by the letter P, and it is the enharmonic trite synemmenon. Thus the semitone between the enharmonic paranete synemmenon and the mese--between 4608 and 4374--will be divided by the enharmonic trite synemmenon, 4491.

Thus the reasoning of this tetrachord has also been explained. But now a diagram of this tetrachord should be set forth joined with the others, namely the hyperboleon and the diezeugmenon. Thus little by little the progression toward completion is being made.

	Diatonic	Chromatic	Enharmonic
	2304 LL	2304 LL	2304 LL
Hyperboleon	Q/T 2592 KK	SSS/TTT	QQ/TT
		2736 HH	
	Q/T 2916 FF	S/T 2916 FF	2916 FF
			δ 2994 EE
	S/T 3072 DD	S/T 3072 DD	δ 3072 DD
Diezeugmenon	Q/T 3456 CC	SSS/TTT	QQ/TT
		3648 BB	
	Q/T 3888 Y	S/T 3888 AA	3888 AA
			δ 3992 Z
	S/T 4096 X	S/T 4096 X	δ 4096 X
	Q/T 4608 O	Q/T 4608 O	Q/T 4608 O

Synemmenon Tetrachord

Diat.	Chrom.	Enhar.
3456 V	3456 V	3456 V
Q/T 3888 T	SSS/TTT	QQ/TT
	4104 S	
4374 Q	S/T 4374 Q	4374 R
		δ 4491 P
4608 O	S/T 4608 O	δ 4608 O

X. The meson monochord through the three genera.

After all that has been said, I do not deem it necessary to spend too much time in the other tetrachords; for the other tetrachords, the meson and the hypaton, should be fitted in according to the example of those already divided.

Thus we will draw out the meson tetrachord in the diatonic genus as follows: I take a third part of the mese, O (4608); that is 1536. I add this to O, and the result is

6144. Let this be H, the hypate meson, which holds a diatessaron consonance in relation to the mese. This is divided thusly into two tones and a semitone: I take an eighth part of the mese (4608), which is 576. I add this to the mese, which makes 5184. This number is the diatonic lichanos meson, that is M. An eighth part of M is likewise taken, which is 648. I add this to M, and it makes 5832. Let this be I, the diatonic parhypate meson, which stands a tone from the diatonic lichanos meson, but two tones from the mese. Thus there remains a semitone located between the diatonic hypate meson and the diatonic parhypate meson, that is, between 6144 and 5832.

We divide the tetrachord of the mese and the hypate mese in the chromatic genus according to the following plan: I take the difference between the mese and the diatonic lichanos meson: 5184 - 4608 equals 576. I divide this in half, which will make 288; and I add this to the larger number 5184, thus making 5472. Let this number be N, the chromatic lichanos meson. Thus two semitones remain, one between the chromatic lichanos meson and the chromatic parhypate meson--between 5472 and 5832--and another between the chromatic hypate meson and the hypate meson--between 5832 and 6194.

We divide the enharmonic genus as follows: since the diatonic or chromatic parhypate meson (5832) stood at the distance of two tones from the mese, this number will be the enharmonic lichanos meson in the enharmonic genus notated with the letter L, nonetheless standing two tones from the mese. We divide the semitone which remains between the enharmonic lichanos meson and the hypate meson--between 5832 and 6144--into two diesis as follows: I take the difference of 5832 and 6954; this is 312. I divide this in half, which makes 156. I add this to 5832, which results in 5988. Let this number be K, the enharmonic parhypate meson. Thus there are two diesis, one between the enharmonic lichanos meson and the enharmonic parhypate meson--between 5832 and 5988--and another between the enharmonic parhypate meson and the hypate meson--between 5988 and 6144.

Thus the meson tetrachord has been divided. Now this is placed in diagram form and added to that of the other tetrachords:

	Diatonic	Chromatic	Enharmonic
	2304 LL	2304 LL	2304 LL
Hyperboleon	Q/T 2592 KK	SSS/TTT 2736 HH	QQ/TT 2916 FF
	Q/T 2916 FF	S/T 2916 FF	2994 EE
	S/T 3072 DD	S/T 3072 DD	3072 DD
Diezeugmenon	Q/T 3456 CC	SSS/TTT 3648 BB	QQ/TT 3888 AA
	Q/T 3888 Y	S/T 3888 AA	3992 Z
	S/T 4096 X	S/T 4096 X	4096 X
	Q/T 4608 O	Q/T 4608 O	Q/T 4608 O
Meson	Q/T 5184 M	SSS/TTT 5472 N	QQ/TT 5832 L
	Q/T 5832 I	S/T 5832 I	5988 K
	S/T 6144 H	S/T 6144 H	6144 H

Synemmenon Tetrachord

Diat.	Chrom.	Enhar.
3456 V	3456 V	3456 V
Q/T 3888 T	SSS/TTT 4104 S	QQ/TT 4374 R
Q/T 4374 Q	S/T 4374 Q	4491 P
S/T 4608 O	S/T 4608 O	4608 O

XI. The division of the hypaton monochord through the three genera, and a presentation of the complete diagram.

Now the hypaton tetrachord must be divided through the three genera. I take half of the hypate meson (6144), which is 3072, and I add this to the same and produce 9216, the proslambanomenos. This holds a diapente consonance in relation to the hypate meson. Now I take a third part of this same hypate meson (6144), which is 248, and I add this to the same producing 8192. This number is B, the hypate hypaton. Thus the hypate meson to the proslambanomenos is a diapente consonance, but the hypate meson to the hypate hypaton is a diatessaron.

I now take an eighth part from the hypate meson (6144); that will be 768. If this be added to the hypate meson, it makes 6912, which is the diatonic lichanos hypaton, E. This number holds the proportion of a tone in relation to the hypate meson. Likewise an eighth part should be taken from 6912; that is 846. This added to the same number makes 7776, which is the diatonic parhypate hypaton, C. This number stands at the distance of a tone from the diatonic parhypate hypaton, and at the distance of two tones from the hypate meson. Thus a semitone remains between the diatonic parhypate hypaton and the hypate hypaton--between 7776 and 8192. And such is the diatonic genus in this tetrachord.

We divide the chromatic genus as follows: I take the difference between the hypate meson and the diatonic lichanos hypaton: 6912 less 6144 equals 768. I divide this in half so I can make two semitones, and this makes the number 384. I add this to 6912 so that three semitones might be made. This will produce the number 7296, which will be the chromatic lichanos hypaton, F, standing at the distance of three semitones in relation to the hypate meson. Thus there remain two semitones, one between the chromatic lichanos hypaton and the chromatic parhypate hypaton--between 7296 and 7776--and another between the chromatic parhypate hypaton and the hypate hypaton--between 7776 and 8192.

There remains the enharmonic genus, the division of which takes place in accordance with the above example. Since the diatonic or chromatic parhypate hypaton found in the number 7776 stands at the distance of two tones from the hypate meson, this same note will be the enharmonic lichanos hypaton in the enharmonic genus. For the enharmonic lichanos hypaton should be separated from the hypate meson by two whole tones. Thus a semitone remains from the diatessaron consonance, and this semitone is between the enharmonic lichanos hypaton and the hypate hypaton--between 7776 and 8192. We divide this into two diesis as follows: I take the difference between the enharmonic lichanos hypaton and the hypate hypaton; 8192 - 7776 equals 416. I take half of this, which is 208, and add it to 7776, thus making 7984, which is D, the enharmonic parhypate hypaton. Thus there are two diesis, one between the enharmonic parhypate hypaton--between 7776 and 7984--and another between the enharmonic parhypate hypaton and hypate hypaton--between 7984 and 8192. The last tone is contained between the hypate hypaton and the proslambanomenos, that is, between 9216 and 8192.

Thus the hypaton tetrachord has been divided according to the three genera, the diatonic, chromatic, and enharmonic. If this be added to the above tetrachords, the hyperboleon, the diezeugmenon, the synemmenon, and the meson, it will produce the complete and perfect diagram of the monochord ruler divided through all three genera.

	Diatonic	Chromatic	Enharmonic
	2304 LL	2304 LL	2304 LL
Hyperboleon	O/T 2592 KK	SSS/TTT 2736 HH	OO/TT 2916 FF
	O/T 2916 FF	S/T 2916 FF	ꝺ 2994 EE
	S/T 3072 DD	S/T 3072 DD	ꝺ 3072 DD
Diezeugmenon	O/T 3456 CC	SSS/TTT 3648 BB	OO/TT 3888 AA
	O/T 3888 Y	S/T 3888 AA	ꝺ 3992 Z
	S/T 4096 X	S/T 4096 X	ꝺ 4096 X
	O/T 4608 O	O/T 4608 O	O/T 4608 O
Meson	O/T 5184 M	SSS/TTT 5472 N	OO/TT 5832 L
	O/T 5832 I	S/T 5832 I	ꝺ 5988 K
	S/T 6144 H	S/T 6144 H	ꝺ 6144 H
Hypaton	O/T 6912 E	SSS/TTT 7296 F	OO/TT 7776 C
	O/T 7776 C	S/T 7776 C	ꝺ 7984 D
	S/T 8192 B	S/T 8192 B	ꝺ 8192 B
	O/T 9216 A	O/T 9216 A	O/T 9216 A

Synemmenon Tetrachord

Diat.	Chrom.	Enhar.
3456 V	3456 V	3456 V
O/T 3888 T	SSS/TTT 4104 S	OO/TT 4374 R
O/T 4374 Q	S/T 4374 Q	ꝺ 4491 P
S/T 4608 O	S/T 4608 O	ꝺ 4608 O

XII. Rationale of the above diagram.

In the above form, the proslambanomenos to the mese holds the diapason consonance, as does the mese to the nete hyperboleon. The proslambanomenos to the nete hyperboleon, however, holds the bisdiapason. The diatessaron consonance is found from the hypate hypaton to the hypate meson, the hypate meson to the mese, the mese to the nete synemmenon, the paramese to the nete diezeugmenon, the nete diezeugmenon to the nete hyperboleon, and others so situated that we can sound a complete tetrachord between the consonances.

And so that the order of all notes according to the three genera be reconsidered for the sake of more clarity, there are only five tetrachords: first and lowest, the hypaton, of which the hypate hypaton is the first note and the hypate meson the last; second, the meson, of which the hypate meson is the first note and the mese the last; third, the synemmenon, of which the mese is the first note and the nete synemmenon the last; fourth, the diezeugmenon, of which the first note is the paramese and the nete diezeugmenon the last; and fifth, the hyperboleon, whose first note is the nete hyperboleon and this tetrachord is terminated in the final hyperboleon.

XIII. Concerning fixed and movable notes.

Some of all these notes are completely fixed, others are completely movable, whereas others sound neither completely fixed nor completely movable.[32] The completely fixed notes are the proslambanomenos, the hypate hypaton, the hypate meson, the mese, the nete synemmenon, the paramese, the nete diezeugmenon, and the nete hyperboleon. These therefore are the same in all three genera, and they change neither according to place nor name whether they contain a pentachord or a tetrachord. (A pentachord, for example, is from the proslambanomenos to the hypate meson or from the mese to the nete diezeugmenon, whereas a tetrachord is from the hypate hypaton to the hypate meson, or from the hypate meson to the mese.)

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32. Boethius goes further in his classification of notes than the preserved Greek sources, which classify notes as merely fixed or movable; cf., for example, Cleonide *Isagoge Harmonica* (JanS. 183-184), and Ptolemy *Harmonics* ii. 5. 53.

Those notes are movable which are changed according to each individual genus, such as the diatonic and chromatic paranete and lichanos, or the enharmonic trite and parhypate. For that which is at one time the diatonic paranete hyperboleon is at another time the chromatic paranete hyperboleon, and at still another time the enharmonic trite. The diatonic and chromatic paranete diezeugmenon are also different, nor is the enharmonic diezeugmenon the same as the trite in the other genera. Furthermore, the diatonic and chromatic paranete synemmenon and the enharmonic trite synemmenon are not the same notes as those which are the trite in other genera. Also the diatonic lichanos meson and the chromatic lichanos meson are not at the place, and the enharmonic parhypate meson is not found to be the same as the parhypate of any other genera. The diatonic lichanos hypaton and the chromatic lichanos hypaton do not retain the same place and numbers; and the enharmonic parhypate hypaton is found to be different from the parhypate of the other genera.

The notes which are neither completely fixed nor completely movable are those which remain the same in the diatonic and chromatic genera, but are changed in the enharmonic. This may be explained as follows: the diatonic trite hyperboleon and the chromatic trite hyperboleon in the above diagram are both in the same number: 2916; but if we examine the enharmonic genus, we discover another trite, that is, 2994. Therefore, the sound which was the same in two genera is changed in the third. The same is valid for the diezeugmenon tetrachord; for the diatonic and chromatic trite diezeugmenon are one and the same note, but the enharmonic trite diezeugmenon is different from these notes. In the synemmenon tetrachord the diatonic and chromatic trite synemmenon are also the same, while the enharmonic trite synemmenon is different. Moreover, the diatonic parhypate meson and the chromatic parhypate meson are notated by the same number, but just as in the case of the above trite, the parhypate in the enharmonic genus is found next to the hypate meson, and its pitch and its effect are different from the parhypate of the other genera. Again the diatonic and chromatic parhypate hypaton are the same, but it is not the same if it is sought in the enharmonic genus.

But so that this matter of the partly movable notes might become clearer, let us return to the hyperboleon tetrachord. In this tetrachord the note which is the trite hyperboleon in the diatonic and chromatic genera is changed in the enharmonic genus and becomes the paranete. Likewise that note which is the trite hyperboleon in the diatonic and chromatic genera is changed in the enharmonic genus and becomes the paranete. Likewise that note which is called the trite diezeugmenon in either the chromatic or diatonic genus is called paranete in the enharmonic. The note which was the trite synemmenon in the chromatic or diatonic genus changes to the paranete in the enharmonic genus. The note considered the parhypate meson in the chromatic or diatonic genus is discovered to be the lichanos meson in the enharmonic genus. And finally, the note called parhypate hypaton in the diatonic or chromatic genus is named the lichanos hypaton in the enharmonic.

Thus these are the fixed notes: proslambanomenos, hypate hypaton, hypate meson, mese, nete synemmenon, paramese, nete diezeugmenon, nete hyperboleon. The movable notes are those which we call lichanos or paranete in either the diatonic, chromatic or enharmonic genus. The notes which are not completely movable are those which we call parhypate or trite in the diatonic and chromatic genera, but lichanos or paranete in the enharmonic genus.

XIV. Concerning the species of consonances.[33]

Now we must discuss the species of the prime consonances, those being the diapason, diapente, and diatessaron. A species is a certain position contained within any numerical proportion which yields a consonance, having its own form according to any one of the genera, for example the diatonic.[34]

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33. Chapters XIV-XVII of this Book contain Boethius' modal theory. The most difficult, misunderstood, and disputed passage of Boethius' musical work. The following text will be carefully related wherever possible, to its Greek sources (cf. Commentary, Chapter I, pp. 359-361) and the essential relationships between the charts, the text, and the Greek sources will be discussed. It is hoped that some understanding of what this passage in and of itself is trying to say will be thus achieved. For further concerning this passage see Commentary, Chapter IV, pp. 446-451.

34. Here the text begins to draw on its key source of modal theory: Ptolemy *Harmonics* ii. 3-11. 49-80. This definition of species is taken with minor differences from *Harmonics* ii. 3. 49. For the sake of comparison I give Duering's German translation of the passage: "*Gattung ist eine besondere, in geeigneten Zahlenverhaeltnissen ausgedruechte Stellung der fuer jedes Tetrachord charakteristischen Toene.*" (Duering, *Ptolemaios und Porphyrios ueber die Musik*, 1934, p. 63).

If we place the diezeugmenon tetrachord between the hyperboleon and meson tetrachords, and then take away the synemmenon tetrachord, there will be fifteen notes. If the proslambanemenos is removed from these, fourteen will remain. These are disposed as follows:

[B]	A.	hypate hypaton[35]
[c]	B.	parhypate hypaton
[d]	C.	lichanos hypaton
[e]	D.	hypate meson
[f]	E.	parhypate meson
[g]	F.	lichanos meson
[a]	G.	mese
[b]	H.	paramese
[c']	I.	trite diezeugmenon
[d']	K.	paranete diezeugmenon
[e']	L.	nete diezeugmenon
[f']	M.	trite hyperboleon
[g']	N.	paranete hyperboleon
[a']	O.	nete hyperboleon

Thus there is a diapason consonance from the hypate to the paramese; from the same paramese to the hypate meson is a diapente, and from the mese to the hypate meson is a diatessaron. Therefore the diapason will consist of eight notes, the diatessaron of four, and the diapente of five. By virtue of this the diatessaron will have three species,

35. Cf. Ptolemy Harmonics ii. 3. 49. arrangement of these fourteen notes as A-P; I is omitted in Ptolemy's arrangement, and thus his K-P are identical with Boethius' I-O.

the diapente four, and the diapason seven; for there will always be one less species than the number of notes.[36] So that we might begin the others from the mese,[37] the diatessaron consonance has three species in the following manner: one species will be from G (the mese) to D; a second from F to C; and a third from E to B.[38] Thus the species of the diatessaron proceed up to this point; for up to this point the species have contained two notes of the same diatessaron, as GD contains E and F, FC contains E and D, and EB contains E and D.[39] But if I add the

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36. Cf. Ibid. Ptolemy's account for the number of species is somewhat different: "Deshalb gibt es in jedem Intervall gleich viele Gattungen wie Tonschritte." (Duer. Ptolemaios und Porphyrios, p. 64).

37. This apparent attempt to be systematic by beginning the other consonances from the mese contributes to some inconsistencies which follow. First, he cannot begin the other consonances from the mese and derive all their species, and second, by not beginning on the same notes as Ptolemy he arrives at different species of the diapente, one of which is not a diapente species at all (Cf. n. 40).

38. These diatessaron species are intervallically identical with those of Ptolemy Harmonics ii. 3. 50; Boethius merely transposes them down an octave.

39. Friedlein's text at this point (p. 338, l. 23) reads "C and D" rather than "E and D," as do most MSS of the text. But C is not found in the diatessaron GD, whereas E is; and the argument requires that two notes of EB be contained in GD. Moreover, Bamberg HI. N. 19, probably the oldest extant MS of this treatise, Munich 14523 (10th. c.),

diatessaron DA to these, it will be different from GD; for it will contain only one note of the consonance GD, that is, only D. Thus it exceeds the consonance GD. And by reason of this the diatessaron is said to have three species.

The other consonances follow a similar pattern. But the diapente will have four species, as follows: one from H to D, another from G to C, another from F to B, and another from E to A.[40] The diapason will have seven species

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and Munich 6316 (11th c) all read "E and D" (Friedlein, apparatus criticus, 338). Therefore "E and D" is obviously the correct reading of this passage.

40. The text errs in computing these species of diapente by not beginning the process from the same note which Ptolemy uses (Ptolemy Harmonics ii. 3. 50). Cf. the following chart (intervals given in descending order of pitch):

Ptolemy (omits I)			Boethius		
1. GM	(a-e)	TTST	1. HD	(b-e)	TTTS
2. FL	(g-d)	TSTT	2. GC	(a-d)	TTST
3. EK	(f-c)	STTT	3. FB	(g-c)	TSTT
4. DH	(e-b)	TTTS	4. EA	(f-b)	STTS

First of all, the intervallic contents of the successive species are not identical; but perhaps this text is merely showing the possibilities beginning from H, since it states "alia" rather than "secunda," "tertia," etc. Indeed in the following passage where diapente species between fixed notes are discussed, the same notes as Ptolemy are used; however, the numbering is the opposite of Ptolemy (cf. n. 43). Secondly, the last species of this text is not a diapente at all, but a tritone.

Boethius' source was here probably considering H as the mese, which would make A the proslambanomenos. Thus the description would yield the four species of diapente in the order used in the following passage (cf. n. 43).

in the following manner: one from O to G; a second from N to F; a third from M to E; a fourth from L to D; a fifth from K to C; a sixth from I to B; and a seventh from H to A.[41]

Thus it is clear from what has been said that a diatessaron is contained only once (semel tantum) between immovable and fixed notes. For if I begin from the hypate hypaton, the first diatessaron in this order will be AD, that is, from the hypate hypaton to the hypate meson. The others, BE and CF, are limited by movable notes; for both the parhypate hypaton and the parhypate meson, as well as the lichanos hypaton and the lichanos meson, have been proved to be movable. But if we begin a diatessaron consonance from the hypate meson, it will be that species of diatessaron terminated in fixed notes. This consonance will be DG, that is, the hypate meson to the mese, which is the first species of diatessaron.[42] The others, EH and FI, are by no means fixed; for the parhypate meson, the lichanos meson, and the trite diezeugmenon have not proved to

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41. These diapason species are identical with those of Ptolemy Harmonics ii. 3. 50.

42. Agrees with Ptolemy Harmonics ii. 3. 49.

be fixed notes. Likewise, if we begin a diatessaron from the paramese, it will be that species of diatessaron which is encompassed by the fixed notes HL, that is, from the paramese to the nete diezeugmenon, and this is the first species of diatessaron. The other possibilities, which are IM and KN, are limited by movable notes, for the trite diezeugmenon, the paranete diezeugmenon, and the trite hyperboleon have already been discussed as movable notes.

The diapente consonance has two such species which are contained between fixed notes. If we begin a diapente from the hypate meson, the result is DH, that is, from the hypate meson to the paramese, which is the first species of diapente. Another such diapente is GL, that is, from the mese to the nete diezeugmenon, and this is the fourth species of diapente.[43] The other diapentes, EI and

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43. These two fixed diapente species are the same as those listed by Ptolemy (Harmonics ii. 3. 49). However, this text numbers them in reversed order of Ptolemy, i.e., DH, the "first" of Boethius' text, is the fourth of Ptolemy, whereas GL the "fourth" of Boethius' text is the first of Ptolemy (GM; cf. n. 40). There is some justification, however for this text's numbering of these species, e.g. Cleonide, Isagoge Harmonica (JanS. 196) and Bacchius, Isagoge (JanS. 308) state that the first species of diapente is from the hypate meson to the paramese, which is Boethius' DG, and the fourth is from the mese to the nete diezeugmenon, which equals Boethius' GL.

FK, are in no case bounded by fixed notes; for the parhypate, the lichanos, the trite, and the paranete have all been proved to be movable. But the rationale will be the same if the species of these consonances be considered from the nete diezeugmenon toward a lower pitch; for they will be contained by these same immovable notes which were discussed above. Whether we compute a consonance toward the mese, the paramese, or even from the nete hyperboleon, a stretching out (districtio) of two notes which are fixed will not be able to happen.

The order of the diapason consonance, whether it be taken from the hypate hypaton to the paramese or from the nete hyperboleon to the meson, will hold only three of these species which are contained between fixed notes. Beginning from the hypate hypaton, one of these species is AH, that is, from the hypate hypaton to the paramese, and this is the first species of diapason. Another is DL, that is, from the hypate meson to the nete diezeugmenon, and this is the fourth species of diapason. Finally there is GO, that is, from the mese to the nete hyperboleon, and this is the seventh species of diapason.[44] The outermost notes of the other species are in no way constituted as these three are; for the parhypate, lichanos, trite and paranete, as discussed above, are not fixed sounds. Moreover, if we would begin from the nete hyperboleon, the

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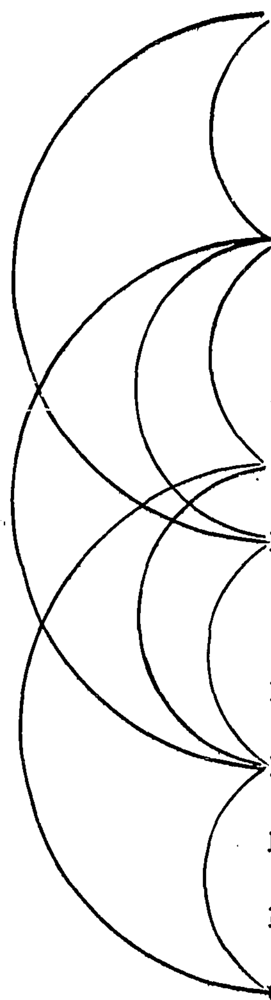
44. These diapason species are again identical with those of Ptolemy (Harmonics ii. 3. 49.), but their numbering is opposite. Indeed the numbering here is opposite from that given by Boethius above (p. 271), where AH is listed as the seventh species and GO as the first. But again there is justification for this numbering in the Greek sources: e.g., Bacchius (JanS. 308-309), Gaudentius (JanS. 346), and Cleonide (JanS. 197-198) list the first species of diapason as falling between the hypate hypaton and the paramese i.e., Boethius' AG, whereas the seventh falls between the mese and the nete hyperboleon, i.e., Boethius' GO. (DL is the fourth counting from either direction.) Ptolemy himself is aware of this alternate numbering, but rejects it because it does not fit his theory regarding the systema teleion, the diapason species, and the tropes (Harmonics ii. 5. 53.).

It would seem that though Boethius' source bases this discussion mainly on the Harmonics of Ptolemy, it is nevertheless at least aware of this other tradition regarding the numbering of the species; and it chooses (or happens) to use this numbering even though it is in conflict with Ptolemy and indeed with part of the present presentation. Nevertheless it must be emphasized that Ptolemy is the key source for the contents of Chapters XIV-XVII. Boethius' source, following Ptolemy, does not assign any names (e.g. Dorian, Phrygian, etc.) to the octave species as do Gaudentius, Bacchius, and Cleonide. This is, as in Ptolemy, reserved for the discussion of the modes which follows.

order of the species would interweave similarly through these same notes. All of this is grasped more clearly in the following diagram:



A. Hypate hypaton
B. Parhypate hypaton
C. Hypaton lichanos
D. Hypate meson
E. Parhypate meson
F. Lichanos meson
G. Mese
H. Paramese
I. Trite diezeugmenon
K. Paranete diezeugmenon
L. Nete diezeugmenon
M. Trite hyperboleon
N. Paranete hyperboleon
O. Nete hyperboleon

XV. <u>The introduction of the modes and a diagram of the notational signs through the singular modes and notes.</u>

Therefore the elements of music called "modes" are made from these species of the diapason consonance; but they are also called "tropes" or "tones."[45] Moreover the systems[46] of the tropes are in the total series of notes, and they are different according to their lowness or highness. Their system is, as it were, the complete body of modulation consisting of the conjunction of consonances whether they be diapason, diapason and diatessaron, or bisdiapason. But the system of a diapason is from the proslambanomenos to the mese with all the other notes between these two, or from the mese to the nete hyperboleon with the intervening notes, or from the hypate

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45. The word "mode" as used here has little relation to the medieval and modern concept of mode. Boethius is literally translating the Greek word τροπος into the Latin word *modus*. The words *tropus* and *tonus*, which modes "are also called," are merely Latin transliterations of the Greek words τροπος and τόνος. Boethius is *not* confusing the diapason or octave species, the medieval and modern "modes," with the Greek tropes. He does not say that the modes "are" the species of the diapason, but rather they "are made from" or more literally "exist from" the species of the diapason consonance (*Ex diapason consonantiae speciebus existunt qui appellantur modi*). This idea is taken from Ptolemy *Harmonics* ii. 9. 60. and ii. 11. 64-65. Cf. n. 53.

46. "*Constitutio*" is Boethius' literal translation of the Greek word συστημα (see n. 47).

meson to the nete diezeugmenon with those notes which these outside notes contain. The system of the synemmenon (the diapason and diatessaron) is made from the proslambanomenos to the nete synemmenon with all the notes between these. Finally, the bisdiapason is considered from the proslambanomenos tc the nete hyperboleon along with all the notes interposed between these.[47] But if someone should move these total systems up or down in pitch according to the species of the diapason consonance discussed above, then he would make the seven modes.[48] The names of these modes

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47. This passage concerning systems is a rather condensed paraphrase of Ptolemy Harmonics ii. 4. 50-51. Ptolemy there argues that the systema teleion (bisdiapason) is the only possible system which will contain all the species of the diapason, and therefore the other systems (the synemmenon and diapason systems) are superfluous. However, the present text chooses to ignore this argument, probably because the charts which are so essential to its understanding of the modes include the synemmenon system.

48. For example, the first diapason species of Chapter XIV was GO (a'-a), or:

T T S T T S T. Thus the trope
or mode is: a g f e d c b a g f etc. to a.

The second diapason species was NF (g'-g), or:

T S T T S T T. Thus the
trope is: b a g f# e d c# b a g f# etc. to b.

The third diapason species was ME (f'-f) or:

S T T S T T T. Thus the trope
is: c# b a g# f# e d# c# b a etc. to c#. [continued]

are as follows: hypodorian, hypophrygian, hypolydian, dorian, phrygian, lydian, mixolydian.[49] Their order proceeds in the following manner: let the order of notes be arranged in the diatonic genus from the proslambanomenos to the nete hyperboleon, and let this be the hypodorian mode. If someone should raise the proslambanomenos one tone in pitch, and further diminish the hypate hypaton by this same tone and make all the other notes higher by a tone, then the total order would come out higher than it was before the transposition of a tone. This whole disposition made higher by a tone will be the hypophrygian mode. But if the notes in the hypophrygian mode are similarly transposed up a tone, then the hypolydian mode is born. Moreover, if someone transpose the hypolydian

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Just as in Chapter XIV, the first species begins from the highest note (in pitch), the second from the second (N), the third from the third (M), etc. Thus to arrive at these modes or tropes, the total system is moved "up or down in pitch according to the species of the diapason consonance" (cf. Chapter XVI, n. 53).

49. Concerning the application of these names to the tropes rather than the diapason species see Ptolemy Harmonics ii. 10. 62-63.

mode up a semitone, then the dorian mode is made.[50] And the other modes are made by continuing this upward transposition. But so that the form of these modes might be recognized by the eye and not only grasped by the mind, a description handed down from ancient musicians should be given.[51] But since the ancient musicians notated each single note with a different sign, it seems that the previous diagram of the notes should be called to mind. In this way the considering of this diagram of the modes should be made easier by using these signs already known as such.[52]

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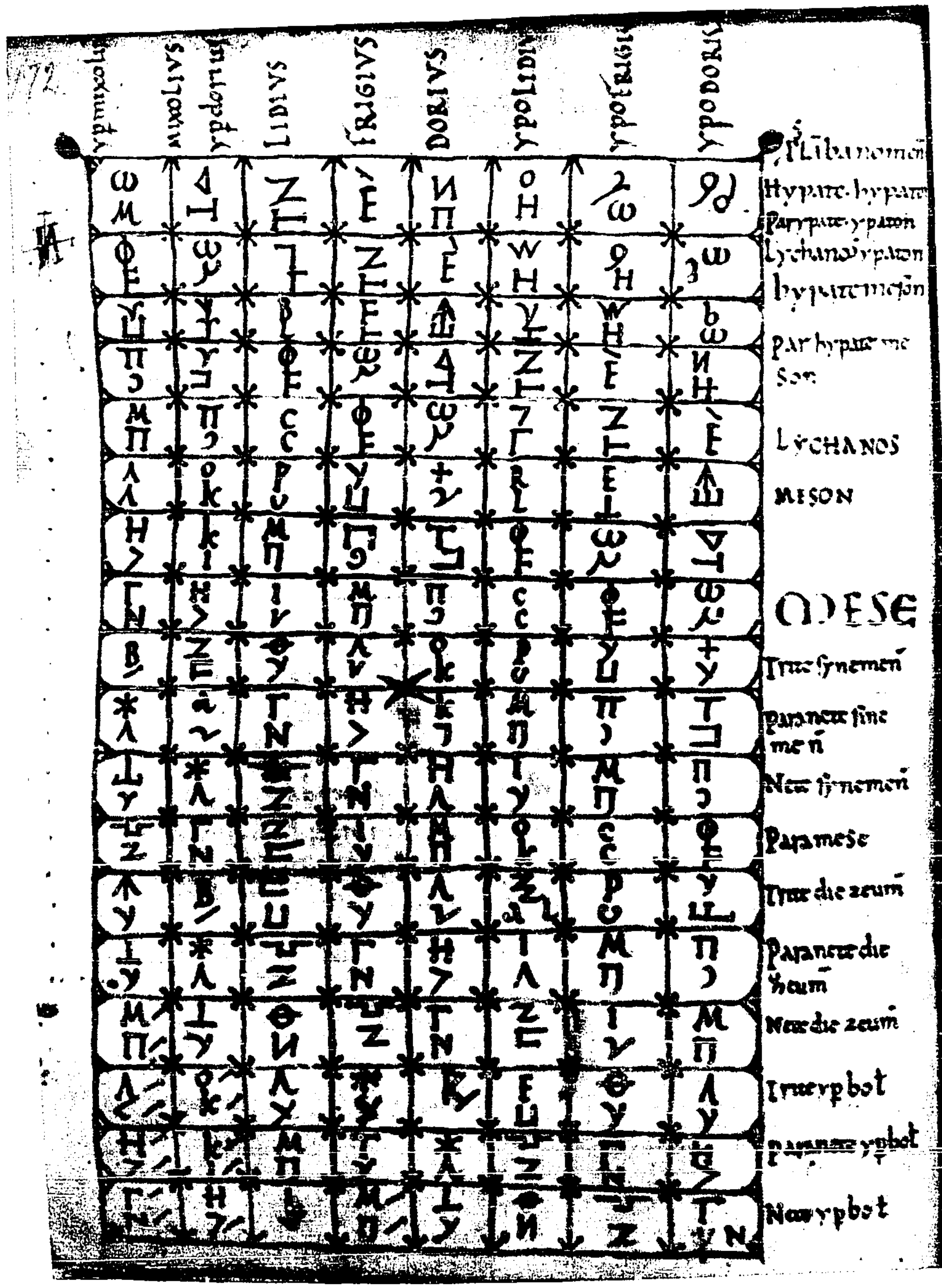
50. The order in which these four modes (hypodorian, hypophrygian, hypolydian, dorian) proceeds is taken from Ptolemy Harmonics ii. 10. 63., where Ptolemy gives the intervallic spacing for these four modes, and only these four, just as discussed here. After these three transpositions have been given, the remaining ones quite logically follow.

51. This "description handed down from ancient musicians" survives in no other treatise.

52. Since the notation of the previous diagram (Chapter III) gave only the symbols for the lydian mode, many of the symbols on this chart remained unknown to the reader, especially those below the lydian proslambanomenos (7 ⊢) and above the lydian hypate hypaton (|' ≺). Nevertheless the basic logic of the chart, especially in conjunction with the chart of the next chapter, is evident merely with a knowledge of the symbols for the lydian mode--i.e., the systematic transposition is evident. With relatively few exceptions these symbols concur with those given in Alypius, Isagoge (JanS. 369-383; cf. Chapter III, n. 18).



(Cologne, Stadtarchiv, W. 331, 172)

XVI. Diagram containing the order and differences of the modes.

The above diagram contains the names ascribed to the notes along with their notational signs, and the additional words designate the mode, whether lydian, phrygian, or dorian. But since we said that these modes are found in the species of the diapason consonance, we should indeed now describe them as such in the diatonic genus. In this way the intelligence should immediately learn, by seeing, just what the order of these modes is.[53]

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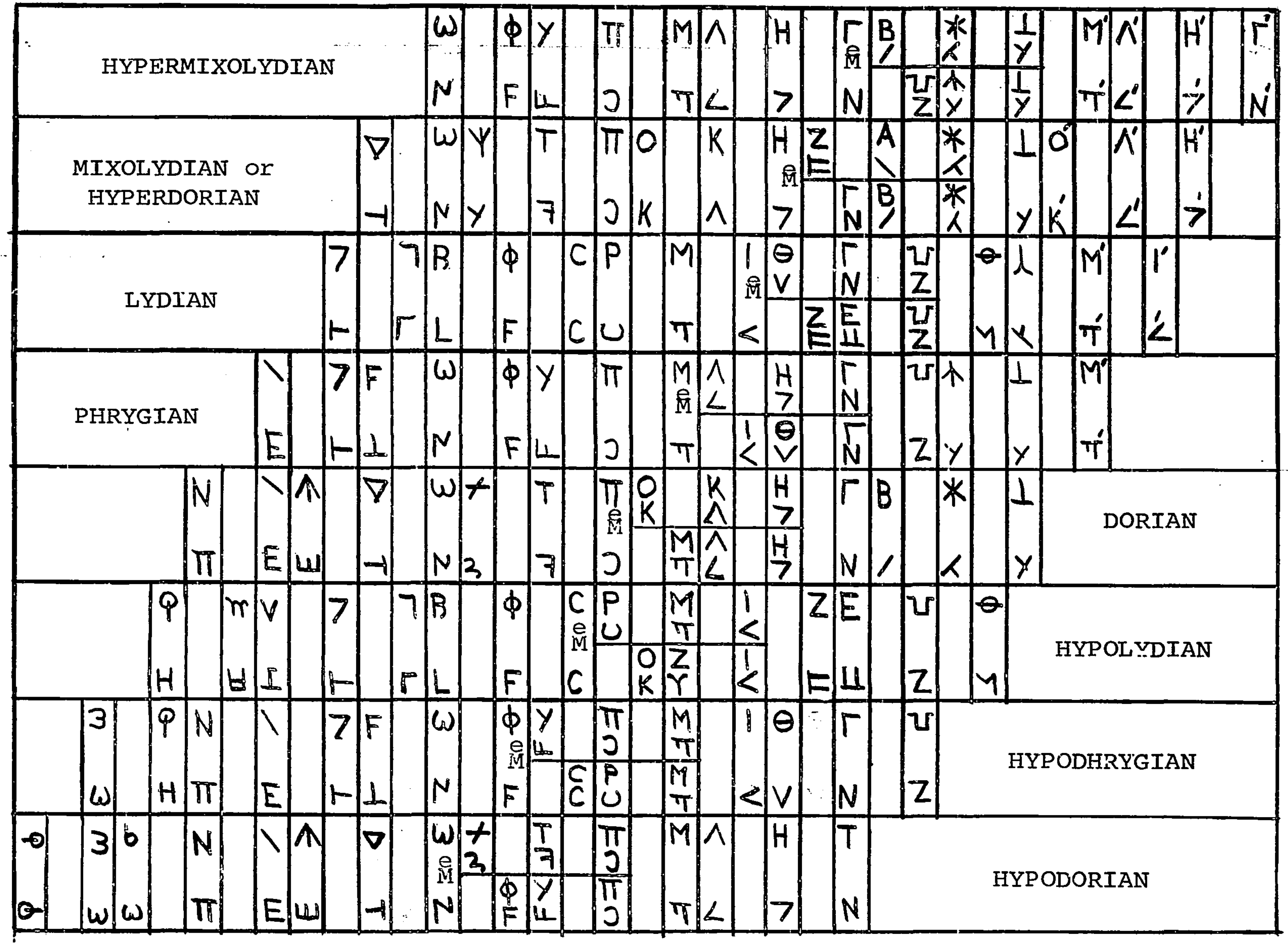
53. (The three notes written conjunct to the mese (M^e) of each mode in the following chart are the synemmenon tetrachord.) The expressed purpose of the following diagram is to show the relation of these modes to the diapason species. The "intelligence" must see three characteristics in this diagram in order to "learn" the order of these modes: (1) the diagram is intervalic, i.e., spaces occur between notes whose interval is a tone, whereas semitones are given no such spaces (cf. following chapter); (2) spaces marked ω/Ɲ or B/L and Γ/N or E/Ш are filled in all modes in the diezeugmenon system, thus forming a fixed middle octave; (3) the mese of each mode is marked M and the range of these mese further outlines the fixed middle octave (cf. following chapter, n. 55). Thus if one recalls the diapason species of Chapter XIV (pp. 268-271), and keeps these three characteristics in mind, the order of these modes indeed becomes clearly seen. (Moreover, the ambiguity with regard to numbering makes little difference, for the species remain the same regardless of numbering.) The first diapason species was from the nete hyperboleon to the mese (O to G), thus Γ/N to ω/Ɲ of the hypodorian mode forms


HYPERMIXOLYDIAN
MIXOLYDIAN or HYPERDORIAN
LYDIAN
PHRYGIAN
DORIAN
HYPOLYDIAN
HYPODHRYGIAN
HYPODORIAN

XVII. Rationale of the above diagram.

We have said that there are seven modes, but it would not seem incongruent if an eighth mode were added. But we will discuss the reason for this addition a little later.[54] Now we must consider the little spaces created between the regularly drawn lines of the diagram: some of these spaces have signs for a musical note, whereas others have nothing at all, for example, in the hypermixolydian mode, the first space contains the sign ω, the third space Φ, but the second space is empty. This empty space shows that there is a tone between these two notes. On the other hand, since no space separates the note of the third space Φ, from that of the fourth space Y, and the column is continued without interruption

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this species, the traditional Greek hypodorian species. The second species was from the paranete hyperboleon to the lichanos meson (N to F), thus Γ / N to ω / N of the hypophrygian mode forms this species, the traditional Greek hypophrygian species. E / Ш to B / L of the hypolydian mode (the same notes as Γ / N and ω / N of the hypodorian and hypophrygian modes) form the third octave species, and thus the order of the modes in relation to the diapason species proceeds through the remaining modes. Concerning this theory of a middle octave and the modes altering the species see Ptolemy Harmonics ii. 11. 64-65.

54. See pp. 290-91, n. 56.

this reveals that a semitone is the difference between them. The proof of this is as follows: if ω is the proslambanomenos, φ the hypate hypaton, and γ the parhypate hypaton, then there is necessarily the distance of a tone between the proslambanomenos ω and the hypate hypaton φ, but the distance contained between the hypate hypaton φ and the parhypate hypaton γ is necessarily a semitone. This same rule applies to the rest of the diagram. Thus if a whole space separates the signs of two notes, we know that there is the interval of a tone between them; but if the column is continuously filled with notational signs and not interrupted by a space, then we will recognize that the interval is a semitone.

Having anticipated this matter, if two rows in the bisdiapason chart are compared to each other and the one proslambanomenos is lower than the other proslambanomenos, or if any other note of one row is lower than the corresponding note in the other, and of course if both rows are in the same genus, then that total row is necessarily lower than the other. In this way we can tell which row is the lower one.

But this is accomplished easier from that middle note which is the mese. When comparing two rows of the bisdiapason consonance, if the mese of one be lower than that of the other, then this whole row is also lower. For if the other notes be similarly compared, they will be found to be no lower. Thus if one middle note appears to be a tone higher or lower than the middle note of another, all of the other notes, if compared to their corresponding notes, will appear to be a tone higher or lower, assuming that both rows are in the same genus. But considering four middle notes, if the first to the fourth is the interval of a diatessaron, whereas the first to the second and the second to the third stand at the distance of a tone, then the third to the fourth will make the difference of a semitone. For example, let four middle notes be assumed: A, B, C, and D: A:D equals a sesquitertian proportion, that is, a diatessaron; A:B and B:C equal tones; there remains C:D, which holds the interval of a semitone.

If there are five middle terms the results are similar; for if the first stands at the sesquialter proportion from the fifth, and the first to the second, the second to the third, and the third to the fourth are all tones, then the fourth to the fifth makes a semitone.

Those modes whose middle notes approach the position of the proslambanomenos form the lower modes, whereas those nearer to the nete hyperboleon make the higher modes. In the above diagram of the modes the first proslambanomenoi are located on the left side whereas the right side ends with the extreme nete. Therefore the hypermixolydian mode will be highest of all, whereas the hypodorian will be the lowest.

We will now trace out the remaining modes, beginning from the lowest one, the hypodorian, and indicate their distance from this one. The mese of the hypodorian mode (ω) is a tone from the mese of the hypophrygian mode. This is easily observed in this way: if someone compare the note ω of the hypophrygian to the mese (φ) of this same mode, then he will see that the mese of the hypodorian mode is the same as the lichanos meson of the hypophrygian mode; for φ and ω are separated by a tone, as indicated by the space between them on the chart. Moreover, the mese of the hypolydian mode is a tone away from that of the hypophrygian mode. For ω , the mese of the hypolydian, is a tone away from φ--the lichanos meson of the hypolydian, but the mese of the hypophrygian. The mese of the hypolydian mode (C) is the distance of a semitone from the mese of the dorian. This will be acknowledged because a mere line rather than a space separates the perpendicular column of the hypolydian mese and the perpendicular column of the dorian mese. Whence it follows that the mese of the hypodorian is a complete diatessaron consonance away from the mese of the dorian. This is proved as follows: ω is the mese in the hypodorian mode, but it is the hypate meson in the dorian mode, and the hypate meson stands at the distance of a diatessaron consonance from the mese in any mode or genus.

The mese of the dorian (Π) is a tone from that of the phrygian (M); for Π , the mese in the dorian mode, is the lichanos meson in the phrygian mode. Furthermore, the mese of the phrygian mode (M) is a tone away from the mese of the lydian mode (|); for M , the mese of the phrygian, is the lichanos meson of the lydian. The mese of the lydian mode (\ ) is a semitone away from the mese of the mixolydian (⊢⊣); for the row containing the lydian meson is separated by a line rather than a space from the row containing mixolydian mese.

The mese of the mixolydian mode (⊢⊣) holds the difference of a tone in relation to that of the hypermixolydian (Γ); for ⊢⊣ , the mese of the mixolydian, is the lichanos meson of the hypermixolydian. Whence it follows that the dorian mese is a diatessaron consonance away from the mixolydian mese. This is proved as follows: ΤΤ , the dorian mese, is the mixolydian hypate meson, and the hypate meson holds a diatessaron consonance in relation to the mese in any mode. Moreover, there is a diapente consonance from the dorian mese (ΤΤ) to the hypermixolydian mese (Γ); for ΤΤ , the dorian mese, is the lichanos hypaton in the hypermixolydian column, and if the lichanos hypaton be compared to the mese in the diatonic genus, the result is a diapente consonance in any mode.[55]

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55. This process of tracing the distance of the modes from the mese and comparing the mesai themselves probably grew out of Ptolemy's theory of the "dynamic mese" (<u>Harmonics</u> ii. 11. 64-65. Boethius makes no specific reference

The reason for the addition of an eighth mode--the hypermixolydian--becomes evident in the following. Assume the following bisdiapason consonance:

A B C D E F G H I K L M N O P

A:H contains a diapason consonance, for it consists of eight notes. Thus we said that this is the first species of the diapason. The second is B:I; the third C:K; the fourth, D:L; the fifth, E:M; the sixth, F:N; the seventh G:O. Therefore there still remains H:P, and since it completes the total series, it is added to the others. This then is the eighth mode, which Ptolemy added at the top.[56]

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to this theory as such, but the basic theory is evident through the chart preceding this chapter, with its central fixed octave and marked mesai (see n. 53), plus the emphasis on the mese in the present chapter.

56. Ptolemy does state the possibility of an eighth mode, hypermixolydian by name and "added at the top" (*Harmonics* ii. 9. 63) and even discusses it in terms of letters (*Ibid.* 64), although the order of his letters is different. However, he rejects its validity because its intervallic structure (i.e., its diapason species) is "harmonically identical" with the first mode (*Ibid.* 13. 59, 9. 60-63)

The interpretation of Boethius' series of letters should probably be taken literally from the text itself. In reference to A:H, Boethius clearly states: "We said this was the first species" (*Primam igitur diximus esse speciem diapason eam, quae est A:H*). Thus he is again (cf. pp. 273-74 n. 44) using the numbering of the species which is opposite that taken from Ptolemy, where G:O was the first species and A:H the seventh (cf. pp. 270-71 n. 41). Therefore A:H

XVIII. How musical consonance can be clearly judged by the ear.

An instrument can be made quite simply and quickly so that the rationale of these consonances may be considered very clearly. Let a ruler be set forth--AD--and let two hemispheres, which the Greeks call magada, be added to this ruler in such a way that the line leading from curve E to

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here equals the diapason species of the mixolydian mode--from the hypate meson to the paramese (b-b). Ptolemy himself, when similarly computing these modes in letter, begins with the "highest" mode--therefore the mixolydian--and calls it A. In this context Ptolemy states that the eighth mode might be added next to A (the top) or next to C (the bottom), just so a tone separates them. But earlier he had stated that the eighth mode is an octave higher than the hypodorian, the lowest mode; hence this text describes it as "the eighth mode, which Ptolemy added at the top." But H:P is not the diapason species of the hypermixolydian mode; for its species is from the proslambanomenos to the mese (a-a; cf. diagram of Chapter XVI, p. 283), whereas H:P is an octave higher than the hypate hypaton to the paramese (b-b). Thus this text leaves some clarity to be desired with regard to the species of this eighth mode. Nevertheless the basic argument seems clear: a bisdiapason will contain eight diapason species; therefore there may be eight modes or tropes; Ptolemy places this eighth mode "at the top, i.e., an octave above the hypodorian, a tone higher than the mixolydian.

B will form a right angle with AD; the line leading from curve F to point C should also form a right angle. Let these hemispheres be fitted and finished very carefully and thoroughly at every point and let them be prepared equal to each other for this use. Let a string equal on all sides be stretched over AEFD. Now if I want to discover how a diatessaron consonance sounds, I do as follows. I divide EF--the line from E where the string touches a hemisphere to F where the other end of the string touches a hemisphere--into seven parts, and I place a mark at the fourth part of these seven. This mark is K. Therefore EK to KF is a sesquitertian. Thus if I place a hemisphere equal to the other hemispheres at K, and if EK and KF are both in turn struck with an additional plectrum, the interval of a diatessaron will resound, whereas if they are both struck at the same time, I thus come to know a diatessaron consonance.[57] If we

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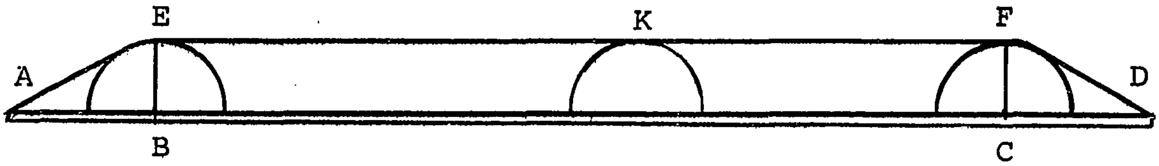
57. Alterutra vicissim. . . diatessaron distantia consonabit, sin vero simul utrasque percussero, diatessaron consonantiam nosco. This is a clear reference to harmony as simultaneous sounds as distinct from interval as successive sounds (cf. Commentary, Chapter IV, pp. 440-443.)

should want to make a diapente, I divide the total into five parts, and I will give three parts to one portion and two to the other. Thus, with the hemisphere in position according to the manner discussed above, I will judge the consonances and dissonances. Moreover, if I should want to try a diapason consonance, I divide the total into three parts and distribute two parts to one side and one to the other; thus by striking them alternately or at the same time I come to know what sounds consonant or dissonant. The triple proportion, which is born from intermingling consonances, is produced as follows: if we partition the total into four parts, and then if the whole length of string is divided into three and one of these parts, the hemisphere positioned at this point will produce the consonance and dissonance of the triple proportion.

4 : 3



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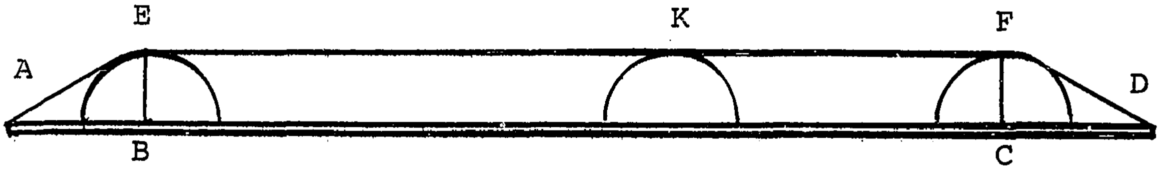

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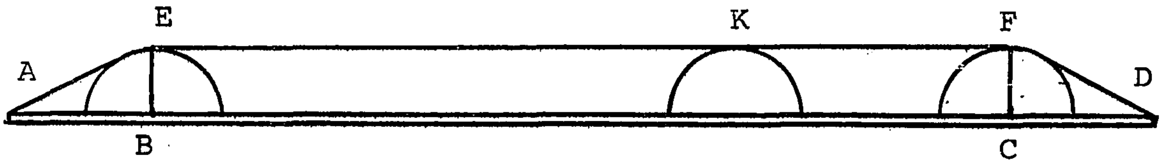

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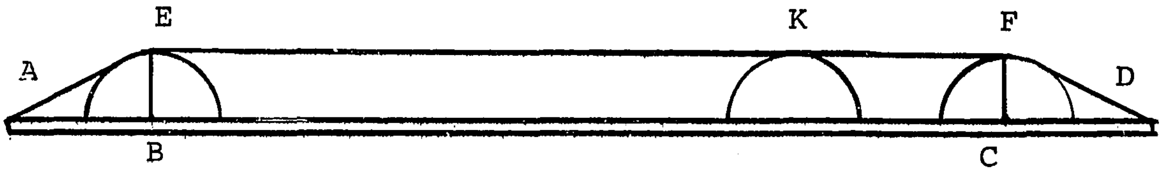

BOOK V

I. Introduction

Following the division of the monochord ruler I deem it necessary to add a discussion of those points in which the ancient musical scholars show a diversity of opinion. It is necessary to possess a keen power of discrimination concerning all the things discussed in this work; and that which has been omitted from the scheme of this work must be filled in through the use of ordinary erudition. For example, another monochord division is possible in which not only one string is divided according to the proportions we have proposed, but rather eight strings, as in the case of the kithara. In this way the complete ratio of the proportions may be discerned right before the eyes, in whatever number of strings might be necessary.

II. Concerning the potency (vis) of harmony, the instruments used in judgment of harmony, and to what extent it is necessary to trust the senses.

But we will discuss this matter a little later.

Now it is necessary to discuss the potency of harmony; for we have filled four books organizing its basic elements, but we have saved the description of its nature and potency for the course of this fifth book.[1] Harmony is the faculty of considering the differences between high and low sounds using the reason and senses. For the senses and reason are considered instruments of this faculty of harmony. The sense focuses its power on something confused but yet approximate to that which it senses. But reason judges truth, and searches out the ultimate difference. Thus the sense finds something confused but close to the truth, but one accepts the truth from reason. Indeed reason alone finds the truth, though it receives a confused but approximate likeness of the truth; for the

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1. Boethius never returns to the matter of the kithara as he predicted in the first sentence of this chapter. The remainder of Book V is a somewhat paraphrased translation of Ptolemy Harmonics, Book I. At this point Boethius begins translating Chapter i. 3-5 of this source. Henceforth each chapter heading will be noted, and the corresponding chapter of Ptolemy will be cited. Thus Ptolemy is also the key source of the discussions and criticisms of Aristoxenus, Archytas, and "the Pythagoreans" in this book. Nevertheless, wherever possible, references to these writers will also be noted.

senses take in nothing of truth itself, but come only to an approximation. Reason is the final judge.

If someone, for example, drew a circle with his hand, the eye might judge it to be a true circle; but the reason would know that it is not that which it appears to be. This happens because the sense is oriented toward matter, and it grasps the species in those things which are in flux and imperfect, and which are not determinate and refined to the last degree (ad unguem expolitae), just as matter itself is. Thus confusion accompanies the senses, but matter does not impede the mind or reason. Rightness and truth accompany reason, for reason most ably corrects or completes that which is mistaken or missing in sense perception. But that which the sense as an unskillful appraiser acknowledges as true, and in reality is incomplete, confused, and less than truth, would have little consequence taken as a singular instance; but if errors be collected and multiplied, they will make a very great and noticeable difference. If, for example, the sense judges two tones to stand at the distance of a tone, and likewise thinks a third sound stands a tone away from one of these notes, and this is not the true and whole distance of a tone; and again if the sense regards the distance between a third and a fourth note that of a tone, and there it errs again, for it is not the distance of a tone; and should it regard the fifth note as a semitone from the fourth, and it judges this faultily; the error seems small taken as a singular instance. But if that which the sense left out of the first tone plus that which was mistaken in the second and third tones and the fourth semitone be collected together in one succession, it would not contain a diapente consonance from the first to the fourth note; and this must occur if three whole tones plus a semitone be added together. Thus the error which appears minor in the singular tones becomes quite evident if collected in a consonance.

So that it might be clearly seen that the senses bring things together in error and by no means ascend to the truth of reason, let us consider the following: it is not difficult for the senses to find a line longer or shorter than a given line. But this initial perception by the senses will not be able to find a line longer or shorter by some definite quantity; this the skillful finding power of reason will have to determine. Suppose the problem be to double a given line, the sense might nevertheless be able to succeed. But if the problem be to find a line three times as long, or a third as long, or four times as long, or a fourth as long, will not this be impossible for the senses unless the fullness of reason partakes in the deduction? Thus the importance of the senses decreases, because through such a process the place of reason becomes more important. If someone asks you to seek out the eighth part of a given line, or to give a line eight times its length, then you will have to take half of the line, then half of the half to make a fourth, then half of the fourth to arrive at the eighth; or in the opposite case, you will have to double the line, then double the double and arrive at a line four times the length of the original, then double this to arrive at a line eight times longer than the original. In such a case where so many things must be figured and counted, the senses can do nothing; for their whole judgment is quite unprepared and superficial, and thus does not reveal truth and perfection. Therefore all judgment should not be given to the ears, but reason should also be used, for reason rules and tempers the erring senses as though they, wavering and failing, were leaning on a walking cane.[2]

Thus just as each art has its particular instruments, some which inform rather confusedly, like the adze (*aciscu-lus*), and others which disclose the truth, like the compass, thus also the potency of harmony has two parts of judgment: the first part grasps the differences of given sounds through the senses, and the other part considers the true quantity and measure of these same differences.

III. *Concerning the harmonic ruler, or what the Pythagoreans, Aristoxenus, and Ptolemy have said the application of harmony is.*[3]

This type of instrument which is used in searching out the differences between sounds is called a harmonic ruler. The opinions of various scholars is in great discord concerning this matter. For those who followed the disciples of Pythagoras have said the purpose of harmony is that all fitting things be delegated to

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2. This simile is Boethius' addition to Ptolemy's argument.

3. Ptolemy *Harmonics* i. 2. 5-6.

reason.[4] It is as though the senses sow the seeds of knowledge, and the reason causes them to grow.

Aristoxenus, to the contrary, said that the reason is merely an attendant and secondary element, and that all things are circumscribed by sensual judgment; and the movement of melody should be harmonized and organized by the senses.[5]

Ptolemy, however, presents another type of application for harmony. He holds that there is nothing irreconcilable between the ears and the reason. The solution to the problem of harmony according to Ptolemy is that the senses inform, and then the reason decides the proportion. Thus lest the senses contradict reality, the application of harmony joins these two faculties into a union. And this theory refutes those of Aristoxenus and the Pythagoreans: for Aristoxenus would not trust the reason, but believed only the senses, while the Pythagoreans hold to the reliability of reason in harmonic proportions because they have no faith in the senses.

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4. See, for example, Pseudo-Plutarch De Musica 1144 F; but cf. Book I. ix, pp. 57-59.

5. Aristoxenus Harmonics, ii. 33-34.

IV. Concerning the opinions of Aristoxenus, the Pythagoreans, and Ptolemy as to the makeup of high and low sound.[6]

Everyone agrees that sound is the percussion of the air, but Aristoxenus and the Pythagoreans hold different opinions concerning the difference between high and low sounds. Aristoxenus holds that the difference between high and low sounds is qualitative,[7] while the Pythagoreans hold that it is quantitative.[8]

Ptolemy seems to agree with the Pythagoreans in this matter. He thinks that high and low sounds consist of quantity rather than quality; for a tighter and finer body emits a high sound, whereas a longer and looser body produces a low sound. (Although here nothing is said concerning the tightening or loosening of these bodies; for when something is loosened, its vibration becomes less frequent and coarser,

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6. Ptolemy Harmonics ii. 3. 6-9; this chapter is a highly condensed paraphrase of the source.

7. Although "qualitative" forms a good corollary to "quantitative," it is a rather superficial and inadequate description of Aristoxenus' theory of pitch; cf. Aristoxenus Harmonics i. 10-13.

8. See, for example, Sectio Canonis, Introduction (JanS. 148-149); Book IV. i. pp. 211-212.

whereas if something be stretched tighter, its vibration becomes more frequent and keener.)[9]

V. Concerning Ptolemy's opinions on the differences of sounds.[10]

Having presented this matter, Ptolemy divides the differences of sounds as follows. Some sounds are in unison, whereas others are not. Those sounds are in unison which produce one pitch, either high or low. Sounds are not in unison when one is lower and another is higher. Some of these non-unison sounds are joined by a common boundary; for they are not disconnected, but lead from a low to a high pitch in such a way that they seem continuous. Other non-unison sounds are separated from each other by an intervening silence. Those sounds joined by a common boundary are like a rainbow; its colors are so close to each other that there is no definite boundary. Thus if one color is to be distinguished from the

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9. This parenthesis represents a concession to Aristoxenus, who held that tension and relaxation were the "causes" of high and low "quality" of sounds (Harmonics i. 10-13).

10. .olemy Harmonics i. 4. 9-10.

other, for example, where the red ends and the yellow begins, you will find that the colors change through continuous mutation in such a way that there is no definite intervening space between them which separates them. This can also be the case in sounds: for example, if someone plays a string and then tightens it while he plays, at first a low sound comes forth, but when it is tightened, the same sound becomes higher, and thus a high and a low note are made from one continuous sound.

VI. Which sounds are fitting for harmony.[11]

Thus some non-unison pitches are continuous, and others are discrete. Continuous pitches are of such a nature that the distinction between the two pitches is united through a common limit, and one cannot point to a definite place which belongs to the high note and another which belongs to the low. Discrete pitches, however, have definite locations, just like unmixed colors whose difference may be seen by means of the place to which they are confined.

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11. Ptolemy *Harmonics* i. 4. 9-10.

Continuous non-unison pitches have no part in the harmonic faculty; for they are in themselves dissimilar, but produce no single definite pitch. Discrete pitches, however, are the objects of the harmonic discipline; for the difference of these distant and dissimilar pitches can be studied.[12]

Those pitches which can be joined together to make melody are called emmeleis, but those which cannot be joined together to make melody are called ekmeleis.

VII. The numbers which the Pythagoreans established for the proportions.[13]

Pitches are called consonant if they produce an intermingled and sweet sound; they are called dissonant if they do not. This is Ptolemy's opinion concerning the difference of sounds.

But now it seems necessary to discuss what other musicians have proposed as the disposition of consonances. The Pythagoreans hold that the diapente and diatessaron

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12. Cf. Book I, xii. pp. 63-64.

13. Ptolemy Harmonics i. 5. 11-12; although the first sentence of this chapter is the last sentence of the preceding chapter in Ptolemy.

are the simple consonances, and from these they join together one diapason consonance. There is also the diapente and diapason, and the bisdiapason, the former the triple, and the latter the quadruple. They hold that the diapason and diatessaron is not a consonance, for it falls into neither multiple nor superparticular proportions but in multiple superpartient proportions. The proportion of this interval is 8:3. If someone were to place a 4 between these numbers, it would produce these terms: 8:4:3. Eight to 4 produces a diapason consonance, whereas 4:3 makes a diatessaron. But 8:3 consists of a multiple superpartient proportion. What a multiple superpartient proportion is should be known from the book on arithmetic as well as the discussion of the second book of this work.[14] The Pythagoreans placed consonances in the multiple and superparticular types of inequality, as discussed in the second and fourth books of this work; but they separated consonances from the superpartient and multiple superpartient types of inequality. The manner in

14. <u>De Institutione Arithmetica</u> i. 31; and Book II. iv. pp. 108-109.

which the Pythagoreans joined the diapason with the duple proportion, the diatessaron with the sesquitertian, the diapente with the sesquialter should be recalled from the second and fourth books of this *Principles of Music*.[15]

VIII. *How Ptolemy attacked the Pythagoreans with regard to the number of proportions.*[16]

Ptolemy completely disagreed with that demonstration of the Pythagoreans which we have expounded in various ways in the above mentioned books of this work, namely that they join the diatessaron with the sesquitertian proportion and the diapente with the sesquialter proportions but apply no consonances whatsoever to the other superparticular proportions even though they are of the same genus.

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15. Concerning the "Pythagorean" (Nicomachus, Eubulides, Hippasus) disposition of the consonances, see Book II. xviii-xx. pp. 141-149. Concerning the associations of certain consonances with certain proportions see Book II. xxi-xxvii. pp. 149-161 , and Book IV. ii. pp. 213-221.

16. Ptolemy *Harmonics* i. 6. 13-14.

IX. Ptolemy's demonstration that the diapason and diatessaron is a consonance.[17]

He proves that the diatessaron and diapason produces a definite consonance as follows: since the diapason consonance produces a conjunction of sounds in such a way that the sound seems to come from one and the same string (even the Pythagoreans agree here), if a consonance were added to this, it would preserve a complete and unviolated consonance; for it is added to the diapason consonance as if it were one string. Consider the diapason consonance contained between the hypate meson and the nete diezeugmenon: these two pitches sound together and united in such a way that one sound, as if from one string rather than two mixed together, is heard. Thus if we were to join any consonance whatsoever to this diapason consonance, it would remain wholly consonant; for the diapason is united as if it were one sound and string. Thus if two upper diatessarons were joined to the hypate meson and the nete

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17. The theory presented in this chapter is implicit throughout Ptolemy Harmonics i. 7. 15-16; although it is logically based on Ptolemy's classification of musical intervals (cf. Chapter XI. pp. 310-312).

diezeugmenon, that is, if the nete hyperboleon were joined to the nete diezeugmenon, and the mese to the hypate meson, both would form a consonance with each other, and moreover the mese would form a consonance with both the nete diezeugmenon and the hypate meson, and the nete hyperboleon would form a consonance with both the nete diezeugmenon and the hypate meson. The same would be the case if two lower diatessarons were added, that is, the hypate hypaton would hold a diatessaron consonance in relation to the hypate meson, and likewise the paramese in relation to the nete diezeugmenon. And the hypate meson would form a consonance with both the hypate meson and the nete diezeugmenon, as would the paramese with both the nete diezeugmenon and the hypate meson, but in such a way that the lower string would hold a diatessaron consonance in relation to the string closest to it but a diapason and diatessaron in relation to the string farthest from it (that is, from the hypate hypaton to the hypate meson is a diatessaron, but from the hypate hypaton to the nete diezeugmenon is a diatessaron and a diapason). Likewise, the higher nete hyperboleon forms a diatessaron consonance with the closer nete diezeugmenon, but it forms a diatessaron and diapason with the hypate meson.[18]

X. Concerning the property of the diapason consonance.[19]

This is likely to happen because the diapason is often one sound, and a type of consonance that makes one single pitch. This consonance is like the number 10; for if some number within 10 be added to it, the number is preserved just as it was before, whereas with other numbers this is not the case. For if you add 2 to 3, you directly produce 5, and the species of the numbers has been changed. But if you add the same to 10, you will produce 12, and the 2 has been preserved in conjunction with the 10. The same applies to the number 3 and others numbers treated in this manner.

In the same way the diapason consonance can be united to any other consonance, and it preserves the consonance rather than changes it; and it does not yield a dissonance from a consonance. For just as a diapente consonance joined

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18. These computations from specific pitches are Boethius' contribution to Ptolemy's argument.

19. This chapter seems to be a "trope" by Boethius on Ptolemy's theory of the diapason, or the "equal-sounding" interval.

to a diapason preserves a diapason and diapente consonance, of course in the triple proportion, thus if a diatessaron also be joined to the diapason, it produces another consonance. Thus according to Ptolemy, the diapason and diatessaron should be added to the list of consonances; it is founded in the multiple superpartient type of inequality, for its proportion is a double superbipartient proportion, that is, 8:3. Eight contains three twice and two other parts of it, that is, two unities.

XI. *How Ptolemy set forth the consonances.*[20]

Ptolemy differs with the opinion of the Pythagoreans as follows: Here is the order by which he traces out the proportions and numbers of consonances. He says that pitches (*voces*) are either unison or non-unison. Some non-unison pitches are equal-sounding, some are consonant, some are melodic (*emmelis*), some are dissonant, and some are un-melodic (*ekmelis*).

Unison pitches are those which, vibrating individually, produce one and the same sound. Equal sounding

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20. Ptolemy *Harmonics* i. 7, 15-16.

pitches are those which, sounding at the same time, yield one and a seemingly simple sound from two sounds, for example, the diapason and its double, the bisdiapason. Consonant pitches are those which give a composite and intermingled, but nevertheless pleasant sound, for example, the diapente and the diatessaron. Those pitches are melodic which are not consonant but can nevertheless be rightly fitted to melody, for example the notes which join the consonances together. Dissonant pitches are those which do not mix sounds together and affect the senses unpleasantly. Un-melodic pitches are those which are not considered in conjunction with consonance. (We will discuss this type a little later in the division of the tetrachord.)[21]

Since equal sounding pitches come closest to unison pitches in these comparisons, it is necessary that the numerical inequality which is closest to equality be added to the numerical equality. The double proportion is quite near to numerical equality; for it is both the first species of multiple proportions, and, when the larger

21. Boethius never returns to this topic.

number is placed above the smaller number, it transcends the smaller number by the smaller number itself: two transcends one by one, which is equal to the very same unity. Thus the duple proportion is rightly fitted to equal-sounding pitches, that is, to the diapason. But the bisdiapason is twice the duple proportion, that is, the quadruple.

But those first and largest proportions which divide the duple proportion ought to be fitted to those consonances which divide the equal-sounding diapason. Whence it follows that the diapente is joined to the sesquialter, whereas the diatessaron is joined to the sesquitertian. Thus these consonances joined with equal-sounding consonances produce other consonances, for example, the diapente and diapason in the triple proportion, and the diatessaron and diapason in 8:3 proportion.

Melodic pitches are those which divide the diapente and the diatessaron, for example, the tone and the other proportions concerning which we will speak a little later in connection with the division of tetrachords; for the others are indeed the simple parts of tetrachords.

XII. Which intervals are equal-sounding, which are consonant, and which are melodic.[22]

The diapason and bisdiapason are equal-sounding, since by their temperament and nature they produce a seemingly single and simple sound. The first proportions of the superparticular genus are consonant: the sesquialter and the sesquitertian, or the diapente and the diatessaron. The diapason and diapente and the diapason and diatessaron are also consonant; for they are joined and united from equal sounding and consonant intervals. Those which remain are melodic, that is, those which are able to be placed between these consonances, for example, the tone is the difference between a diapente and a diatessaron. And just as the equal-sounding intervals are brought together from the consonant intervals, for example, the diapason from the diatessaron and the diapente, so the consonant intervals are brought together from those intervals called melodic, for example, the diatessaron and the diapente from tones and the other proportions which we will discuss

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22. Ptolemy *Harmonics* i. 7. 16.

later.[23] But to grasp how the proportion of all these sounds can be assembled, one should read the description of the string stretched over a hemisphere in the last part of the fourth book:[24] for in that place is found the equal-sounding diapason and the bisdiapason, the simple consonances of the diapente and the diatessaron, the composite consonances of the diapason and diapente and the diapason and the diatessaron, and the melodic sounds consisting of the difference of a tone.

XIII. How Aristoxenus considered the intervals.[25]

We should now briefly discuss the opinion of

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23. This reference would refer to Chapters xx-xxx of this book, which Boethius never completed.

24. See Book IV, xviii, pp. 291-293; this reference to the last chapter of the fourth book would represent, for Boethius, Ptolemy Harmonics i. 8. 16-19, for in this chapter Ptolemy demonstrates intervals on a monochord much like Boethius did at the last of Book IV. However, the last chapter of Book V does not present the diapason and diatessaron as this chapter claims.

25. This chapter represents a highly condensed paraphrase of Ptolemy Harmonics i. 9-10, 19-24. Boethius chose not to translate these chapters at any length because he had already offered the same criticisms and arguments which Ptolemy uses (cf. n. 27).

Aristoxenus concerning these things. For he placed little value on the rational process but yielded to aural judgment. Thus he does not present these same intervals in conjunction with numbers so that he might assemble their proportions but rather takes the difference between them, so that he might investigate not the intervals themselves but rather the difference between them.[26] He quite carelessly thought himself to know the difference between sounds to which he assigned no magnitude of measure. Thus he proposed that the diatessaron consists of two tones and a semitone, the diapente of three tones and a semitone, and the diapason of six tones, which was demonstrated in the above books to be impossible.[27]

XIV. Description of an octochord which demonstrates that the diapason is less than six tones.[28]

Ptolemy teaches that the diapason falls within six tones using a division of a type of octochord in the following manner. Let there be eight strings stretched forth:

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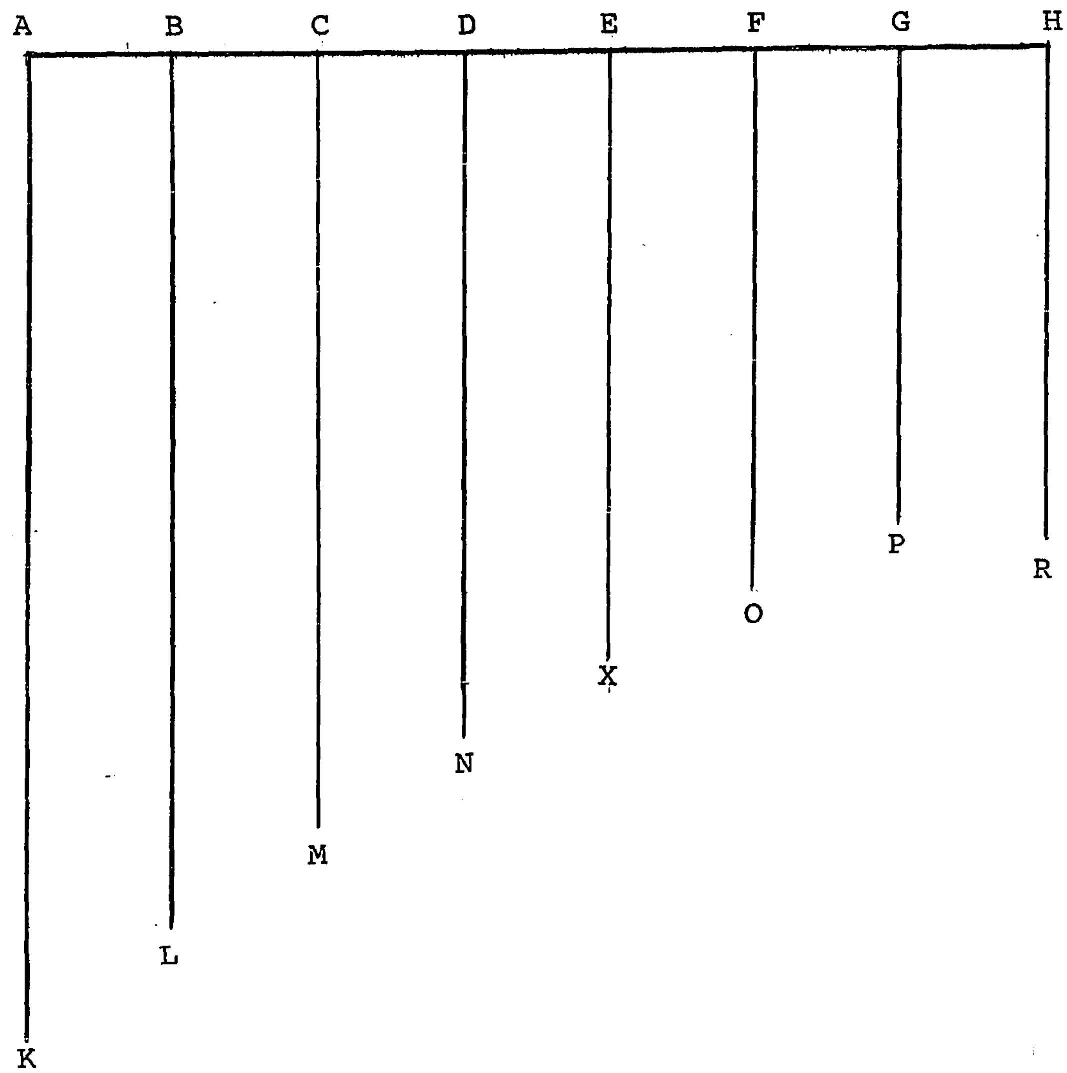
26. Aristoxenus *Harmonics* i. 15.
27. See Book II. xxxi pp. 167-170 ; and Book III. iii. pp. 178-180.
28. Ptolemy *Harmonics* i. 11. 25-28.

A, B, C, D, E, F, G, and H. And let AK be the sesquioctave proportion of BL, BL that of CM, CM that of DN, DN that of EX, EX that of FO, and FO that of GP. Thus these will be six tones. Furthermore, let the string H be divided in the middle at R, and thus AK will be the duple of HR. Thus when AK and HR are struck at the same time, they yield the consonant sound of the equal-sounding diapason. But if someone strikes GP, it will always be a little sharper, and therefore six tones transcend the diapason consonance. For if AK and GP sounding together resounded a diapason, then the diapason consonance would consist of six tones. But if these do not form a consonance, and AK and HR do, and if HR be higher than GP, then the diapason consonance would exceed six tones. But since AK and HR form a consonance, and the same HR is found to be lower than GP, there is no doubt that six tones exceed a diapason consonance. And thus it is also possible to prove with the senses that the diapason consonance falls within six tones. Therefore, without a doubt, the error of Aristoxenus is incontestably proved.



XV. <u>That a diatessaron consonance is contained in the tetrachord.</u>[29]

Now the division of the tetrachords must be discussed. The diatessaron consonance is made of four strings, and, in

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29. Boethius inserts this chapter as a transition to the tetrachord divisions which follow. It does not seem to be a translation of Ptolemy.

fact, it is called "diatessaron" for this reason. Thus so that the tetrachord might be made, when the two outer strings have been set in place and they are sounding the diatessaron consonance, it is necessary that two strings be set out in the middle, and these strings in turn form three proportions with themselves and the outer two strings.

XVI. How Aristoxenus divided the tone and the genera, and a diagram of the same.[30]

Aristoxenus divided the diatessaron into the three genera using the following rationale.[31] He divided the tone into two parts and called this the semitone. He divided the tone into three parts and called this third part the diesis of the soft (*mollis*) chromatic genus. He divided the same into four parts, and this fourth part plus its own half, that is, plus an eighth part of a whole, he called the diesis of the hemiolic chromatic genus. Thus since these are so arranged and since th· division of the genera according to him is two-fold, one genus is softer

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30. Ptolemy *Harmonics* i. 12. 28-30.
31. Aristoxenus *Harmonics* i. 24-27.

(*mollius*), while another is sharper (*incitatius*). The softer is the enharmonic genus, whereas the sharper is the diatonic. The chromatic genus occupies a place between these, partaking of the soft and the sharp. Thus according to this order there arises six different types of intermingled genera: one enharmonic genus; three chromatic genera, the soft chromatic genus, the hemiolic chromatic genus, and the tonic (*toniaei*) chromatic genus; and the remaining two are the soft and sharp diatonic genera.

The division of all this according to Aristoxenus is as follows. Since it has been said that the fourth part of a tone is called the enharmonic diesis and since Aristoxenus does not compare the sounds themselves among themselves but rather measures the difference of the sounds and intervals, according to Aristoxenus the tone consists of 12 unities. Thus a fourth part of the tone, the enharmonic diesis, will be 3. Moreover, since the diatessaron consonance is joined together from two tones and a semitone, the whole diatessaron will be made from 2 x 12 + 6. But since, as often happens, we will want to deduce eighth parts of a tone, the total diatessaron should be represented by the number 60; in this way we should arrive at these eighth parts, not in integral numbers but rather in numbers which are to some extent plural. Thus the tone will be 24, the semitone 12, the fourth part of a tone (the enharmonic diesis) 6, and the eighth part 3. When the eighth part is joined to a fourth part, thus producing the hemiolic chromatic diesis, 6 is added to 3, and this will make 9.

Having thusly constructed the three genera (enharmonic, chromatic, and diatonic), they seemed to Aristoxenus to have these properties: he called some compact (_spissus_) and others non-compact (_non spissus_). Those genera whose two lower proportions do not surpass in size the one higher proportion opposite them are compact; whereas the genera with two proportions which can transcend the one remaining proportion are non-compact. Thus the enharmonic and chromatic genera are compact, whereas the diatonic is non-compact.

The enharmonic genus according to Aristoxenus is divided 6, 6, 48, so that between the lowest string and the next to lowest string there is a fourth part of a tone (since the tone is founded in 24 unities), and this fourth part is called the enharmonic diesis. Moreover the second interval between the second to lowest string and the third string is a similar fourth part of a tone. The remainder from the total proportion 60 is found between the third to lowest string and the highest fourth string, and that remainder is 48. Thus the two proportions placed at the lower end, 6 and 6, do not surpass the one remaining at the higher end, 48.

The soft chromatic genus produces the division 8, 8, 44, so that 8 and 8 might be third parts of tones. For the tone, as has been said, is 24, and a third part of a tone is called the soft chromatic diesis.

The hemiolic chromatic diatessaron is divided 9, 9, 42; for the hemiolic chromatic diesis is an eighth part of a tone plus a fourth part: an eighth part of 24 is 6, therefore 6 plus 3.

The tonic chromatic genus according to Aristoxenus yields the division 12, 12, 32, so that it consists of single semitones in two intervals plus that which remains. And in all of these divisions the two proportions next to the lower note do not surpass in size the remainder which is placed at the top. For they, as has been said, are the compact genera. Thus the enharmonic and chromatic genera are compact.

The diatonic division is also two-fold. The division of the soft diatonic genus is 12, 18, 30, so that 12 might be a semitone, 18 a semitone plus a fourth part of a tone, and 30 the remainder. Eighteen plus 12 makes 30, and this is not surpassed by that part which remains.

The sharp diatonic genus is divided in such a way that it might have a semitone and two whole tones, that is, 12, 24, 24. Twenty-four plus 12, that is 36, is not surpassed by the part remaining above, but rather it surpasses the remainder.

Therefore the division of the tetrachords according to Aristoxenus which will be shown in the following description has already been discussed.

Enharmonic	Soft Chromatic	Hemiolic Chromatic
48	44	42
6	8	9
6	8	9
60	60	60

Tonic Chromatic	Soft Diatonic	Sharp Diatonic
36	30	24
12	18	24
12	12	&&12
60	60	60

XVII. How Archytas divided the tetrachords and a description of the same.[32]

Archytas, giving all to reason, neglected to observe the sense of hearing not only in the first consonances but even followed the reason in the division of the tetrachords. But he did so in such a way that he did not effectually extricate that which he sought, nor did the rationale set forth by him agree with the senses. For he held that there were three genera: the enharmonic, the chromatic, and the diatonic. He established the same lowest and highest notes in these genera, making 2016 the lowest sound in all genera and 1512 the highest. He placed 1944 as the note next to the lowest in the three genera, so that it would hold a sesquitwenty-seventh (28:27) proportion in relation to 2016. Then he placed below the highest note the third note from the lowest in the enharmonic genus; this should be 1890, which is joined to 1944 by a sesquithirty-fifth (36:35) proportion. Moreover, there would be a sesquiquartian proportion (5:4) between 1890 and

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32. Ptolemy *Harmonics* i. 13. 30-31; this is the only source of Archytas' tetrachord division which has survived.

the highest 1512. Likewise he positioned the third note from the lowest, or second from the highest in the diatonic genus; this should be 1701, which is joined to 1944 in a sesquiseventh (8:7) proportion. The same 1701 forms a sesquioctave with the highest 1512. Then he placed the third note from the lowest, or second from the highest in the chromatic genus, which should hold the proportion 256:243 in relation to 1701, the third note from the lowest in the diatonic genus; this number is 1792, which is placed second to the highest. And the note second from the highest in the diatonic genus (1701) to the note second from the highest in the chromatic genus (1792) holds the 256:243 proportion. The following description shows the form of these divided tetrachords according to the opinion of Archytas.

Enharmonic	1512	1890	1944	2016
Chromatic	1512	1792	1944	2016
Diatonic	1512	1701	1944	2016

XVIII. How Ptolemy considered the tetrachord divisions of Aristoxenus and Archytas.[33]

But Ptolemy criticizes both of these tetrachord divisions. He rebukes Archytas first of all because the second to highest note in the chromatic genus--1792--forms a superparticular proportion with neither the note above it--1512--nor the note below it--1944, while Archytas considered superparticular proportions to be of such dignity that he even accepted them in the rationale of consonances. Furthermore the sense comprehends the first proportion in the lowest part of the chromatic genus as larger than Archytas makes it; for he makes it a sesquitwenty-seventh proportion in the chromatic genus--1944:2016, while it ought to have been a sesquitwenty-first (22:21) according to usual modulation. Likewise that proportion in the enharmonic genus, which retains the first position from the lowest according to the division of Archytas, is such that it turns out much smaller than it ought to be in the other genera; however, he makes it equal to that of the other genera, since he places the first sesquitwenty-seventh as the lowest proportion in the three genera.

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33. Ptolemy *Harmonics* i. 14. 32.

He reproaches Aristoxenus because he would have placed the first and second proportions from the lowest note in the soft and hemiolic chromatic genera in such a way that the sense could practically never distinguish a difference between them. For the first proportion in the soft chromatic genus--the first division of Aristoxenus--is 8, whereas the same in the hemiolic chromatic genus is 9. But 8 and 9 are separated by unity. The total tone consists of 24 unities according to his position, and unity is thus 1/24 part of a tone. Therefore between the first lower proportions of the soft and hemiolic chromatic genera is the distance of 1/24 part of a tone, which because of the smallness of this difference, can not possibly be sensed by the hearing. Furthermore, he rebukes Aristoxenus because he would have made only two divisions of the diatonic genus, since he divided it into a soft and a sharp genus; for other species of diatonic genera could also be discovered.

XIX. The manner in which Ptolemy said the division of tetrachords should be accomplished.[34]

Ptolemy divides the tetrachords in accordance with

34. Ptolemy *Harmonics* i. 15. 33.

a different rationale. He begins with the principle that the notes between the two outer sounds should be fitted in such a way that they proceed through superparticular proportions, nevertheless unequal; for a superparticular proportion cannot be equally divided. Moreover, the division should be such that every comparison which is made to the lowest string should be smaller than the other three which are contained in the higher strings. But the divisions which we call compact ought to be such that the two proportions which are lowest should be smaller than the proportion which remains at the top; but in the non-compact genera, two notes are nowhere surpassed by the remaining one, as in the diatonic genus.

XX. How inequality of proportions is made from equality.[35]

[The Principles of Music II. vii. Multiple and

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35. Boethius never completed these concluding chapters of Book V. Nevertheless he left the titles, and the basic theory that would have been contained in these chapters can be surmised by comparing the titles with theory already discussed in The Principles of Music and the theory contained in the last two chapters of Ptolemy's Harmonics, Book I. The source for each of these chapters will thus be cited in brackets under each heading, and a brief outline of the theory which each chapter would have contained will be presented.

superparticular proportions are the first departures from equality, and Ptolemy, like the Pythagoreans, restricts musical speculation to those two types of inequality even in the division of the tetrachord.]

XXI. How Ptolemy divided the diatessaron into two parts.

[Ptolemy *Harmonics* i. 15. 33-34. 4:3 can be divided into three pairs of superparticular proportions: (5:4) x (16:15); (6:5) x (10:9); and (7:6) x (8:7).]

XXII. Which genera are compact and which are not; and how the proportions are fitted to the division of the enharmonic genus by Ptolemy.

[Ptolemy *Harmonics* i. 15. 34, 35-36. Genera are compact (*pyknon*) if the first interval is larger than the sum of the two remaining intervals, while they are non-compact (*apyknon*) if the first interval is smaller than the sum of the two remaining. Therefore compact genera divide the smaller superparticular proportions of XXI into two superparticular proportions, while non-compact genera divide the larger proportions of XXI into two superparticular proportions.

Ptolemy Harmonics i. 15. 34-35. The enharmonic genus is divided as follows: (5:4) x (24:23) x (46:45); rationale: (24:23) x (46:45) equals 16:15 of XXI.]

XXIII. Ptolemy's division of the soft chromatic genus.

[Ptolemy Harmonics i. 15. 34-35. The soft chromatic genus (chroma malakon) is divided as follows: (6:5) x (15:14) x (28:27); rationale: (15:14) x (28:27) equals 10:9 of XXI.]

XXIV. Ptolemy's division of the sharp chromatic genus.

[Ptolemy Harmonics i. 34-35. The sharp chromatic genus (chroma syntonon) is divided as follows: (7:6) x (12:11) x (22:21); rationale: (12:11) x (22:21) equals 15:16 of XXI.]

XXV. Disposition of Ptolemy's compact genera with numbers and proportions.

[Ptolemy Harmonics i. 15. 35.

Enharmonic:	(5:4)x(24:23)x(46:45)	106260:132825:138600:141680
Soft Chrom.	(6:5)x(15:14)x(28:27)	106260:127512:132620:141680
Sharp Chrom.	(7:6)x(12:11)x(22:21)	106260:123970:135240:141680]

XXVI. Ptolemy's division of the soft diatonic genus.

[Ptolemy *Harmonics* i. 15. 35-36. The soft diatonic genus (*diatonon malakon*) is divided as follows: (8:7) x (10:9) x (21:22); rationale: (10:9) x (21:20) equals 7:6 of XXI].

XXVII. Ptolemy's division of the sharp diatonic genus.

[Ptolemy *Harmonics* i. 15. 35-36. The sharp diatonic genus (*diatonon synaton*) is divided as follows: (10:9) x (9:8) x (16:15); rationale: (9:8) x (16:15) equals 6:5 of XXI].

XVIII. Ptolemy's division of the tonic diatonic genus.

[Ptolemy *Harmonics* i. 15. 35-36. The tonic diatonic genus (*diatonon toniaion*) is divided as follows: (9:8) x (8:7) x (28:27), called "tonic" because its first interval consists of a true "tone." Rationale: 9:8 itself forms a tone, but no superparticular proportion forms 4:3 in conjunction with 9:8; 9:8 was just considered in conjunction with 10:9, but not in conjunction with 8:7, and if (9:8) x (8:7) be subtracted from 4:3, 28:27 remains.]

XXIX. Disposition of the various [non-compact] genera with numbers and proportions.

[Ptolemy *Harmonics* i. 15. 37.

Soft Diat.: (8:7)x(10:9)x(21:20) 504:576:640:672

Sharp Diat.: (9:8)x(8:7)x(28:27) 504:576:648:672

Tonic Diat.: (10:9)x(9:8)x(16:15) 504:560:630:672]

XXX. Ptolemy's division of the equal diatonic genus.

[Ptolemy *Harmonics* i. 16. 38-40. The equal diatonic genus (*diatonon homalon*) is divided as follows: (10:9) x (11:10) x (12:11), that is, "arithmetically" equal, 12:11:10:9. This chapter would have also stated that the diatonic divisions are the most pleasing genera, and it would have concluded with the following comparison of the Equal Diatonic division with the Pythagorean Diatonic division (*diatonon ditonion*):

Equal Diat.:(10:9)x(11:10)x(12:11) 18:20:22:24
576:640:704:768

Diatonic: (9:8)x(9:8)x(256:243) 192:216:243:256
576:648:729:768]

PART II

COMMENTARY

CHAPTER I

THE SOURCES OF *THE PRINCIPLES OF MUSIC*

All the works of Boethius with the exception of the *Commentary on Cicero's Topics*, the *Opuscula Sacra*, and the *Consolation of Philosophy* are basically translations. But these are by no means translations in the modern sense of the word; for Boethius always feels free to paraphrase and reorganize the original text to some extent, and he is not always careful to indicate where the translation ends and his own commentary begins. Nevertheless these are works which are largely dependent on pre-existent Greek sources. Thus any discussion concerning a given work of Boethius must begin with an explication of the work or works which served as Boethius' source. Only thereafter may the contents of the work at hand be discussed in its proper light. Therefore this commentary on Boethius' *The Principles of Music* will begin with an examination of the Greek and Latin sources Boethius used in compiling this work; for these sources determine Boethius' aesthetic and mathematical theory. The aesthetic and mathematical theory in turn give birth to the musical theory presented in Boethius' work. The commentary will finally examine briefly the unique place which The Principles of Music held in the Middle Ages.

The problem of discovering the Greek works from which Boethius translated and compiled The Principles of Music is not as simple a task as it would be with regard to The Principles of Arithmetic; for Boethius acknowledges the fact that this work is a translation of Nicomachus' Introduction to Arithmetic.[1] Cassidorus is more frustrating than helpful concerning this matter; for while he cites the works on arithmetic and geometry as translations of Nicomachus and Euclid, and the lost work on astronomy as a translation of Ptolemy, he calls the work on music a translation of Pythagoras.[2] This is impossible, not only because Pythagoras never wrote such a work, but also because The Principles of Music abounds in citations of authors who lived long after Pythagoras. Cassiodorus probably meant to

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1. De Institutione Arithmetica, ed. Friedlein (Leipzig: Teubner, 1866), "Praefatio," p. 4, l. 31.

2. Cassiodorus Variae, I. 45, Migne Patrologia Latina (Paris: 1865) lxix, 539 C.

say that the work was a translation of Pythagoreans, which nevertheless identifies the work as a translation. No single extant work, however, can be identified as the source of *The Principles of Music*.

If Boethius refrained from citing any one work as the source of his work on music, he by no means hesitated to give credit to authors and sources of particular theories. The following outline presents the chapters of *The Principles of Music* which expressly cite their sources:

Book I.,	Chapter I.	Plato Cicero Statius
	II.	Aristotle
	III.	Ptolemy
	VI.	Ptolemy
	IX.	Pythagoreans
	XX.	Nicomachus
	XXVI.	Albinus
	XXVII.	Cicero
	XXX.	Plato
	XXXI.	Nicomachus
	XXXII.	Nicomachus
Book II.	IV.	*The Principles of Arithmetic*
	VII.	*The Principles of Arithmetic*
	XII.	*The Principles of Arithmetic*
	XIV.	*The Principles of Arithmetic*
	XV.	*The Principles of Arithmetic*
	XVII.	*The Principles of Arithmetic*
	XIX.	Eubulides Hippasus
	XX.	Nicomachus
	XXIII.	*The Principles of Arithmetic*

Book II., Chapter	XXVII	Pythagoreans
		Nicomachus
		Ptolemy
	XXXI.	Aristoxenus
Book III.	I.	Aristoxenus
		The Principles of Arithmetic
	III.	Aristoxenus
	V.	Philolaus
	VIII.	Philolaus
	XI.	Archytas
Book IV.	XVII.	Ptolemy
Book V.	III.	Aristoxenus
		Pythagoreans
		Ptolemy
	IV.	Aristoxenus
		Pythagoreans
		Ptolemy
	V.	Ptolemy
	VII.	Pythagoreans
	VIII.	Ptolemy
		Pythagoreans
	IX.	Ptolemy
	X.	Ptolemy
	XI.	Ptolemy
	XIII.	Aristoxenus
	XIV.	Ptolemy
	XVI.	Aristoxenus
	XVII.	Archytas
	XVIII.	Ptolemy
		Archytas
		Aristoxenus
	XIX.	Ptolemy

Besides these specific citations given by Boethius, the following works can be identified as possible sources for given sections of The Principles of Music:

Book I., Chapter	III.	Nicomachus, Manual IV
	VIII.	Nicomachus, Manual XII
	XII.	Nicomachus, Manual II
	XIII.	Nicomachus, Manual II
	XXII.	Nicomachus, Manual XII
Book II.	III.	The Principles of Arithmetic
	V.	The Principles of Arithmetic
	VIII.	The Principles of Arithmetic
Book III.		None
Book IV.	I.	Sectio Canonis
	II.	Sectio Canonis
	III.	Alypius, Isagoge
	XIV.	Ptolemy, Harmonics II
	XV.	Ptolemy, Harmonics II
	XVI.	Ptolemy, Harmonics II
	XVII.	Ptolemy, Harmonics II
Book V.	I-XIX.	Ptolemy, Harmonics I

Based on these outlines the following generalizations can be made: Books I, II, and V present the most thorough documentation; Books III and IV present comparatively few specific citations; Nicomachus and The Principles of Arithmetic appear to be the principal sources of Books I and II; and Ptolemy appears to be the principal source of Book V. If Book V of The Principles of Music is compared with Book I of Ptolemy's Harmonics, it becomes immediately apparent that Boethius' Book V is basically a translation of Ptolemy's Book I. Therefore at least the last book of The Principles of Music is dependent on a pre-existing Greek source. No such existing work, however, can be identified as the source for the first four books.

Nicomachus is cited in Books I and II in much the same way that Ptolemy is cited in Book V, but these books are not a translation of any extant work of Nicomachus. On the one hand, certain sections of Book I appear to be rather literal translations of Nicomachus' Manual,[3] but these are passages in which Nicomachus is not specifically cited. On the other hand, the chapters in which Nicomachus is specifically cited are impossible to relate to extant works of Nicomachus. Thus the problem of works by Nicomachus which might have been available to Boethius but which have not survived to the present must be solved before Boethius' Nicomachus source can be established.

The main musical work by Nicomachus which has survived is his Manual of Harmonics,[4] a work which is little more than a superficial introduction to the general area of what the Greeks considered to be the scholarly discipline of music.

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3. See Book I. iii. p. 48 , n. 22; viii. pp. 56-57, nn. 29-32; xii. p. 63 , n. 37; xxii. pp. 85-89 , n. 54.

4. Nicomachus Ἁρμονικον ἐγχειριδιον , Karl Jan, Musici Scriptores Graeci (Hildesheim: Georg Olms [reprint of 1894 edition], 1962), pp. 237-265.

This work is addressed to a noble lady, written on her request,[5] and in the introductory sentences Nicomachus acknowledges the fact that this exposition of the musical discipline is somewhat limited.[6] Furthermore, he promises a more complete musical work as soon as he has the time to compose it.[7] This promise is repeated throughout the work in conjunction with specific discussions: in Chapter III he promises more concerning the harmony of the spheres;[8] at the end of Chapter XI he promises more concerning the addition of notes, their inventors, and the times and circumstances of their inventions;[9] Chapter XII promises more concerning musical proportions,[10] as well as further discussion of the octave, its merit, and that it consists of five tones and two semitones rather than six tones.[11] Nicomachus' Manual concludes with an apology for the brevity of the work, and again promises a much more complete work on music.[12]

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5. Nicomachus Manual I (JanS. 237, 15).
6. Ibid. (JanS. 238, 6 ff.).
7. Ibid.
8. Ibid. iii. (JanS. 242, 11 ff.).
9. Ibid. xi. (JanS. 260, 4 ff.).
10. Ibid. xii. (JanS. 261, 18).
11. Ibid. (JanS. 264, 1 ff.).
12. Ibid. (JanS. 265)

A comparison of these promises with the chapters of The Principles of Music which specifically cite Nicomachus as their source proves most helpful in identifying Boethius' Nicomachus source. Book I, Chapter XX cites Nicomachus concerning the additions of strings, giving their inventors and the circumstances of their invention. This discussion is found nowhere in the Manual, but it does fulfill the promise of Manual, Chapter XI. Book I, Chapters XXXI and XXXII cite Nicomachus concerning his theory of consonance and the merits of various consonances, especially the diapason. Again no such discussion is found in the Manual, but such a discussion would result from the second promise given in Manual, Chapter XII. The theory attributed to Nicomachus in Book II, Chapters XX and XXVII, is absent from the Manual, but this theory could be considered answers to the promises of Manual, Chapter XII, concerning proportions and the diapason. It seems logical to conclude that Nicomachus did write the larger musical work which he promised, and that this work was Boethius' source for the theory he relates to Nicomachus.

Further evidence for the existence of another more complete musical work by Nicomachus are the Nicomachus

Fragments,[13] short passages concerning music which could be a part of a larger work on music. These *Fragments* aid to some extent in supplying some cited and uncited Nicomachus texts as possible sources for *The Principles of Music*. Boethius' first Nicomachus citation discusses the additions of strings and their inventors as follows:

Four strings	Mercury
Fifth string	Toroebus
Sixth string	Hyagnis
Seventh string	Terpander
Eighth string	Lycaon of Samos
Ninth string	Prophrastus of Periotes
Tenth string	Histiaeus of Colophon
Eleventh string	Timothius of Melesia

It must first be pointed out that the account of the first eight strings does not agree with *Fragment I* and the *Manual*.[14] But the account of the ninth, tenth, and eleventh strings agrees essentially with *Fragment IV*.[15] (Boethius accredits the addition of the ninth to Prophrastus of Periotes whereas Nicomachus' *Fragment IV* accredits this addition to Theophrastus of Periotes; nevertheless, this disagreement could be the result of confusion in orthography.) Moreover, this

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13. Nicomachus *Fragments*, Jan *Musici Scriptores Graeci*, pp. 266-282.
14. See Book I. xx. pp. 72-73, n. 43.
15. See Book I. xx. pp. 77-78 , nn. 50-52.

fragment concludes with a discussion of the genera and the number of notes in the Greater Perfect System, the exact same contents which follow Boethius' discussion of the additions of notes and their names.[16]

The discussion of notes and their related stars, Book I, Chapter XXVII, is closely related to _Fragment III_; for the comparisons given by Boethius and by _Fragment III_ are identical, whereas the comparisons of Venus and Mercury found in _Manual_, Chapter III, agree with neither Boethius nor _Fragment III_. Nicomachus had indeed promised more concerning the harmony of the spheres when he wrote his larger work,[17] and _Fragment III_, the probable source of Boethius' similar discussion, could be the fulfillment of that promise.

None of the other theory for which Nicomachus is specifically cited appears in the _Fragments_. However, a series of numbers used repeatedly by Boethius--192:216:256:266--is used by Nicomachus in _Fragment II_,[18] although it must be acknowledged that Nicomachus also uses multiples of

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16. Nicomachus _Fragment IV_ (JanS. 274-275).
17. Nicomachus _Manual_ iii. (JanS. 242, 11 ff.).
18. Nicomachus _Fragment II_ (JanS. 267-271).

these numbers which nowhere appear in Boethius' treatise.

The principal source for the first part of Book II is clearly The Principles of Arithmetic. This work on arithmetic is, in fact, such an integral part of The Principles of Music that the musical work appears to be a direct continuation of the arithmetical work. However, The Principles of Arithmetic is nothing other than a translation of Nicomachus' Introduction to Arithmetic, and thus one might suppose that the first Books of The Principles of Music were based on a work by Nicomachus which was a direct continuation of his Introduction to Arithmetic. The title of the musical work promised in the Manual would have been Isagoge or Introduction to Music;[19] and a thorough work on music by Nicomachus would necessarily have been dependent on his Introduction to Arithmetic, just as Boethius' The Principles of Music is dependent on his The Principles of Arithmetic. Therefore, since Book II is such a logical and necessary outgrowth of Book I, and since Nicomachus--through both cited and uncited sources as well as through his Introduction to Arithmetic--is

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19. Nicomachus Manual i. (JanS. 238, 8).

without doubt the prevailing source for the theory presented in Books I and II, it seems logical to conclude that the larger musical work promised by Nicomachus in the Manual served as the principal Greek source from which Boethius compiled at least the first two Books of The Principles of Music.

Before proceeding to arguments concerning the source of the third and fourth Books of The Principles of Music, it will be helpful to discuss the other cited sources in these first two Books and examine the context in which they appear. But for the sake of stylistic comparison the secondary sources of Book V must first be briefly discussed.

Book V is a translation of Ptolemy's Harmonics,[20] Book I. But in Book V three sources other than Ptolemy are cited: the Pythagoreans, Aristoxenus, and Archytas. Upon close comparison of these citations with primary source of Book V, it becomes clear that Boethius did not use primary sources by Pythagoreans, Aristoxenus, and Archytas for the theory attributed to them; for the chapters of Book V which cite these sources are merely translations of chapters of Ptolemy which cite these sources.

20. Ptolemy, Harmonics , ed. Ingemar Duering (Goeteberg: Wetterbren & Kerbers Foerlag, 1930).

Moreover, Boethius uses citations in Book V in a rather consistent manner. Although Book V is essentially a translation of Ptolemy, Ptolemy is by no means cited in every chapter, and in no place is Ptolemy specifically cited as the source from which this Book was compiled. So long as Boethius is presenting theory which might be considered generally consistent with the basic tenets of Pythagorean-Ptolemaic musical theory, no citation whatsoever is given.[21] However, where differences of opinion between the Pythagoreans and Ptolemy arise, or where the theories of the Pythagoreans and Ptolemy can be used to refute theories of Aristoxenus, at these places--and generally only at these places--does Boethius attribute theories to specific authors.[22] Since this can be established as the style of citation in the last Book for which the Greek source is still extant, this same pattern can be applied to the first four Books for which no complete source is extant.

The citations of authors other than Nicomachus in Books I and II can be divided into two types: rhetorical and

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21. See, e.g., Book V. ii, xii.
22. See, e.g., Book V. iii-v, viii-xi, xi, xiii, xvi-xix.

theoretical. The citations of Plato and Aristotle contained in the first two chapters of Book I are clearly rhetorical, and they might well be taken over from the original Greek source. The rhetorical citation of Statius in Chapter I, Book I, could be considered the logical addition of a well-read Roman. The same could be said of the two quotations from Cicero contained in Book I, although both of these quotations are expressing a difference of opinion and are not properly rhetorical. Two theoretical citations of Book I, those to Albinus,[23] can definitely be considered Latin additions of Boethius to his basic Greek source. But the second of these Albinus citations presents further evidence that Boethius was probably working from a Nicomachus source. After discussing Albinus' Latin names for the notes Chapter XXVI concludes: _Sed nobis in alieno opere non erit inmorandum_[24] ("But let us not be lingering in another work"). The immediate reaction of the reader to such a statement is to smile somewhat at Boethius' presumptuous audacity. But if the sentence and the Albinus citation

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23. Book I, xii., xxvi.; concerning Albinus see p.64 n. 38. The theory credited to Albinus bears a marked similarity to theory presented by Martianus Capella; see also p. 93 n. 57.

24. _De Institutione Musica_. i. 26 (ed. Friedlein, p. 219, ll. 1-2).

are viewed in their full context, they present a definite clue that Boethius is working from one Greek source. Chapter XXVI, containing this Albinus citation, is surrounded by chapters containing theory which relates very strongly to Nicomachus: Chapters XX-XXIII are definitely related to Nicomachus, Chapters XXIV and XXV could be considered logical outgrowths of the preceding, and Chapter XXVII is definitely related to Nicomachus' *Fragment III*. Thus the Albinus citation occurs in the midst of theory which points very definitely to Nicomachus, it interrupts the sequence of thought to some extent, and Boethius states: *Sed nobis in alieno opere erit inmorandum*. The probable meaning of the sentence becomes quite evident: "Let us quit lingering over this work of Albinus and get back to our main source, namely Nicomachus."

In Chapter XXX, Book I, Plato's theory of how consonance physically occurs is presented. Boethius may well have known this theory through his reading of *Timaeus*. But Nicomachus' refutation of Plato's theory is presented in the following chapter. Therefore the style of citation is analogous to that of Book V. Here the question is one of personal opinions; and just as Boethius, while translating Ptolemy, specifically cited Ptolemy in refutation of Archytas' and Aristoxenus' tetrachord divisions in Book V, Boethius here, probably working from a Nicomachus source, cites Nicomachus in refutation of Plato's theory of consonance. Thus both of these chapters are probably based on one Greek source, and the style of citation points to Nicomachus as author of that source.

Two other theoretical citations occur in the first two Books, those to Eubulides and Hippasus[25] in Chapter XIX of Book II. But again the manner of presentation points to the conclusion that Boethius used no primary source for these references. In Chapter XVIII of Book II Boethius cites Nicomachus concerning the merit of respective consonances. In Chapter XIX he cites Hippasus and Eubulides presenting another opinion. Finally, Chapter XX returns to Nicomachus' ranking of consonances. Based on Boethius' style of citation in Book V, for which the primary source is still extant, one could argue that Boethius was here

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25. Concerning Eubulides and Hippasus see Book II. xix., p. 143, n. 26-27.

translating a work by Nicomachus: these are the first specific citations of Nicomachus; the theory presented up to this point has been consistent with Pythagorean-Ptolemaic position; here a definite difference of opinion is being expressed; thus Eubulides and Hippasus are cited in the same manner that Archytas and Aristoxenus are cited in Book V.

Finally, the Nicomachus citation occurring in Chapter XXVII of Book II is again stylistically identical with citations to Ptolemy in Book V. Here Nicomachus is said to hold an opinion different from that of Ptolemy, namely that the diatessaron and diapason is not a consonance, and when such a difference of opinion arises in The Principles of Music, stylistic consistency demands a citation. Ptolemy is cited elsewhere in Books I and II concerning this same matter. Boethius no doubt began with a basic plan of translation for The Principles of Music; since he was surely well acquainted with the theory of Nicomachus and Ptolemy, when the matter of the diapason and diatessaron arose in the first two Books, he made reference to the translation of Ptolemy which was to come in the last Book.

Thus the first two Books of The Principles of Music contain very convincing evidence for arguing that they are based on the lost Introduction to Music of Nicomachus. Nicomachus is cited more than any other source in these books, and the style of citation to Nicomachus in these Books is the same as that to Ptolemy in Book V. The other Greek sources which are cited could easily have been a part of Nicomachus' work, and again the style of their citation is the same as citations to sources other than Ptolemy in Book V. Finally, the dependence of theory contained in Books I and II on The Principles of Arithmetic again clearly indicates a Nicomachan source.

The problem of identifying the source or sources of Books III and IV is considerably more difficult than that of Books I, II, and V. Three aspects of these two Books must be examined and discussed in order to present arguments concerning their primary source: (1) the manner in which citations of specific sources occur; (2) the presence of uncited sources; and (3) the interrelationship between these two books and the remaining three.

Book III makes reference to only three authors: Aristoxenus, Philolaus, and Archytas. All citations of

Aristoxenus are refutations of his position that a semitone is half of a tone. Thus it is unlikely that his work served as the source for Book III. The two references to Philolaus,[26] though not refutations, can be considered little more than interesting insertions, for their theory is in no way essential to the basic contents of Book III. Thus Philolaus could not be considered a likely source for Book III. Finally the citation of Archytas[27] is in reference to a basic geometric axiom which is necessary to prove certain arguments presented in _The Principles of Music_; but this argument is presented here as being inadequate for proving the particular point. Thus Archytas could not be the principal source for the arguments which occur in Book III. If one compares these references with those to persons other than Nicomachus in the first two Books and to persons other than Ptolemy in Book V, they are seen to be presented in exactly the same style. Thus again one should probably conclude that Boethius used no primary source for the specific citations which occur in

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26. Concerning Philolaus see Book III, v. p. 182, n. 8.

27. Concerning Archytas see Book III, xi., pp. 194-195, n. 11.

Book III, but rather that Book III was based on some Greek work which also made reference to these authors.

No uncited sources have been identified as texts from which all or parts of Books III were compiled.

An examination of the interrelationship between the theory of Book III and that of Books I and II proves much more helpful in determining the probable source of this Book. From the very first chapter of Book III a dependence on _The Principles of Arithmetic_ and the mathematical theory of Book II is evident. Chapter I makes specific reference to the work on arithmetic--that is, the translation of Nicomachus; Chapter II refers to the numerical series for a divided diatessaron first presented in Books I and II--a series again found in Nicomachus, _Fragment II_. Chapter III cites the six continuous tones computed in the last chapter of Book II; Chapter IV makes reference to the number containing the comma discussed in Book II; and finally, Chapters XII and XIII contain arguments which would be impossible to prove were it not for the axioms concerning proportions presented in Chapter IX of Book II, and these axioms are specifically cited as proof for these arguments. In short, the contents of Book III would have little meaning to the reader had not the basic mathematical axioms of Book II been first presented. Thus it seems logical to conclude that Book III is a direct continuation of Book II, and that the source for both is the same.

Evidence in Book II points very strongly to a Nicomachus source. Nicomachus was cited in Book II, but no such citation occurs in Book III. Nevertheless, it has been shown that Boethius makes references to the specific authors he is translating only when a personal opinion of one author is not consistent with the general position of Pythagorean-Ptolemaic musical theory. The contents of Book III are completely consistent with this position, and thus no reference to Nicomachus, if he were the source, would seem necessary. Therefore, since the specific citations which do occur in Book III are in the same style as those to persons other than Nicomachus in Books I and II and to persons other than Ptolemy in Book V, since there is no other extant work which can be identified as the source of this book, and since Book III is so integrally connected with the theory of the two preceding Books, the most logical conclusion seems to be that Boethius, in Book III, is continuing his translation of the same source which was used in Books I and II, namely the _Introduction to Music_ of Nicomachus.

Upon superficial examination Book IV seems to be a pasticcio of Greek sources. Only one specific citation occurs in the whole of Book IV, a reference to Ptolemy (Chapter XVII) concerning the possibility of an eighth mode. But three radically different Greek works can be identified as possible sources for this work. Chapters I and II are an expanded translation of the introduction and first nine propositions of _Sectio Canonis_. Chapter III demonstrates a close relation to parts of Alypius' _Introduction to Music_. Finally, the modal theory contained in Chapters XIV through XVII is clearly related to that of Ptolemy's _Harmonics_, Book II. Nevertheless, none of these sources are specifically cited. Ptolemy does receive specific citation, not in conjunction with the modal theory itself, but rather in conjunction with the addition of an eighth mode which Ptolemy himself refuted.[28] Thus again

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28. Ptolemy _Harmonics_ ii. 9. 60-63.

this citation seems to be consistent with the other specific citations which occur throughout this work; that is, it would seem that this eighth mode is not consistent with the general position of Pythagorean-Ptolemaic music theory, and thus it is merely a citation of Ptolemy's personal opinion.

Again an examination of the interrelationship between this book and the remaining four proves helpful in identifying its possible source. The heart of Book IV is the division of the monochord ruler. Furthermore the whole of The Principles of Music has been steadily moving toward this division. Chapters XI and XXVII of Book I specifically promise this division, as does the last chapter of Book III. Moreover, this division is the final fulfillment of all the intricate mathematical theory which has preceded, for it is the application of this mathematics to a string and the final demonstration to the ears that these mathematical proportions contain the harmonies which they have been deductively demonstrated to hold.

Chapter XXVII of Book I also promises further discussion concerning fixed notes, movable notes, and notes which are in part fixed and in part movable. Such a discussion appears nowhere in the first three Books, but does occur in Chapter XIII of Book IV.

Another fulfilled promise occurs in Chapter II of Book IV; for here it is conclusively demonstrated that a superparticular proportion cannot be equally divided, and that, if multiplied, a superparticular proportion produces neither a superparticular nor a multiple proportion. This axiom, basic to the division of the monochord, was presented dogmatically in Chapter X of Book II and inconclusively in Chapter XI of Book III, and both of these chapters promise the conclusive demonstration of the premise which occurs in Chapter II of Book IV. Moreover, Chapter II of Book IV is part of the expanded translation of _Sectio Canonis_. It is conceivable that Boethius could have made these promises concerning this matter to be discussed in Book IV in the same way he makes promises concerning theories of Ptolemy to be discussed in Book V. But if this thesis is accepted, one would have to conclude that Boethius departed from his source for Books I, II, and III and took up a new source for Book IV. It would seem much more logical to argue that the _Sectio Canonis_ text and that for the division of the monochord were both found in the same source which served as the basis for the first three Books; for this theory is the logical outgrowth of the first three Books, whereas the theory attributed to Ptolemy--the diatessaron and diapason is a consonance--is in direct contradiction to the premise of the first four Books that consonances are found only in multiple and superparticular proportions. Moreover the proofs concerning various intervals found in the last chapters of Book II, though not definitely identifiable as a translation of *Sectio Canonis*, strongly resemble the proofs found in Propositions IX through XIII of *Sectio Canonis*.[29] Therefore *Sectio Canonis* could have been one of the basic sources from which Boethius' source was compiled.

The Greek notation presented in Chapter III of Book IV is such an integral part of Book IV that it is unlikely that it came from a separate source. The notation and particularly the accompanying descriptions are closely related to the *Introduction to Music* of Alypius.[30] But if

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29. See Book II. xxiii. p. 152, n. 30.

30. Alypius Εισαγωγη μουσικη, Jan *Musici Scriptores Graeci*, pp. 367-406.

Boethius used the Alypius source directly, it is likely that he would have cited Alypius. Alypius is by no means a Pythagorean, and his modal theory is not consistent with that of Ptolemy; for Alypius presents no less than fifteen possible tropes in each genera. Ptolemy, on the other hand, argues that there are seven tropes, presents the possibility of an eighth, but rejects it in the end. If Boethius was basing Chapter III of Book IV on Alypius and if he were consistent with his style of citation throughout The Principles of Music, he would have at least said that Alypius presented numerous other modes which will not be discussed in The Principles of Music. Alypius is a comparatively late Greek musical theorist,[31] and thus he probably based his text on the same text, or similar texts, as did the source of the notational theory presented in Chapter III of Book IV.

Rather than arguing that Boethius worked from the Alypius treatise or some similar text in Chapter III, it would seem much more logical to argue that the source for the notational theory was the same as that for the monochord

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31. Fourth century A.D.

division; for the notation is used in Chapter IV, the beginning of the monochord division, to signify the names of the notes computed on the monochord ruler. The notational theory is also necessary to comprehend the modal theory presented in Chapters XIV through XVII of Book IV. Thus this theory seems to be a logically necessary part of Book IV as a whole rather than an interesting insert based on Alypius' treatise.

Finally the modal theory of Chapters XIV through XVII and its relation to a possible Ptolemaic source must be considered. The theory presented in these chapters is without doubt directly related to Ptolemy.[32] But if one considered these chapters a Boethian translation of, or even commentary on the modal theory of Ptolemy, he would be forced to conclude that Boethius was a rather inept translator of the Greek language, a conclusion which is wholly untenable when Boethius' translations of Nichomachus' _Introduction to Arithmetic_ and the logical works of Aristotle are considered. Although the theory of these chapters is basically Ptolemaic, constant confusion appears

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32. See Book IV, xiv-xvii. pp. 267-290 , nn. 33-56.

in the text with regard to the numbering of the species of consonances, and the addition of an eighth mode is falsely attributed to Ptolemy. No competent Greek scholar such as Boethius would likely make such a confused translation of this theory; for Ptolemy's text is straightforward, well organized, and in no way lends itself to such a translation or even interpretation. Moreover, notational charts are presented in Boethius' text which are essential to any understanding of the modal system. On the other hand, no such charts appear in the Harmonics of Ptolemy. Finally, Ptolemy is cited in such a way that his opinion seems to be an exception to the theory presented in these chapters. Therefore the source for these chapters was probably a person who was thoroughly acquainted with the traditional numbering of the species,[33] but who was also acquainted with Ptolemy's convincing reorganization of Greek modal theory. This person was thus trying to incorporate both theories into his modal discussion.

This person could have been Boethius, but it is much more likely that it was Nicomachus. The contents of Book IV

33. See Book IV, xiv. p. 274, n. 44.

are without doubt Pythagorean in character, and Nicomachus is the only Pythagorean to make reference to Ptolemy.[34] Moreover Nicomachus' reference to Ptolemy is in direct relation to Ptolemy's modal theory; and in this reference Nicomachus demonstrates a knowledge that the modes are related to the species of consonances, that each is composed of fifteen notes, and that the mese plays an important functional role in these modes. Such knowledge is clearly the prerequisite to Chapters XIV through XVII of Book IV.

Further evidence that this theory is not taken directly from Ptolemy is the inclusion of the Lesser Perfect System, of Synemmenon System, in the discussion of the modes or tropes.[35] Ptolemy rejects this system because it will not contain all species of the diapason, and Nicomachus' *Fragment IV* is clearly aware of this fact. A more traditional theorist such as Nicomachus would have probably kept the Synemmenon System in his modal discussion.

The final chapter of Book IV again clearly points to

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34. Nicomachus *Fragment IV* (JanS. 275, 7-15).
35. See Book IV. xv. , pp. 278-279, n, 47.

a Pythagorean source. In this chapter various consonances are tested on the monochord: the diatessaron, the diapente, the diapason, and the diapason and diapente. If this chapter were taken from a Ptolemaic source, the diapason and diatessaron consonance would also have been tested, just as Ptolemy himself does in a similar passage.[36] But this has been one of the theoretical problems which has been discussed throughout The Principles of Music: Nicomachus holds that the diapason and diatessaron is not a consonance; Ptolemy holds that it is. Therefore this last chapter is clearly consistent with Nicomachus.

Books I through IV are a complete unity. They are definitely closely interconnected, and they maintain a consistent Pythagorean position in all their contents. Book V begins something new: for there the aural sense plays an important role in determining which intervals are consonant, whereas in Books I through IV the mathematical reason is always the final judge. The diatessaron and diapason sounds consonant, but it is rejected in the first four Books because it does not fall in multiple or

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36. Ptolemy Harmonics ii. 2. 47.

superparticular proportions. In the last Book it is accepted because any consonance added to the "equal-sounding" diapason produces another consonance. The opening sentence to Book V clearly implies the beginning of something new and different: "Following the division of the monochord ruler I deem it necessary to add a discussion of those points in which the ancient musical scholars show a diversity of opinion." This sentence implies that what has preceded was more or less of one opinion, namely that of the Pythagoreans. The only Pythagorean who could have compiled the basic text on which the first four Books were based is Nicomachus; for only this second century scholar could have had access to the writings of such theorists as Hippasus, Philolaus, Eubulides, Archytas, Aristoxenus, Ptolemy, and such a treatise as Sectio Canonis.

In summary, Nicomachus is cited in the first two Books when an opinion of his is not a generally accepted position of Pythagorean-Ptolemaic musical theory; other parts of Book I can be positively identified as translations of Nicomachus' Manual; in this same Manual Nicomachus promises a more complete treatise on music, and it is entirely probable that much of the *Manual* would have been incorporated into this larger work; the first four Books of *The Principles of Music* are so closely interrelated that they could have been compiled from one single work; these same four Books are equally interrelated with *The Principles of Arithmetic*, a work which is a translation of Nicomachus' *Introduction to Arithmetic*; no reference to Nicomachus occurs in Books III or IV, but in these Books no theory which could be described as Nicomachus' personal opinion arises, and thus, consistent with the general style of citation throughout this work, no specific citation to Nicomachus would have been necessary; the works of Boethius written in this period are primarily translations of Greek sources, and the works on the *Quadrivium* are cited by Cassiodorus as being translations: therefore, the most consistently logical and acceptable thesis concerning the origin of the first four Books of *The Principles of Music* is that they are a Boethian translation of Nicomachus' lost *Introduction to Music*. Indeed these four Books are thereby all the more significant, for they, and they alone, preserve the basic contents of this last and most complete Greek Pythagorean treatise on music.

Book V has already been identified as an incomplete translation of Ptolemy's Harmonics, Book I. However, Boethius left the remaining chapter titles for this Book, and therefore the conclusion of Book V can be easily constructed by comparing these titles with the concluding chapters of Ptolemy's Book I.[37] In this Book Boethius' style and ability as a translator can be established. At times Boethius translates quite literally,[38] and at these times Boethius' skill as a translator of Greek is proven beyond any doubt. At times Boethius presents more or less paraphrases of the original texts,[39] while at other times he merely presents a highly condensed version of the essential argument being presented.[40] Boethius tends to render a rather literal translation when the basic premises of Ptolemy's position are involved or when the discussion becomes more technical and mathematical. Boethius tends

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37. See pp. 327-331.
38. Book V. ii. xvi-xix.
39. Book V. iii. v-vii., xi , xii , xvii.
40. Book V. iv, viii , xiii ; see also xii , p. 314, n. 24.

to paraphrase the original text when the basic premises are being developed and carried to their conclusions. He is likely to present only the essence of the original argument if the basic content of the text has been covered in one of the preceding Books. At two points in Book V Boethius inserts chapters which are not at all present in Ptolemy's work. The first of these, Chapter X, is a commentary on Ptolemy's theory of the diapason, and the latter, Chapter XV, merely serves as transition.

It is logical to conclude that Boethius used this same style and technique in translating the first four Books from Nicomachus. Thus the basic premises expressed in these four Books are probably more or less exact translations of Nicomachus.[41] The same can be said of the more technical and mathematical sections.[42] The developments of these basic premises are probably paraphrases.[43] Since most of the highly condensed paraphrases of Book V cover arguments presented in the first four Books, it is unlikely

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41. E.g. Book I, iii-ix , Book IV. i., iii.

42. E.g. Book I, xvi-xviii , Book II. ii-xviii , Book III. i-xvi , Book IV. ii. (cf. following n. 44), v-xi.

43. E.g. Book II. xxi-xxviii. (see xxvii, p. 161) "But although Nicomachus said much about this matter, nevertheless we have presented here most briefly the same things which the Pythagoreans affirm."),

that many translations of this type occur in Books I through IV. On the other hand, passages which might be transitions or commentaries similar to those which occur in Book V might have been added by Boethius to the basic source of the first four Books.[44]

The contents of Book V, even if completed, would have formed a most illogical conclusion to The Principles of Music. Moreover, there are clear indications throughout The Principles of Music that Boethius did not intend to stop after merely translating this first of Ptolemy's Harmonics. In Book I, Chapter II, Boethius promises to return to a discussion of the music of the universe and the music of the human being.[45] These types of music are presented in the introductory chapters as being most high in the scale of being, and it would seem inconsistent to discuss musical proportions at such length without afterwards rising to the music of the human being and the music of the universe. Nevertheless, these promises are nowhere

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44. Book I. xv., Book II, i. are examples of transitions. The numerical examples in the proofs from Sectio Canonis, Book IV. ii , may well be Boethian commentaries on the original text.

45. See Book I. ii , pp. 46-47, nn. 18, 21.

fulfilled in the existing five Books of Boethius' work. But they would have been most thoroughly completed by a translation of Book III of Ptolemy's Harmonics.[46] Similarly Chapter II of Book V promises more discussion of the kithara, whereby "the complete ratio of the proportions may be discerned right before the eyes, in whatever number of strings might be necessary."[47] This passage without question refers to Chapter XVI of Ptolemy's Harmonics, Book II. Thus the original plan of The Principles of Music becomes evident: a work consisting of seven Books, the first four of which were based on Nicomachus' Introduction to Music, and the last three of which were to be based on Ptolemy's Harmonics. Thereby The Principles of Music would have been the most comprehensive musical work of Classical antiquity. One can but marvel at the ambition of the comparatively young Boethius in planning a work of such scope; but at the same time one can only regret that this ambition was never fulfilled.

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46. See Ptolemy Harmonics iii. 5-16. 95-111.

47. Although the promise of further discussion occurs in Book V. ii, this description of what is to be discussed occurs in Book V. i.

But Boethius considered music and the other mathematical disciplines a preparation for the study of philosophy. His grounding in the basic theory of these disciplines is obvious from the surviving works on these disciplines which he completed. Boethius, having mastered these disciplines himself, probably considered his philosophical task of translating Plato and Aristotle more important that completing this treatise on music. Therefore the Middle Ages inherited a translation of Nicomachus' Introduction to Music and an incomplete translation of Ptolemy's Harmonics. Rather than the complete translation of Ptolemy's Harmonics they received translations of Aristotle's logical works.

CHAPTER II

THE AESTHETICS OF THE PRINCIPLES OF MUSIC

The aesthetic theory presented in *The Principles of Music* is essentially a theory of epistemology: that which is most knowable is most harmonious; only quantities, magnitudes, forms, *essences*, are knowable, and the discipline of music concerns itself with essences consisting of related quantities; those related quantities which are easiest to *know* are the proportions, the essences, the most harmonious sounds.

This commentary on Boethius' aesthetics will be based exclusively on the aesthetic theory contained in *The Principles of Music*. The preceding chapter argued that this work essentially consists of a translation of Nicomachus' *Introduction to Music* and an incomplete translation of Ptolemy's *Harmonics*. The aesthetic theory contained in this work will thus be largely determined by the aesthetic position of these two authors. Later in his life, Boethius translated much of Aristotle, and thus the aesthetic position of his later works is somewhat different from the essentially

Pythagorean and Platonic position found in The Principles of Music. Nevertheless, this discussion of Boethius' aesthetics will limit itself to the theory contained in this comparatively early work; for this work, rather than the later Aristotelian works, has had the greatest influence on musical thought.[1]

Any discussion of the aesthetic position of The Principles of Music must begin with an explication of music's place in the quadrivium; for every theory which is part of this aesthetics is basically dependent upon music's place in the quadrivium and the general nature of these four mathematical disciplines. Boethius was the first scholar to call the Classical mathematical disciplines the "quadrivium";[2] however, he was by no means

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1. Cf. Edgar de Bruyne Etudes d'esthetique Medievale (Brugge: De Tempel, 1946), vol. 1, pp. 3-34. Leo Schrade, "Das propaedeutische Ethos in der Musikausschauung des Boethius," Zeitschrift fuer Geschichte der Erziehung und des Unterricht, XX (1930), pp. 179-215; "Die Stellung der Musik in der Philosophie des Boethius als Grundlage der ontologischen Musikerzeihung," Archiv fuer Geschichte der Philosophie, XLI (1932), pp. 368-400; and "Music in the Philosophy of Boethius," Musical Quarterly, XXXIII (1947), pp. 188-200.

2. See The Principles of Arithmetic I. i. pp. 23-30.

the first to group these four disciplines together and discuss them as necessary prerequisites to any philosophical education. The concept that arithmetic, music, geometry, and astronomy formed the basic areas within mathematical speculation can be traced back to the earliest Pythagoreans. Plato developed this concept when outlining the proper educational system for his ideal state,[3] and Nicomachus, Boethius' principal source, presented the concept much as it had been developed by Plato.[4] Since Boethius is working in this same tradition, this concept of music as a mathematical science (*scientia*) determines the basic aesthetic position of *The Principles of Music*.

Philosophy, the ultimate goal of every student, requires an ability to reason concerning things which are truely and properly said "to be."[5] Nothing which partakes in corporeal substance can properly be said "to be," for corporeal substance is in constant flux, and anything which partakes in corporeal substance is subject to radical

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3. Plato *Republic* 323-334.
4. Nicomachus *Arithmetic* i. 1-2.
5. *The Principles of Arithmetic* I. i. p. 24; Nicomachus *Arithmetic* i. 1.

change.[6] Therefore, the student should turn away from things which are in such flux and turn his attention to things which exist in and of themselves, things whose being is changeless and eternal. Such things are magnitudes, forms, multitudes, equalities, and proportions, that is, essences.[7] Some such essences are found in corporeal substance, and, in that they partake in corporeal substance, they appear to change as matter itself changes. But reason can abstract essences from corporeal substances, and thereby reason can come to know them in and of themselves, that is, as incorporeal essences.[8]

The essences which the reason should first come to know are of two types: multitude and magnitude. Multitude is that type of essence which is composed of parts, and these parts are not continuous with each other; that is, a beginning and end of each part can be identified, for example, a chorus, an orchestra, or a mass of similar objects.

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6. The Principles of Arithmetic I. i. p. 24; Nicomachus Arithmetic i. 1.

7. The Principles of Arithmetic I. i. p. 24; Nicomachus Arithmetic i. 1.

8. The Principles of Arithmetic I. i. p. 24; Nicomachus Arithmetic i. 1; cf. The Principles of Music II, ii. pp. 105-106.

Magnitude, on the other hand, is continuous; that is, it does not consist of parts, for example, a stone, or a cube. Each of these types of essences is in turn divided into two types. For multitude can be considered in and of itself, for example 3, 60, 100; or it can be considered in relation to something else (relatae ad aliquid), for example, a half, a double, a third, or a triple. Magnitude on the other hand can be motionless, for example, the stars and the heavenly spheres. Therefore four disciplines exist within these two types of essences, and each becomes the subject of one discipline among the quadrivium. Arithmetic studies that multitude which exists in and of itself, while music studies multitude which exists in relation to something else. Geometry studies immovable magnitude, while astronomy studies magnitude in motion.[9]

Each of these disciplines has its beginning in the corporeal, but through these disciplines the reason can forsake the world of changing matter and rise to the world of incorporeal essences.[10] After these disciplines have

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9. The Principles of Arithmetic I. i. pp. 25-26, cf. The Principles of Music II. iii. pp. 106-108.

10. The Principles of Music I. i. p. 43-44 ; I. ix. pp. 57-59.

been mastered--and only thereafter--the reason can begin to pursue truth in the light of truth alone. After these disciplines have been mastered the student is prepared to enter the realm of Platonic philosophy.[11]

Therefore music is of value because through it the reason comes to know essences consisting of related quantities. But this is by no means the only reason the student should study music. Through music one does come to know truth, but music--and music alone of the four disciplines--gives pleasure and pain as well as truth. One's actions can be greatly influenced by music: the calm can be enraged, the enraged can be made calm. Therefore music is not only concerned with the investigation of truth, it is related to morality as well. Music is such an integral part of human nature that men and women of all ages and all races are affected by it.[12]

> For this reason the power of the mind ought to be directed toward fully understanding by knowledge what is inherent in us through nature. Thus just as erudite scholars are not satisfied by merely seeing colors and forms without also investigating their properties, so

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11. <u>The Principles of Arithmetic</u> I. i. p. 27.
12. <u>The Principles of Music</u> I. i. pp. 31-44.

> musicians should not be satisfied by merely finding pleasure in music without knowing by what musical *proportions* these sounds are put together.[13]

Therefore the final goal of Boethius' aesthetics is *to know* musical proportions, on the one hand because they are truths, on the other hand because they are part of our human nature and give us pleasure or pain. These proportions, these harmonies, are primarily experienced in sound (*musica instrumentis constituta*), but they are also evident in the human being (*musica humana*) and the universe itself (*musica mundana*).[14] For the order of the human body is proportionally harmonious, and the incorporeal substance of the reason and the corporeal body are joined together in a harmony, "as it were, a careful tuning of low and high pitches in such a way that they produce one consonance."[15] Moreover the harmony of the universe is clearly evident in the diversity of yearly seasons, the harmony of the elements, and finally the harmonies of the stars and heavenly spheres.[16]

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13. *Ibid*. pp. 43-44.
14. *Ibid*. I. ii. pp. 44-48.
15. *Ibid*. p. 47.
16. *Ibid*. pp. 44-46.

These three types of music, like the concept of the *quadrivium*, were implicit in Classical Platonic and Pythagorean thought, but Boethius is the first to give them such clear classification in Latin.[17] The movement up the scale of knowledge is clearly evident in these divisions: music of instruments, music of the human being, music of the universe, and thereafter, in the realm of Platonic dialectic, the harmony of the pure forms themselves. Thus through the experience of music the reason ascends from the corporeal to the incorporeal.[18]

Nevertheless music begins in sound, and thus the theory of sound presented in *The Principles of Music* is an essential part of Boethius' aesthetics. Book I, Chapter III, defines sound generally as "a percussion of the air, which percussion remains undissolved until it reaches the ear." Book I, Chapter VIII, refines this definition to fit music: "Thus sound is the melodious inflection of the voice, that is, fitted to song in a single tune." Both these definitions

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17. Cf. *Ibid.*, pp. 44-46 , nn. 16, 19.
18. Cf. Augustine *De Musica* vi.

were taken from Nicomachus, and were thus probably standard Pythagorean definitions of sound.[19]

The early Pythagoreans, and even Plato himself, thought that high and low sounds were caused respectively by fast and slow motions.[20] This doctrine was refined somewhat, probably by Archytas,[21] and sound came to be considered in terms of fast and slow frequency as well as fast and slow motion. Book I, Chapter III, describes the occurrence of sound as follows:

> But one should not think that every time a string is struck, only one sound is produced, or only one percussion is present in these sounds; but as often as the air is moved, the vibrating string will have struck it. But since the great speeds of sounds are connected, one hears no interruption, and one sound, either low or high, affects the sense.[22]

Book I, Chapter XXXI, citing Nicomachus, describes sound in the same terms: "One pulse alone does not emit a simple musical sound; but rather a string, once struck hits the air again and again in order to produce a sound." Thus the essential aspect of sound for Boethius is _frequency_ of

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19. _See The Principles of Music_ I. iii. p. 48 , n. 22, I. viii. p. 57, n. 29.

20. See _Ibid_. I. xxx. p. 97 , n. 62.

21. Hermann Diels, _Die Fragmente der Vorsokratiker_, ed. Kranz (9th ed.; Berlin: Weidmann, 1960), I. 47. Bl, pp. 431-435.

22. _The Principles of Music_ I. iii. p. 49.

percussion. Since Nicomachus is cited in conjunction with this theory, Boethius probably translated this definition rather literally from Nicomachus.[23]

Where frequency of percussion is found, there number is also present. And where two sounds occur, they can be discussed in terms of numerical proportions.[24] Thus the essence of musical sounds are related quantities or numerical proportions. But some sounds are pleasing when mixed together, whereas others are unpleasant. Musical sounds travel in waves, much like the waves in a pond of water caused by a stone.[25] Musical consonance occurs when a mixture of a high and a low sound falls "uniformly and pleasantly on the ears."[26] If, on the other hand, the two sounds are "harsh and unpleasant" when they fall on the ears, then the two sounds are dissonant.[27] The frequencies of the two sounds determine whether the sounds are consonant or dissonant; for if the frequencies of the sounds heard are commensurate, then the ear is pleased by the sounds and

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23. Ibid. I. xxxi. pp. 98-99, n. 63.
24. Ibid. I. iii. p. 50 ; IV. i. p. 212.
25. Ibid. I. xiv. pp. 65-66.
26. Ibid. I. viii. p. 57.
27. Ibid.

perceives them to form a consonance. If the frequencies of the two sounds are not commensurate, then the ear is displeased and identifies the sounds as dissonance.[28] Therefore the proportions of these commensurate sounds must be found if the student is to know the essences of those which give pleasure.

The senses thus play a very important role in the musical experience; for they identify which intervals are consonant. In fact, Book II, Chapter XVIII, presents a classical statement of the correspondence theory of truth: "For just as every single thing is in itself, so it is perceived by the sense."[29] Therefore Boethius is by no means opposed to experiencing the sensuous pleasure of music. Nevertheless, the task of finding the proportions, the truths, of musical consonance is beyond the senses. For the senses are "neither equal in all men, nor at all times equal within the same man";[30] and the senses have great difficulty perceiving the most minute things as well as the excessively great things, for example, a very soft sound

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28. _The Principles of Music_ I. xxxi. p. 98.
29. _Ibid_. II. xviii. p. 141.
30. _Ibid_. I. ix. p. 58.

or an excessively loud sound.[31] Therefore although the senses identify consonances and take pleasure in them, they are nevertheless incapable of presenting musical essences to the mind; this task must be relegated to the reason.

The respective roles of the senses and the reason in musical judgment form the key aesthetic position which unifies the theories of Nicomachus and Ptolemy as presented in _The Principles of Music_. The Pythagorean position of the first four Books of this treatise was taken primarily from Nicomachus, a very late Pythagorean, and thus it can not be considered identical with that of the earliest Pythagoreans.[32] The Nicomachan position concerning this matter is best stated in Book I, Chapter IX:

> For this reason the Pythagoreans take a certain middle path. For they measure consonances themselves with the ear, but the distances by which consonances differ among themselves they do not entrust to the ears, the judgments of which are unclear, but to rules and reason--as though the sense were an obedient servant, and the reason were truly a commanding judge.[33]

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31. _Ibid._; cf. V. ii. pp. 295-299.
32. _Ibid._ I. ix. p. 48, n. 33.
33. _Ibid._

This same chapter acknowledges the fact that the senses are necessary in music, for if it were not for the senses, no sounds whatsoever would have been heard. They are therefore the "first principle" in music, but a first principle in the manner of an "exhortation," exhorting the student to make a reasoned investigation into what is pleasing to the ears.[34]

> He [Ptolemy] holds that there is nothing irreconcilable between the ears and the reason. The solution to the problem of harmony according to Ptolemy is that the senses inform, and then the reason decides the proportion. Thus less the senses contradict reality the application of harmony joins these two faculties into a union.[35]

Thus the basic position concerning the senses and reason throughout _The Principles of Music_ is the same: consonances are first perceived and identified by the senses, but this perception is sometimes inaccurate and confused, and thus not adequate for a judgment of truth; thus the final judgment of the proportions, the essences, of harmonious sounds must be given to the reason. _The Principles of Music_,

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34. _Ibid._
35. _The Principles of Music_ V. iii. p. 300.

drawing from sources by both Nicomachus and Ptolemy, repeatedly attacks Aristoxenus, who, since he relied on the ears rather than reason, arrived at different conclusions concerning the distances and proportions of musical intervals.[36]

Pythagoras is accredited with having first discovered the mathematical proportions of the intervals which the ears had recognized as consonances. He accomplished this by weighing hammers which were resounding musical consonances in the hands of blacksmiths.[37] Thus he discovered that the diapason (the octave) consisted of a 2:1 proportion, the diapente (the fifth) of a 3:2 proportion, the diatessaron (the fourth) of a 4:3 proportion, and the tone--the difference between a diatessaron and a diapente--of a 9:8 proportion. These proportions then become the musical essences which the reason must manipulate in order to discover the proportions of all other musical intervals. Moreover, although Pythagoras' discovery must be considered an inductive proof of these proportions, nevertheless the identity of these

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36. E.g., II. xxxi ; III. i , iii ; V. iii-iv , xiii-xiv.

37. _The Principles of Music_ I. x. p. 62, n. 35.

proportions with the respective consonances can also be proved deductively by the reason alone.[38] Thereafter the reason is convinced of the truth of these essences.

Hence the reason can immediately demonstrate that the octave consists of a fourth plus a fifth, for 3:2 plus 4:3 equal 4:2, the proportions of an octave.[39] Moreover, the reason discovers that the twelfth, another consonance, consists of an octave plus a fifth and is found in a 3:1 proportion, for 3:2 plus 2:1, a fifth plus an octave, equals 3:1, a twelfth.[40] The reason discovers that the double octave, since it consists of two octaves (4:2 plus 2:1), consists of the 4:1 proportion.[41] The reason can demonstrate that the 9:8 proportion and all similar proportions cannot be equally divided, and thus the semitone cannot possibly be a half of a tone as Aristoxenus thought;[42] and, since a semitone is not a half a tone, the octave consists of five tones and two semitones rather than six tones.[43] Therefore

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38. Ibid. II. xxi-xxiii. pp. 149-156.
39. Ibid. I. xvi. pp. 66-68.
40. Ibid. II. xxvi. pp. 156-157.
41. Ibid.
42. Ibid. III. i. pp.171-176, nn. 1-2.
43. Ibid. III. iii. pp. 178-180.

Aristoxenus abounded in false conclusions because he trusted the senses to give him the truths concerning musical intervals rather than relying on the reason.[44] The reason continues in this same manner to determine the proportions of consonances throughout The Principles of Music and thereby comes to know their essences. Thus a basically consistent position is maintained throughout this treatise.

But the reason does not merely demonstrate the proportions of various intervals in the first four Books; it must also judge the value of the respective consonances. At this point disagreement occurs between the Nicomachus source of the first four Books and the Ptolemy source of the last. For in the first four Books the mathematical reason alone determines the identity and value of consonances, whereas the sense of hearing is allowed to share somewhat in this judgment in the last Book.

Consonance has been said to occur when the frequencies of two sounds are commensurate. This fact further reveals itself in the numerical proportions of the consonances: 2:1,

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44. Ibid. III. i. p. 171, n. 1.

3:1, 4:1, 3:2, and 4:3; for either the smallest number or one part of the smallest number equally divides the larger number in every case. According to the first four Books, the less the larger number departs from the smaller number, or the more "equality" there is between the two numbers, the more harmonious is the consonance produced by the proportions.[45] These proportions of consonances can be divided into two types: multiple and superparticular; in the former the larger number surpasses the smaller number by a multiple of the smaller number, while in the latter the larger number supersedes the smaller number by only one part of the smaller number.[46]

The value of these proportions, and therefore of their respective consonances, is determined by the order in which they are known and the degree of their equality. Once a number or a line has been perceived by the reason, nothing is easier to deduce that its double. After the double, the half is most easily comprehended, and after

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45. The Principles of Music I. vi. pp. 54-56 , xxix. pp. 95-97 ; II. v. pp. 110-111, xviii., pp.141-143.
46. Ibid. II. ii., pp. 105-106.

the half the triple, and after the triple the third.[47] However, proportions of the multiple type are closer to equality than proportions of the superparticular type; for multiples are nothing more than a natural series of numbers related to the first number, that is, one or "unity."[48] Moreover, the smaller number in a multiple proportion is completely contained by the larger number, and therefore a certain completeness and simplicity is evident in multiple proportions.[49] Superparticular proportions, on the other hand, are not related to the first number in a natural numerical series, but rather begin in plurality; and rather than the larger number containing the smaller number completely, it contains a half or a third or a fourth of the smaller number. Nevertheless, superparticular proportions achieve "a division into singular and simple parts."[50] The other simple type of proportion, superpartient (in which the larger number contains the smaller number plus more than one of its parts), departs from the simplicity and

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47. *Ibid.* I. xxxii. pp. 99-100.
48. *Ibid.* I. vi. pp. 54-56.
49. *Ibid.* II. v. pp. 110-111.
50. *Ibid.* I. vi. p. 55.

completeness of the first two types, and is not qualified to contain musical harmonies.[51] Therefore reason, in the first four Books, must conclude that only proportions of the multiple and superparticular types can yield musical consonances; and that consonances produced by multiple proportions are the most harmonious, for they are the first known by virtue of their simplicity and closest to equality by virtue of their completeness.[52] Consonances contained by the superparticular proportions follow these in value, and again the proportions closest to simplicity and completeness are the most harmonious.[53] Therefore the order of consonances according to their respective value must proceed as follows: the octave (2:1), the twelfth (3:1), the double octave (4:1), the fifth (3:2), and the fourth (4:3).[54] Thus the value of the respective consonances is judged by the reason alone in the order in which their essences are known; and just as sounds of commensurate frequencies are pleasing to the ears, so the proportions of

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51. Ibid.
52. Ibid. I. v-vi; II. v, xviii-xx ; IV. i.
53. Ibid. I. v. p. 53.
54. Ibid. II. xviii. pp. 141-143, xx. pp. 145-149.

the most commensurate numbers are pleasing to the reason.[55]

A somewhat different rationale for judging the value of consonances is found in Book V, where Ptolemy rather than Nicomachus is the principal source. Ptolemy is in essential agreement with the Pythagorean musical thought, for he insists that reason must judge the proportions of musical intervals,[56] and that these intervals must be measured in numerical proportions like those of the Pythagoreans rather than merely estimating the difference between two sounds as did Aristoxenus.[57] Nevertheless Ptolemy holds that the Pythagoreans put too much emphasis on the reason in determining which intervals were consonant and in judging their respective values.[58] Ptolemy is clearly aiming his criticism at the early Pythagoreans rather than the Nicomachus source of the first four Books. The first four Books, with Ptolemy, acknowledge that the senses play an important role in musical judgment, namely they recognize consonances.[59] Nevertheless the first four Books hold that

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55. <u>The Principles of Music</u> I. xxxii. p. 99.
56. <u>Ibid</u>. V. ii. pp. 294-299.
57. <u>Ibid</u>. V. iv. pp. 301-302
58. <u>Ibid</u>. V. iii. p. 300 , n. 4.
59. <u>Ibid</u>. I. viii-ix, xxviii; II. xviii; IV. i.

only multiple and superparticular proportions contain musical consonances, and implicit therein would be the phrase "regardless of what the senses judge." In the first four Books the octave, the fourth, and fifth, and the octave and fifth (the twelfth) are acknowledged to be consonances; but the octave and fourth (the eleventh) is expressly denied such classification because it is not a multiple or superparticular proportion,[60] and when the consonances are tested on the monochord, the eleventh is omitted.[61]

Ptolemy would consider this as carrying the role of reason too far in musical judgments; for as both the fifth and the fourth are considered consonances, so should the octave and fifth (the twelfth) and the octave and fourth (the eleventh) be considered consonances, regardless of the latter's proportion. The eleventh sounds consonant to Ptolemy; it is not contained in a multiple or superparticular proportion (6:3, an octave, plus 8:6, a fourth, equals 8:3, a multiple-superbipartient proportion), but it is nevertheless a consonance to Ptolemy. Thus Ptolemy's somewhat

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60. _The Principles of Music_ II. xxvii. pp. 159-161.
61. _Ibid_. IV. xviii. pp. 291-293.

different position is that the ears judge which intervals are consonant, and the reason determines their proportions regardless of which type inequality holds their proportions. He nonetheless exhibits a certain Pythagorean preoccupation with numerical "equality" when he classifies the consonances according to their value. For he classifies all related pitches as unison, equal-sounding, consonant, melodic, unmelodic, and dissonant.[62] Unison pitches are equal in pitch, and thus they are equivalent to equality in number. The equal-sounding octave logically follows unison pitches:

> Since equal sounding pitches come closest to unison pitches in these comparisons, it is necessary that the numerical inequality which is closest to equality be added to the numerical equality. The double proportion is quite near to numerical equality; for it is both the first species of multiple proportion, and, when the larger number is placed above the smaller number, it transcends the smaller number by the smaller number itself: two transcends one by one, which is equal to the very same unity. Thus the duple proportion is rightly fitted to equal-sounding pitches, that is, to the diapason.[63]

The intervals which combine to form the equal-sounding octave, the fourth, and the fifth, are considered the first

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62. Ibid. V. xi. pp. 310-312.
63. Ibid. pp. 311-312.

consonances.[64] Moreover any consonance added to the equal-sounding octave will produce another consonance. Therefore both the twelfth and the eleventh will be consonances.[65] Thus Ptolemy arrives at value judgments somewhat different from those of the first four Books; but he nevertheless justifies these judgments using the same mathematical value as used in Books I-IV--equality.

The reader of Book V is likely to feel that Boethius accepted this tenet of Ptolemy and did indeed himself consider this interval a consonance. But Boethius could accept this proof only after he had added a Pythagorean commentary to the basic text of Ptolemy.[66] For Book V, Chapter X, compares Ptolemy's equal sounding octave with the number 10, which preserves its own identity if any number contained within it is added to it. Such a comparison is found nowhere in the text of Ptolemy, and only after Boethius has added this commentary is the text allowed to proceed and conclusively demonstrate that this interval is a consonance. It is as though the reason could take pleasure

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64. Ibid. p. 312.
65. Ibid. p. 312 ; cf. V. ix. pp. 307-309.
66. Ibid. V. x. pp. 309-310, n. 19.

in this commentary just as the ears might take pleasure in adding any consonance to the equal-sounding octave. It is as though Boethius in Book V were trying to harmonize the musical aesthetics of Ptolemy and Nicomachus, the most convincing musical theorists of antiquity, in the same way that he would later try to harmonize the philosophies of Aristotle and Plato, the most convincing philosophers of antiquity.

To consider Boethius' musical philosophy in modern terms, he would by no means argue that music is an autonomous art expressing abstracts in sound which have no meaning besides the purely aural patterns which emerged from any given musical expression. Boethius would hold that music communicates; on the lowest level it communicates pleasure and pain, emotions, to the ears, and man is affected by pleasing or unpleasant combinations of sounds. But on the highest level music communicates truth; for the reason can abstract mathematical proportions from musical sounds and thereby come to know incorporeal essences. Moreover the reason finds as much pleasure in contemplating these harmoniously related essences as the ears find in the fleeting experience of corporeal sound.

In summary, music has a power which none of the other mathematical disciplines share, the power to affect men with either pleasure or pain and thereby to affect their actions. The ears are greatly pleased by sounds which intermingle as one, but find displeasure in sounds which do not so intermingle. The ears can identify combinations of sounds which are pleasing, but they are incapable of judging the proportions of these sounds or of completely determining which of these combinations are the most pleasant. Therefore the reason must search out the essences of these sounds so it can know them and thereby judge their respective merit. The essences of sounds are related quantities or mathematical proportions. Thus the reason determines that sounds of the most commensurate frequencies are the most pleasing to the ears; and just as commensurate sounds please the ears, so commensurate proportions are pleasing to the reason. The reason in turn can see this same pleasing harmony in its own being and in the structure of the universe itself. Hence it can be led to behold the harmony and proportion in the metaphysical world of pure and incorporeal essences.

CHAPTER III

BOETHIUS' MUSICAL MATHEMATICS

"A thing will be easier to know if it is first discerned in numbers."[1] This statement by Boethius would serve as the creed for a Pythagorean musical theorist. The preceding chapter of this commentary demonstrates that the ultimate goal of Boethius' musical aesthetics is to know musical essences, and musical essences are expressed as related quantity. Thus the musical theorist's task is to demonstrate in what numerical proportions the essences of musical consonances are found, and, based on these proportions, deduce the proportions of all other musical intervals. The ears take pleasure in music, and they themselves identify which intervals are consonant; but the reason must <u>objectify</u> each interval which occurs in musical experience so that they may be known, and "a thing will be easier to know if it is first discerned in numbers." This relentless striving toward objectification of all musical

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1. <u>The Principles of Music</u> II. xx. p. 145.

experience is the principal force in Boethius' musical thought; and Boethius' musical theory is totally dependent upon the musical mathematics which serves to objectify every possible sound.

Therefore music must be considered as a mathematical discipline before the purely musical theories presented in The Principles of Music can be explicated. But before music can be studied as a mathematical discipline, arithmetic must first be mastered; for arithmetic is by nature prior to all the other disciplines.[2] Moreover, since arithmetic is the study of quantity in and of itself, it must clearly be mastered before the related quantities of music can be studied.[3] Therefore Boethius presupposes a knowledge of the basic contents of The Principles of Arithmetic throughout The Principles of Music, although he does review the basic "axioms" necessary for musical proofs in the first chapters of Book II. This commentary on Boethius' musical mathematics must thus begin by discussing certain basic mathematical concepts presented in The Principles of

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2. The Principles of Arithmetic I. i. 27-28.
3. Ibid., p. 29.

Arithmetic; thereafter the musical axioms and the deductive proofs of musical intervals may be examined. Finally the application of the mathematics to musical space as found on the monochord will be discussed.

The first principle of Pythagorean mathematics is "unity," or the monad. The monad, "one," was most often notated as a point, the dyad, or two, by two points (..), the triad, or three, by three points (∴), etc. Numbers, or essences consisting of absolute quantity, were consequently defined as even and odd, prime or noncomposite and composite, perfect and imperfect, and were considered as longitudes and latitudes and as various geometric shapes.[4] Numbers of various types were considered to have different qualities or "virtues" according to their composition and shape. But "numerosity" itself does not begin until the number two; for "unity" is indivisible and, as such, the beginning generator of all numbers.[5] Thus in both The Principles of Arithmetic and The Principles of Music the word "unity" is

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4. De Institutione Arithmetica i. 3-20.

5. Ibid. i. 3: "Numerus est unitatum collectio, vel quantitatis acervus ex unitatibus profusus." Cf. i. 7.

used more than the word "one." Moreover, since unity is the generator of all numerosity, it is itself essentially indivisible.

As unity begins to develope into plurality, as one unity is added to another and two unities are derived (..), then absolute quantity can begin to be considered. But as absolute numerical plurality--numbers in and of themselves--is considered, so does related quantity come into consideration; for as numbers are generated from unity, they may be related back to the unity or related to each other; and thus related quantity is born from absolute quantity. But the first and highest type of related quantity is equality. For equality is ultimate "identity"; and there is only one type of equality, for quantities can be related through equality in only one way, that is, they can be equal.[6] The second type of related quantity is inequality. There are five species of this type of related quantity, three simple and two composite: multiple, superparticular, superpartient, multiple-superparticular, and multiple-superpartient. These are the species of inequality that are discussed in the

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6. Ibid. i. 21.

mathematical theory of The Principles of Music,[7] but these species of inequality are explicated according to mathematical, or geometric, logic; and thus the basic axioms which form the basis for this logic must be examined.

Just as absolute numerical quantity proceeds from unity, so that type of related quantity consisting of inequality proceeds from equally related quantity. Chapter VII of the second book describes this process: three equalities are set forth, and under the first of these terms a number is placed which is equal to the first of the equal terms; a term is placed under the second term which is the sum of the first equal term plus the second equal term, and thus the first departure from unity and equality is accomplished; a term is placed under the third term which is equal to the first term plus two times the second--two being the first departure from equality and unity--plus the third. Thus the first and highest type of inequality is derived: the duple proportion in the multiple species of inequality.

1	1	1
1	2	4

7. The Principles of Music I. iv, II. iv.

Since this is the first departure from unity and equality, it is consequently the most beautiful numerical proportion and holds the greatest musical consonance.[8] By following the same process, but using this first departure from unity and equality, the second most beautiful proportion and second musical consonance is discovered; the triple proportion.[9]

1	2	4
1	3	9

(1=1; 1+2=3; 1+2x2+4=9)

Thus the second departure from equality is accomplished, and each successive departure in the superior multiple type of inequality is made from the preceding departure from equality.

The next most beautiful departure from unity is the superparticular type of inequality, and this type is discovered by applying the same process to an inverted multiple series of inequality. Moreover the denomination of the superparticular proportions is determined by the denomination

8. Ibid. II. xviii, xx.
9. *De Institutione Arithmetica* i. 32.

of the multiple proportion inverted: duple yields sesquialter, triple sesquitertian, quadruple sesquiquartian, etc.

4 : 2 : 1
4 : 6 : 9

4=4; 4+2=6; 4+(2x2)+1=9)

9 : 3 : 1
9 :12 :16

9=9; 9+3=12; 9+(2x3)+1=16)

And in this manner the successive departures from equality are accomplished. The superpartient proportions are produced from inverted superparticular proportions; the multiple-superparticular proportions are produced from noninverted superparticular proportions, and the multiple-superpartient proportions are produced from noninverted superpartient proportions. Therefore, the closer the proportions are to equality, the more beautiful their related quantity and the more harmonious the musical interval they hold. This can be considered the first axiom of Boethius' musical mathematics.

The second axiom of musical mathematics is found in Book II, Chapter VIII, and grows directly out of the first axiom presented in Chapter VII. Since superparticular proportions are produced from transformed multiple proportions, any given multiple number will produce as many continuous superparticular proportions in comparable denomination as the given multiple number itself departs from unity:

1	2	4	8	16		1	3	9	27	81
	3	6	12	24			4	12	36	108
		9	18	36				16	48	144
			27	54					64	192
				81						256

This axiom becomes one of the most important in The Principles of Music, for through it six continuous tones can be computed, and it can be thereby demonstrated that the octave does not consist of six tones.

The third mathematical axiom relating to musical proportions is contained in Chapter IX of Book II. The ultimate goal of this axiom is to demonstrate that smaller numbers contain larger proportions and larger numbers contain smaller proportions, but Boethius' method of proving this axiom is somewhat indirect. If the difference of two numbers equally divides them, the quotients will be in the same proportion as the original numbers:

50:55　　55-50=5

$5\overline{)50}=10$　　$5\overline{)55}=11$

$\therefore$ 10:11=50:55

If the difference between two numbers divides them in such a way that the same remainder is produced in each division, if this remainder be subtracted from the original numbers, the resulting numbers will be a larger proportion than the original numbers:

53:58　　58-53=5　　$5\overline{)58}=11+3$　　$5\overline{)53}=10+3$

$$\begin{array}{rr} 53 & 58 \\ -3 & -3 \\ \hline 50 & 55 \end{array}$$

$\therefore$ 55:50 is larger than 58:53

If the difference between two numbers divides the two numbers in such a way that divisor times the quotients surpasses both dividends by the same quantity, if this quantity be added to the original number, the resulting proportion will be smaller than that of the original numbers.

53:48　　53-48=5　　$5\overline{)53}=11-2$　　$5\overline{)48}=10-2$

$$\begin{array}{rr} 53: & 48 \\ +2 & +2 \\ \hline 55: & 50 \end{array}$$

$\therefore$ 55:50 is smaller than 53:48

This demonstration is obviously aimed at superparticular proportions rather than any other species of inequality;

for the examples are all of the superparticular species. Hence if an equal quantity be added to numbers in a superparticular proportion, the resulting numbers will be a smaller proportion than the original; and, conversely, if an equal quantity be subtracted from such numbers, the resulting proportion will be larger:

$$\begin{array}{r} 3:2 \\ +1+1 \\ \hline 4:3 \end{array}$$

$\therefore$ 3:2 is larger than 4:3

Hence Boethius concludes that smaller numbers contain the larger superparticular proportions, whereas larger numbers contain the smaller superparticular proportions.

Chapter X of Book II hints at several axioms necessary for musical demonstrations, but the remaining axioms are scattered throughout Books II, III, and IV. Nevertheless, in this commentary the remaining axioms should be summarized here with the first three axioms. Chapter X states that multiple proportions multiplied by two produce neither multiple nor superparticular proportions. Thus the fourth axiom necessary for musical demonstrations could be that multiple proportions multiplied by two produce other multiple proportions, and if the product of a multiplication by two does not equal a multiple proportion, then the proportion multiplied was not a multiple proportion.

The fifth axiom would then be that superparticular proportions multiplied by two equal neither multiple nor superparticular proportions; and if the result of a multiplication by two equals a superparticular or a multiple proportion, then the proportion multiplied was not superparticular.

Out of this fifth axiom grows the sixth axiom, one of the most important axioms in musical reasoning. If a superparticular proportion multiplied by two cannot produce another superparticular proportion, then it necessarily follows that a superparticular proportion divided by two can not yield another superparticular proportion. If a superparticular proportion divided by two cannot yield another superparticular proportion, then it necessarily follows that a superparticular proportion cannot be divided into two equal proportions. This axiom is not stated as such until Book III, Chapter XI, and is not conclusively demonstrated until Book IV, Chapter II. Nevertheless this axiom logically follows the fifth axiom, and probably should have occurred in the original text following the fourth and fifth axioms. From this axiom arises the essential proof that the semitone cannot consist of half a tone, for the tone (9:8) is a superparticular proportion and thus cannot be divided into two equal proportions.

The seventh axiom is contained in Book II, Chapter XI, which discusses the results of adding superparticular and multiple proportions. The first two superparticular proportions equal the first multiple, for 4:3 plus 3:2 equals 4:2, the duple proportion. Moreover, the first multiple plus the first superparticular equals the second multiple, 3:2 plus 2:1 equals 3:1; while the second superparticular plus the second multiple equals the third multiple, 4:3 plus 3:1 equals 4:1. In this same manner corresponding superparticular and multiple proportions produce the next multiple proportion. From this axiom the composite structure of various consonances can be mathematically demonstrated.

The eighth and final axiom necessary for demonstration of musical truths is found in Chapter XI of Book II. If a superparticular proportion be subtracted from the next largest superparticular proportion, the remainder will not be as much as half of the subtracted proportion. This axiom becomes necessary to prove that the octave, consisting of a fourth and a fifth, must necessarily be found in the duple proportion.

These axioms properly belong in the discipline of arithmetic,[10] but nevertheless Boethius presents them in The Principles of Music so he can deductively demonstrate the mathematical proportion, the essence, of each musical interval and consonance. However, before these axioms can be used to demonstrate the proportions of musical intervals, three assumptions must be made: (1) consonances occur only in multiple and superparticular proportions, (2) a diatessaron plus a diapente equals a diapason, and (3) the difference between a diatessaron and a diapente is a tone. For the diapente and the diatessaron cannot be deductively demonstrated to equal a diapason until the diapason is established in the duple proportion and the diatessaron and the diapente respectively in the sesquialter and sesquitertian proportions, and the 9:8 tone cannot be deductively demonstrated to exist in the difference between the diapente and the diatessaron until these intervals have been established

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10. Cf. The Principles of Music II. pp. 109-110, 113-117, nn. 4-7, 9-14.

in the two largest superparticular proportions. These assumptions are ultimately dependent upon Pythagoras' inductive proof that these four intervals are found in the proportions 2:1, 3:2, 4:3, and 9:8.[11] Given this inductive proof the consonances are indeed found in multiple and superparticular proportions, the octave consists of a fifth plus a fourth, and the difference between the fifth and the fourth is a 9:8 tone. Book I would have the reader take these basic assumptions on faith; and after these basic tenets are accepted on faith the reason can indeed demonstrate their own truth and the truths of all other musical intervals.

Thus the last chapters of Book I and all of Book III proceed to demonstrate the mathematical essences of the consonances and related musical intervals, building each demonstration on the basic assumptions, the axioms, and the preceding proofs in the manner of geometric logic. The octave is demonstrated to be held in the duple proportion,[12] the fifth in the sesquialter, and the fourth in the

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11. *The Principles of Music* I. x-xi.
12. *The Principles of Music* II. xxii-xxiii.

sesquitertian,[13] the tone in the sesquioctave,[14] the twelfth in the triple, and the double octave in the quadruple.[15] Moreover, the semitone, the remainder after two tones have been subtracted from a diatessaron, is found in the proportion 256:243,[16] and this proportion is by no means half of a tone.[17] The apotome, the remainder after a semitone has been subtracted from a tone, is found in the proportion 2187:2048.[18] The octave is smaller than six successive tones, and the difference between the octave and six successive 9:8 proportions forms the comma, the smallest musical interval, consisting in the proportion 531441:524288.[19] This comma also forms the difference between the semitone and apotome,[20] and therefore it forms the difference between two semitones and a tone.[21] The comma is demonstrated to be larger than 75:74 and smaller

13. Ibid. II. xxiii-xxv.
14. Ibid.
15. Ibid. II xxvi.
16. Ibid. II. xxviii.
17. Ibid. II xxix.
18. Ibid. II. xxx.
19. Ibid. II. xxi, III. iv.
20. Ibid. III. vi.
21. Ibid. III. vii.

than 74:73,[22] while the semitone is demonstrated to be larger than 20:19 and smaller than 19 1/2:18 1/2.[23] The semitone is demonstrated to be larger than three commas amd smaller than four,[24] while the apotome is larger than four commas and smaller than three;[25] consequently the tone is larger than seven commas and smaller than eight.[26] Finally, the consonances of the multiple type are judged to be superior to those of the superparticular type because they least depart from proportions of equality.[27] Had Boethius completed the translation of Ptolemy in Book V, this mathematical logic would have divided the fourth or tetrachord into various combinations of three unequal intervals, all consisting of superparticular proportions.[28]

These deductively proven proportions, considered in numbers so they might be known, thus become the unchanging essences which govern the art of music. These essences in turn are applied to the monochord so that the ear and eye can come to know them as they truly are. But before

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22. Ibid. III. xii.
23. Ibid. III. xiii.
24. Ibid. III. xiv.
25. Ibid. III. xv.
26. Ibid. III. xvi.
27. Ibid. II. xviii-xx., cf. V. xi.
28. See Book V. pp. 327-330.

the application of these proportions to the monochord is considered, the theory concerning mathematical means pertaining to music should be discussed.

Three types of means are discussed in Chapters XII through XVII of Book II: arithmetic, geometric, and harmonic. A mean term is one which divides a numerical proportion. If this term divides the proportion in such a way that an equal difference occurs between each term, the mean is then defined as "arithmetic"; if the term divides the proportion in such a way that equal proportions occur between each term, the mean is defined as "geometric"; if the term divides the proportion in such a way that the differences between the terms hold the same proportion as the two outer terms, the mean is defined as "harmonic."[29]

Arithmetic	Geometric	Harmonic
1:2:3	1:2:4	3:4:6

When three such numbers are related in mathematical proportions, the relation as a whole is defined as arithmetic, geometric, or harmonic "proportionality."[30] Moreover, just as proportions of inequality proceed from equality, so the

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29. The Principles of Music II. xii.
30. Ibid.

formation of these types of proportionality are produced from numerical equality.[31]

The principal purpose for including a discussion of mathematical means in a musical treatise is to demonstrate the types of proportionality which can occur in composite musical consonances. Thus a composite twelfth, consisting of an octave and a fifth, is seen to be an example of arithmetic proportionality: 1:2:3. The double-octave is found to be the only geometrically composed composite consonance: 1:2:4. Finally, the composite octave, the most harmonious of musical intervals and consisting of a fourth plus a fifth, is seen to be founded in harmonic proportionality: 3:4:6. Negatively speaking, these types of means demonstrate that superparticular proportions, for example the tone, can be divided by an arithmetic mean (18:17:16), but no other type of mean term can divide this type of proportion; therefore no superparticular proportion can be divided into two equal proportions (axiom 6). Thus through speculation concerning these means the reason can come to know the various relationships in which consonances and other intervals can be related.

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31. Ibid. II. xv.

But the harmonic mean, based on the fourth, the fifth, and the octave, is particularly suited to speculation concerning musical proportions; for through multiplication of terms in this type of proportionality (3:4:6) times themselves, times each other, and times their differences (2:1) all possible proportions governing musical consonance emerge.[32]

36 24 18 16 12 9 8 6 4 3 2 1

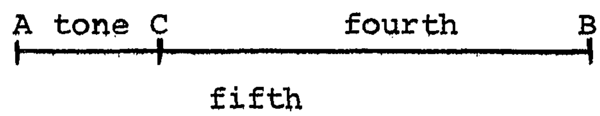
Thus emerge the proportions for the tone (18:16, 9:8), the fourth (24:18, 16:12, 12:9, 8:6, 4:3), the fifth (36:24, 24:16, 18:12, 12:8, 6:4, 3:2), the octave (36:18, 24:12, 18:9, 16:8, 12:6, 8:4, 6:3, 4:2, 2:1), the twelfth (36:18 24:8, 18:6, 12:4, 9:3, 3:1), and the double-octave (26:9, 24:6, 16:4, 12:3, 8:2, 4:1), the most essential proportions considered in speculation concerning musical intervals. Boethius might have also pointed out that the eleventh (8:3) is also found in this series in its root proportion and between 24 and 9; and since this interval grows out of harmonic proportionality, perhaps it could be considered a consonance. Moreover, if Boethius had inserted two tones

32. Book II. xvi. pp. 137-139, n. 22.

into the 36:24 fifth by multiplying 3 times 9 and 4 times 8, thus arriving at the series 36:32:27:24; and had he divided this middle proportion (32:27), consisting of a tone and a semitone since it is the difference between two tones and a fifth, by the proportion of the tone itself (9:8), he would have arrived at the proportion of the semitone, 256:243. Thus even this interval can be born out of harmonic proportionality. Therefore harmonic proportionality, consisting of an octave composed of a fourth and a fifth, is of particular interest to the student of the musical discipline; for all proportions governing musical consonances and other intervals can be derived through mathematical speculation concerning these numbers.

Up to this point Boethius' musical mathematics has been considered purely in terms of numbers. But Boethius objectifies musical intervals in geometric space as well as in arithmetic quantities. This linear objectification is shown most clearly in Book III, Chapters IX-X; for there the intervals are discussed without numbers, purely in terms of adding and subtracting consonances in geometric space. This linear concept of musical intervals is expressed through letters, and thus A to B might be a fifth, B back to

C might be a fourth, and therefore, since the difference between a fifth and a fourth has been mathematically demonstrated to be a tone, the space between A and C must necessarily be a tone.



Therefore the mathematical reason objectifies musical intervals through both arithmetic numbers and geometric letters. This numerical and spacial objectification receives final synthesis in the division of the monochord.

The monochord division, contained in Chapters IV-XII of Book IV, gives each note of each genera a number denoting its numerical quantity in relation to the other notes and a letter denoting its position on the string in relation to the other notes. This division remains perfectly consistent throughout the diatonic genus; for there each tone is mathematically and spacially a 9:8 proportion in relation to the quantity and space computed before it, and every other interval is in the mathematical and spacial proportion of which it has been proven to consist throughout the first four Books. The Pythagorean numerical proportions are perfectly suited for such a division in the diatonic genus, but when these same numbers are applied to the division of the chromatic and enharmonic genera their consistancy is lost. In the diatonic genus, consisting of two tones and a semitone, two 9:8 tones are computed, and each semitone is formed by the remainder between these two tones and the 4:3 fourth. Each semitone in the diatonic genus thus consists of the proportion 256:243, the demonstrated mathematical essence of this interval. But the fourth of the chromatic genus is not divided into two tones and a semitone, but rather into an interval consisting of a tone and a half and two semitones. Rather than form this division as an autonomous division, the text, probably a translation of the Pythagorean Nicomachus, tries to build the chromatic genus from the already divided diatonic genus. Since the diatonic genus already has a semitone at the lower end of each tetrachord, this same semitone (256:243) is assumed as the first semitone in the chromatic genus. Another semitone must follow this semitone and the division must end with an interval consisting of a tone and a half. Thus the text attempts to increase the 9:8 tone at the top of the diatonic genus into an interval consisting of a tone and a half. It does so by equally dividing the difference between the two numbers containing this tone and adding half of that distance to the original tone.[33] For example, if the top tone in the diatonic genus were 18:16, the difference would be subtracted (2) and half this difference would be added to 18; thus 19:16 would be the proportion of the tone and a half. Thus the "half tone" added to the original 9:8 diatonic tone is the proportion 19:18. Book III, Chapter XIII demonstrated that the 256:243 semitone was smaller than 19 1/2:18 1/2, whereas 19:18, the proportion of the semitone added to the tone in this division of the chromatic monochord, is a larger proportion than 19 1/2:18 1/2 (axiom 3). Therefore an interval considerably larger than the semitone has been added to the tone. Moreover, another semitone is said to have been created between the number containing this tone and a half and the number containing the 256:243 semitone in the chromatic genus. But this new semitone must be considerably smaller than the Pythagorean apotome (2187:2048); for the apotome was formed by subtracting a semitone from a tone, whereas this interval was made by subtracting an interval _larger_ than a semitone from a tone

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33. _The Principles of Music_ IV. vii. pp. 243-245, n. 30.

(subtracted in the same sense that the 19:18 interval was added to the top tone in the diatonic genus, and thus the same amount was taken away from the middle tone of the diatonic genus, leaving "part" of a tone). Since this newly computed semitone is smaller than an apotome (which is larger than half of a tone), and the original diatonic semitone is smaller than half a tone, then the sum of the two semitones computed in this division of the chromatic genus cannot possibly equal a 9:8 tone. The root proportions of the intervals in this division are as follows:

256:243	81:76	19:16
semitone	semitone	tone and a half.

Thus the Pythagorean mathematical consistency which had been maintained throughout the first three Books and through the diatonic division in Book IV is broken in this division of the chromatic genus.

The division of the enharmonic genus is similarly based on the previous division of the diatonic genus; but since the enharmonic genus consists of two quarter-tones plus an interval consisting of two tones, the two 9:8 tones of the diatonic genus can be considered as a single interval and the remaining semitone can be divided into two diesis or quarter-tones. Since the first four Books of this treatise are basically Pythagorean, and since the Pythagorean mathematics of these Books is related almost exclusively to the diatonic genus, no numerical proportion has been established as that of the diesis. Thus the text subtracts the difference between numbers containing the diatonic semitone, divides this difference in half, and adds half this difference to the smaller number:[34] for example, 486 (2x243) subtracted from 512 (2x256) leaves 26, half of 26 is 13, which produces 499 when added to 486; thus 512:499:486 produces a semitone divided into two diesis. But in this division the differences are equal, and thus the proportions must be unequal. Therefore there is a minor diesis and a major diesis just as there is a minor and a major (apotome) semitone. Thus the proportions of the enharmonic division are as follows:

512:499	499:486	81:64
minor diesis	major diesis	two tones

Had Boethius completed translating Book V from the first Book of Ptolemy, the reader of The Principles of Music

34. The Principles of Music IV. vii. pp. 245-246, n. 31.

would have learned other divisions for all three genera, divisions which were built around unequal superparticular proportions. But these divisions were denied the Latin reader of *The Principles of Music*, and consequently the musical proportions he learned were those most perfectly suited to the Pythagorean division of the diatonic genus. Nevertheless, he did come to know the numerical essences of the consonances, the tone, the semitone, the apotome, and the comma, for the proportions of these intervals were thoroughly demonstrated by deductive reason. Moreover, he came to know the spacial proportions of these intervals as they were computed on the monochord, and thereby he came to know the sounds of these mathematically proven essences. Thus he came to demand the sounds of these mathematical essences in the music he heard.

CHAPTER IV

BOETHIUS' MUSICAL THEORY

Boethius' chief source for _The Principles of Music_ was the Pythagorean Nicomachus, and his second source was Ptolemy, a theorist who shared many views of the Pythagoreans. Out of these basically Pythagorean sources grew an aesthetics which held that music was determined by essences consisting of related numerical quantities; and out of this aesthetics developed a mathematics which demonstrated the numerical proportions of musical consonances and other intervals. The purely musical theory to be discussed in this chapter is largely a logical development of Boethius' aesthetics and mathematics.

The beginning and the end of purely musical theory as discussed by _The Principles of Music_ in its incomplete state is the tetrachord.[1] The tetrachord, for Boethius as well as for all Greek musical theorists, is the musical unit out of which the complete musical scale is derived. This

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1. _The Principles of Music_ I. xv, V. xix.

musical unit consists of a diatessaron, or fourth, divided into three intervals, generally described in the diatonic genus as a semitone, tone, tone--for example, e, f, g, a. By combining these musical units in various ways, the complete two octave musical scale of Greek theory and that of The Principles of Music is derived.

Book I, Chapter XX, of The Principles of Music describes the way in which tetrachords were combined to produce the Greek two octave system, or Greater Perfect System, and in so doing it relates a somewhat mythological history of the musical scale which is found in no other extant source. Boethius, citing Nicomachus, states that music began very simply, consisting of only four notes; and these four notes were tuned as an octave filled by a fourth and a fifth--for example, e, a, b, e. This invention of the four note disposition is accredited to Mercury, and is said to have remained in this state until the time of Orpheus.[2] The perfect tuning ascribed to these four notes is obviously a result of Pythagorean thinking, for such a tuning would be that in which Pythagoras discovered the proportions

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2. Ibid. I. xx. p. 73, n. 44.

of musical consonances: 12:9:8:6. If these first four strings are rather considered as the tetrachord between the hypate and the mese (e, f, g, a) as the text later describes them,[3] then the next three additions quite logically follow. Therefore these four notes become the so-called "Dorian" tetrachord[4] contained between the hypate and the mese:

e	Hypate
f	Parhypate
g	Lichanos
a	Mese

The fifth note is said to have been added by Toroebus the Lydian, who is elsewhere accredited with having invented the Lydian mode.[5] Thus if the "paramese or trite" be added to the above notes, the so-called "Lydian" tetrachord may be thereby produced:

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3. *Ibid*. p. 74.

4. It is actually improper to ascribe modal titles to tetrachords, for all Greek tetrachords in the diatonic genus consist of the same semitone--tone--tone structure. Nevertheless this terminology is used in the present argument to show the logic of the first three additions, rather than introduce the proper term "species" into the text at this time.

5. *The Principles of Music* I. xx. p. 74, n. 45.

e Hypate
f Parhypate
g Lichanos
a Mese
b♭ Paramese or Trite

A sixth note is said to have been added by Hyagnis the Phrygian, who is likewise elsewhere accredited with having invented the Phrygian mode.[6] Thus if the paranete is added to the above five note disposition, the so-called "Phrygian" tetrachord can be produced:

e Hypate
f Parhypate
g Lichanos
a Mese
b♭ Paramese or Trite
c Paranete

Finally, the addition of the seventh note is attributed to Terpander. Moreover, Terpander is elsewhere accredited with having added a "Dorian nete,"[7] and when the nete is

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6. Ibid., n. 46.
7. Ibid., n. 47.

added to the above six note disposition, two so-called "Dorian" tetrachords are formed.

e	Hypate
f	Parhypate
g	Lichanos
a	Mese
b♭	Paramese
c	Paranete
d	Nete

Thus this "history" of the origins of the first seven notes is completely consistent with other Greek sources, and the contents of Book I, Chapter XX, translated from Nicomachus, may be assumed to be a valid Greek rationale for the formation of this seven note disposition.

Two complete tetrachords are contained in the above seven note disposition, but these two tetrachords must be described as conjunct; for one note, the mese, is positioned in such a way that it is both the last note of the first tetrachord and the first note of the last.[8] Thus an eighth

8. Ibid. I. xxiv.

note was added to this disposition by Lycaon of Samos or Pythagoras[9] so that two "disjunct" tetrachords could be formed, and the classical Greek Dorian octave was thereby produced:

e	Hypate
f	Parhypate
g	Lichanos
a	Mese
b	Paramese
c	Trite
d	Paranete
e	Nete

Two tetrachords are described as disjunct when the two tetrachords are separated by a tone.[10]

Subsequently individual notes and conjunct tetrachords were added to this basic octave, until the complete gamuts of the Synemmenon, or conjunct, and the Diezeugmenon, or disjunct, systems were thusly formed:

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9. *Ibid.* I. xx. p. 75, n. 49.
10. *Ibid.* I. xxv.

a Promlambanomenos
b Hypate Hypaton
c Parhypate Hypaton
d Lichanos Hypaton
e Hypate Meson
f Parhypate Meson
g Lichanos Meson
a Mese
b Paramese
c Trite Diezeugmenon
d Paranete Diezeugmenon
e Nete Diezeugmenon
f Trite Hyperboleon
g Paranete Hyperboleon
a Nete Hyperboleon

a Proslambanomenos
b Hypate Hypaton
c Parhypate Hypaton
d Lichanos Hypaton
e Hypate Meson
f Parhypate Meson
g Lichanos Meson
a Mese
b♭ Trite Synemmenon
c Paranete Synemmenon
d Nete Synemmenon

Thus the Diezeugmenon system, or Greater Perfect System, consists of two octaves, each of which consists of two conjunct tetrachords: the first of the Hypaton and Meson and the second of the Diezeugmenon and Hyperboleon; but this system is disjunct because a tone falls between the mese and the paramese, that is, between the Meson tetrachord and the

Diezeugmenon tetrachord. The Synemmenon system, on the other hand, consists of three conjunct tetrachords: the Hypaton, the Meson, and the Synemmenon.

Thus by adding the musical units called tetrachords together in various ways, the two basic systems of Greek music are formed. But these tetrachords are also divided in various ways so that the three basic genera of Greek music can be produced. The tetrachord is first divided as described above into a semitone, a tone, and a tone (e, f, g, a), and such a division forms the diatonic genus. The tetrachord is also divided into two semitones and an interval consisting of three semitones (e, f, f#, a), and such a division forms the chromatic genus. Finally, the tetrachord may be divided into two diesis, or quarter tones, and an interval consisting of two tones, and this division creates the enharmonic genus (e, x, f, a).[11] Complete systems either Diezeugmenon or Synemmenon, are joined together in tetrachords of these three basic genera, and thus three variations of the two basic scales are formed.[12]

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11. The Principles of Music I. xxi.
12. Ibid. I. xxii.

But both tetrachords and the systems made thereof consist of intervals, and some of these intervals are the musical consonances. Thus the numerical proportions defined by reason as the mathematical essences of the musical discipline must be applied to the tetrachords and the systems if the sounds produced thereby are to be consistent with the known musical essences. Every tone must therefore sound in the deductively proven 9:8 proportion, and every fourth must be founded in the 4:3 proportion. Thereby, when two tones are sounded within a fourth, the remaining interval will form the 256:243 semitone, the mathematically demonstrated proportion of this interval. Moreover, every fifth, composed of three tones and a semitone, must sound according to the proportion 3:2, and every octave, consisting of five tones and two semitones, must sound according to the proportion 2:1. Therefore the mathematically demonstrated proportions of the various intervals and consonances must determine the tuning of the tetrachords and the systems.

The tuning of the various intervals according to their mathematical proportions is accomplished on the monochord. This instrument consists of a string stretched over a sound box or brace upon which a ruler is computed. Since the ruler is divided according to the mathematical proportions of musical intervals, if a movable bridge is placed exactly over a point on the ruler which holds the relative proportion for that particular pitch, the pitch will sound in accordance with the mathematical proportion. Thus the ruler is divided in half, and if the movable bridge is placed over this division, half of the string will sound a 2:1 octave in relation to the sound of the complete string. This point on the ruler will be the mese, for the mese is an octave higher than the lowest note, the proslambanomenos. If the distance between the mese and the end of the string is divided in half and the movable bridge is positioned over this division, the sound produced thereby will be an octave higher than the mese or two octaves higher than the proslambanomenos; therefore this pitch must be the hypate hypaton.[13]

Thus the two octaves of the Diezeugmenon system are tuned,

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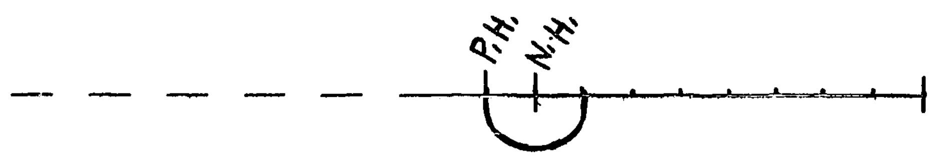
13. The Principles of Music IV. v.

and the other notes may be inserted therein. The space between the hypate hypaton and the end of the ruler is consequently divided into eighths, and one of these eighth parts is added to the hypate hypaton. Thereby a distance of string consisting of nine parts is related to a distance of string consisting of eight parts, and thus if the bridge is moved between these two divisions, the tone, consisting of a 9:8 proportion, will be sounded. Therefore this new division will be the paranete hyperboleon, for this note is a tone below the nete hyperboleon.[14]



If another such 9:8 proportion is computed from this paranete hyperboleon, another interval of a tone will be formed, which will make the trite hyperboleon. If the distance between the nete hyperboleon and the end of the ruler now is divided into three parts, and the distance of one of these parts is added to that of the nete hyperboleon, then the tuning will be set for a 4:3 fourth, and the distance between this new division and that of the last tone, the trite hyperboleon,

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14. <u>Ibid</u>. IV. vi.

will necessarily be the tuned interval of a 256:243 semitone; for when two 9:8 tones are subtracted from a 4:3 fourth, the distance or quantity remaining is that of a 256:243 semitone. Thus the diatonic tuning for the complete hyperboleon tetrachord is set as follows:[15]

Following this same process the complete divisions of the Diezeugmenon and the Synemmenon systems are applied to the ruler,[16] and as the bridge is moved along the string to the various divisions of the ruler, the tuning of the various notes is determined by the proportional division of the ruler.

Thus the complete two octave scale of the diatonic genus is tuned according to the numerical proportions which were logically demonstrated to hold the octave (2:1), the fifth (3:2), the fourth (4:3), the tone (9:8), and the semitone (256:243). The following chart gives the values

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15. Ibid.
16. Ibid. IV. vi-xi. For a complete division of the monochord ruler in diatonic genus, see Appendix.

of these intervals in Cents so that they may be compared to the well-tempered scale.

	Boethius	Modern
octave (2:1)	1200 C	1200 C
fifth (3:2)	701.96 C	700 C
fourth (4:3)	498.04 C	500 C
tone (9:8)	203.91 C	200 C
semitone (256:243)	90.22 C	100 C

The various other intervals produced on the monochord tuned according to these mathematical proportions can be computed by adding the various proportions and values in Cents.

minor third (32:27)	294.13 C	300 C
major third (81:64)	407.82 C	400 C
minor sixth (128:81)	792.18 C	800 C
major sixth (27:16)	905.87 C	900 C

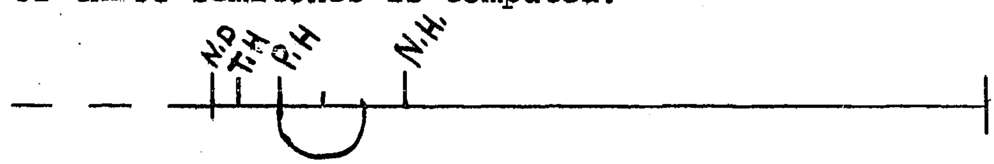
Thus the octave is the same "perfect" interval in both the "Pythagorean" tuning presented in <u>The Principles of Music</u> and the modern "well tempered" tuning. The "perfect" fifth and fourth fall within two Cents of these intervals in the well tempered scale, and the tone is almost four Cents larger than the well tempered tone. The remaining intervals differ from comparable well tempered intervals by at least five Cents, and thus a significant difference occurs between them. The temperament of a representative diatonic tetrachord as tuned by The Principles of Music would be as follows:

e	Hypate		
		(256:243) semitone	90.22 C
f	Parhypate		
		(9:8) tone	203.91 C
g	Lichanos		
		(9:8) tone	203.91 C
a	Mese		498.04 C or 4:3 fourth

The two remaining intervals for which the mathematical proportions were demonstrated in The Principles of Music, the apotome and the comma, occur nowhere in the diatonic scale. Nevertheless their comparable value in Cents would be as follows:

apotome	(2187:2048)	113.69 C
comma	(531441:524288)	27.47 C

The Principles of Music describes the chromatic genus as consisting of two semitones plus an interval composed of three semitones, and the enharmonic genus as consisting of two diesis plus an interval composed of two tones; nevertheless the first four Books of the treatise nowhere discuss the intervals of these genera in any more specific terms. Thus no numerical proportion is given to the diesis, and one would conclude that the division of the chromatic genus would follow the demonstrated proportions of the semitone and the apotome. The chromatic genus might thus be divided into a semitone, an apotome, and an interval composed of a tone plus a semitone, or it might be divided into two semitones plus an interval consisting of a tone plus an apotome. But the division of the monochord in the chromatic genus does not follow the demonstrated proportions of these intervals but rather tries to deduce the chromatic genus from the division of the diatonic genus. Thus the diatonic tetrachord is assumed, and the distance of the last tone is divided in half; this half is in turn added to the distance of the last tone, and thereby the interval said to consist of three semitones is computed:[17]



This new interval does indeed consist of three semitones, in fact, three different semitones; for the original tone consisted of an apotome and a semitone, and the newly computed addition consists of the proportion 19:18, a "semi"

17. The Principles of Music IV. vii. pp. 243-244.

tone which has been discussed nowhere in The Principles of Music. Moreover, since this added "semi" tone is neither an apotome nor a semitone, the interval which remains from the second tone after the new interval is added to the first is neither an apotome nor a semitone. Thus the mathematical consistency which is preserved through the division of the diatonic genus is forsaken in this somewhat contrived division of the chromatic genus.[18] This unsatisfactory division of a chromatic tetrachord would be as follows:

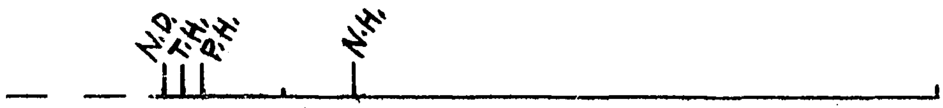
e	Hypate		
		(256:243) semitone	90.22 C
f#	Parhypate		
		(81:76) "semi" tone	110.41 C
f	Lichanos		
		(19:16) minor third	297.41 C
a	Mese		498.04 C

Thus the tuning of the second semitone (110.41 C) is by no means consistent with that of the apotome (113.69 C) or that of the semitone (90.22 C), and thus the sum of the 81:76 "semi" tone plus the original 256:243 semitone is less than a 9:8 tone--200.63 is less than 203.91. Finally, the minor third (297.41 C), consisting of three "semi" tones, is

18. Ibid. p. 244-245, 30.

larger than the diatonic minor third (294.13 C), consisting of two semitones plus an apotome.

Since *The Principles of Music* gives no specific proportional definition of the diesis similar to those for the semitones and the apotome, its division of the enharmonic genus can be said to be lacking any mathematical basis rather than forsaking mathematical consistency. This genus is again based on the division of the diatonic genus; for the two 9:8 tones of the diatonic genus become the enharmonic interval composed of two tones, and the distance of the remaining semitone is divided in half to form the two enharmonic diesis.[19]



Thus the quantities or distances between the three notes containing the diesis are equal, and thus, as in all arithmetic proportionality, the proportions are different.[20] Therefore the tetrachord in the enharmonic genus presents a minor diesis, a major diesis, and a major third.

19. *Ibid.* pp. 245-246, n. 31.
20. *Ibid.* II. xii.

e	Hypate			
		(512:499)	minor diesis	44.64 C
x	Parhypate			
		(499:486)	major diesis	45.58 C
f	Lichanos			
		(81:64)	major third	407.82 C
a	Mese			498.04 C

Thus two new intervals are invented in this division of the enharmonic genus by dividing the diatonic semitone, and the major third of this genus is consistent with the same interval in the diatonic genus (407.82 C).

Boethius presents Archytas' and Aristoxenus' divisions of tetrachords in the three genera in Book V. But since this Book is a translation of Ptolemy and Ptolemy denies the validity of these divisions, Boethius also rejects them.[21] Had Boethius completed translating Book V from the first Book of Ptolemy's Harmonics, The Principles of Music would have contained other possible divisions of tetrachords on the monochord ruler.[22] But since the mathematical ratios of

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21. Ibid. V. xvi-xviii.

22. See pp. 327-331 for titles of unfinished chapters and their corresponding passages in Ptolemy Harmonics i. For comparable temperaments of Ptolemy's divisions see J. Murray Barbour, Tuning and Temperament, A Historical Survey (East Lansing: Michigan State College Press, 1953), pp. 16-21.

these other possible and equally valid divisions were never translated, the reader of The Principles of Music came to know only the Pythagorean tuning of the diatonic genus and somewhat unsatisfactory tunings of the chromatic and enharmonic genera. But the principal goal of Pythagorean musical theory was to define the mathematical proportions of musical consonances, and all other musical intervals were computed from the proportions of these consonances and their differences. Ptolemy, on the other hand, was more interested in discovering all the possible variations into which the tetrachord could be divided, always using superparticular proportions. Thus it could be said that the Pythagoreans were primarily interested in defining the mathematical proportions of pleasing consonances, whereas Ptolemy was primarily interested in defining all the possible mathematical proportions of pleasing melody. Boethius obviously planned to include both traditions in The Principles of Music, thereby demonstrating to the mind of his reader that both pleasing melody and pleasing consonances were founded in the most pleasing mathematical essences, that is, in multiple and superparticular proportions.

Since Boethius did not complete that part of _The Principles of Music_ which discussed the proportions of melodic variations, the prime musical interest of the treatise seems to be in the consonances and the resulting Pythagorean diatonic scale. Thus after the diatonic scale has been divided, the various consonances can be heard according to their mathematical proportions. The theory of consonance or harmony thus becomes one of the most important aspects of musical theory in _The Principles of Music_. The Classical Greek concept of harmony considered consonances in melodic terms; if the mese (a) was sounded following the hypate (e), the result would have been considered a consonance, that is, a melodic consonance. But the theory of harmony presented in _The Principles of Music_ is by no means the Classical Greek concept of _melodic_ consonance, but rather the modern concept of _simultaneous_ sounds. Consonance is generally defined in _The Principles of Music_ as a pleasant "mixture" of sounds or as two sounds which "intermingle."[23] This definition is more or less the

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23. _The Principles of Music_ I. viii, V. vii.

Classical definition of consonance,[24] and might apply to either melodic or simultaneous sounds. However, the more specific definitions of consonance which occur in the first four Books describe consonances as simultaneously intermingling sounds:

> For when two strings, one higher and one lower, are stretched and struck at the same time (simul), and they produce an intermingled sweet sound, and the two sounds agree together as if joined in one, then that is what we call consonance.[25]

This passage is very closely related to a similar passage in Nicomachus' Manual, where consonances are again described in terms of two strings struck at the same time (ἅμα κρουσθέντες).[26] A similar definition contained in Chapter I of Book IV, based on the Sectio Canonis,[27] clearly shows the change from the Classical concept of melodic consonance to the modern concept of simultaneous consonance:

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24. Cf., e.g., Sectio Canonis (Jan, Musici Scriptores Graeci [Hildesheim: Georg Olms (reprint of 1895 edition) 1962] p. 149).

25. The Principles of Music I. xxviii. p. 95.

26. Nicomachus Manual of Harmony xii (JanS. 262).

27. The Principles of Music IV. i. pp. 211-213, nn. 1, 3.

> Consonant sounds are those that sound sweet and intermingled when struck at the same time, while dissonant sounds are those which do not sound sweet and intermingled when struck at the same time.[28]

The original text of this passage as contained in the _Sectio Canonis_ (third century, B.C.) does not contain the phrases "struck at the same time." Nicomachus, using this text as the introduction to the fourth Book of his Introduction to Music, probably added the phrase ἅμα κρουσθέντες to the basic definition of _Sectio Canonis_, which Boethius subsequently translated "_simul pulsae_."

Book IV, Chapter XVIII, finally draws a distinct difference between interval and consonance. If a string is divided into two parts according to the proportion of a diatessaron consonance, and if the two parts are struck alternately, then the _interval_ of a diatessaron becomes known. If, on the other hand, both parts of the string are struck at the same time, the _consonance_ of the diatessaron becomes known.[29] Thus there can be no doubt that the concept of consonance contained in the first four Books, the Books whose principal interest lies in the

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28. _Ibid._, pp. 212-213.
29. _Ibid._ IV. xviii. p. 292, n. 57.

definition of consonances, is one of two notes mixing and intermingling when sounded at the same time; and thus *The Principles of Music* clearly transmitted this Nicomachan notion of simultaneous consonance into the Latin world of the Middle Ages.

One of the most important developments of the aesthetic and mathematical theory discussed in *The Principles of Music* is that of a diastamatic notation. Boethius' aesthetics held that the essences of musical consonances and intervals must be known, and his mathematics consequently objectified these consonances and intervals in knowable numerical proportions. But these proportions were also presented in geometric space[30] and computed on the monochord ruler; and these spacial realizations of musical intervals were marked with letters in geometric fashion. Thus when intervals and consonances which produce a musical scale can be objectified by numerical proportions and determined as a definite point in musical space, at this time, and only at this time, various symbols can be used to denote specific notes and pitches. If the definite pitches of various notes in a musical scale are

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30. *The Principles of Music* III. xi-xii.

not so mathematically defined and objectively determined, then no notation defining specific pitches is possible. Boethius first presents the Greek symbols for the notes of the Lydian mode,[31] guarding "against changing anything handed down from the authority of antiquity";[32] he subsequently uses these Greek symbols in his introductory chapter dividing the monochord[33] and in his discussion of modal theory.[34] But in the complete division of the monochord, Boethius uses Latin letters as symbols of the various pitches, thereby arriving at the following notation of the various notes in the three genera.[35]

	Diatonic		Chromatic		Enharmonic	
Proslamb.	A	A	A	A	A	A
Hypate Hyp.	B	B	B	B	B	B
Parhypate Hyp.	C	c	C	c	D	X
Lichanos Hyp.	E	d	F	c#	C	c
Hypate Meson	H	e	H	e	H	e
Parhypate Meson	I	f	I	f	K	x
Lichanos Meson	M	g	N	f#	L	f
Mese	O	a	O	a	O	a
Paramese	X	b	X	b	X	b
Trite Diez.	Y	c'	Y	c'	Z	x'
Paranete Diez.	CC	d'	BB	c#'	AA	c'
Nete Diez.	DD	e'	DD	e'	DD	e'

31. *Ibid.* IV. iii-iv.
32. *Ibid.* IV. iii. p. 222.
33. *Ibid.* IV. v.
34. *Ibid.* IV. xv-xvi.
35. *Ibid.* IV. vi-xi.

	Diatonic		Chromatic		Enharmonic	
Trite Hyperb.	FF	f'	FF	f'	EE	x'
Paranete Hyperb.	KK	g'	HH	f#'	FF	f'
Nete Hyperb.	LL	a'	LL	a'	LL	a'
Mese	O	a	O	a	O	a
Trite syn.	Q	b♭	Q	a#	P	x
Paranete syn.	T	c'	S	b	R	b♭
Nete syn.	V	d'	V	d'	V	d'

But when discussing the octave species[36] and the modes[37] Boethius presents two other series of letters to notate the pitches in the diatonic genus:

Hypate Hypaton	A	B	A
	B	c	B
	C	d	C
	D	e	D
	E	f	E
	F	g	F
	G	a	G
	H	b	H
	I	c'	I
	K	d'	K
	L	e'	K
	M	f'	M
	N	g'	N
	O	a'	O
		b'	P

Thus no single group of symbols can properly be called "Boethian" notation, but nevertheless Boethius clearly

36. *The Principles of Music* IV. xiv. p. 268.
37. *Ibid.* IV. xvii. p. 290; cf. n. 56.

presents the logical possibility of a musical notation consisting of letters which signify definite pitches.

Boethius' modal theory logically follows the presentation of Greek notation and the presentation of other symbols which can represent the definite notes of a musical scale. The modal theory contained in The Principles of Music consists of two musical entities: the species of the diapason consonance and the tropes, or transposed scales. A "species" is defined as "a certain position contained within any numerical proportion which yields a consonance, having its own form according to any one of the genera, for example the diatonic."[38] Thus a species is a consonance which has a unique intervallic structure by virtue of its position in a given scale. Once a scale has been uniformly tuned according to the mathematical proportions, the possible species of consonances can be identified. Thus the notational system representing the notes from the hypate hypaton (B) to the nete hyperboleon (a') is assumed, and in this system three species of the fourth can be identified, as can four species of the fifth

38. The Principles of Music IV. xiv. p. 267.

and seven species of the octave.[39] The numbering of these species is somewhat ambiguous in The Principles of Music,[40] but nevertheless their unique intervallic structures can be easily identified. The seven species of the octave in turn determine the transpositions of the modes or tropes[41] so that these seven species might be able to occur between two given notes of the same pitches an octave apart. The octave species themselves were given names such as "Dorian," "Phrygian," and "Lydian" by some earlier Greek theorists,[42] but The Principles of Music, like its Ptolemaic source, assigns these names to the transposed scales or tropes rather than to the species themselves.[43]

Thus the Greek notation is again assumed, and two charts are presented which illustrate how the complete Diezeugmenon and Synemmenon systems are transposed in pitch so that a central octave of these transpositions can contain all the possible species of the octave.[44] The second of

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39. Ibid. pp. 268-269.
40. Ibid., pp. 270, 272, and 274, nn. 40, 43, 44.
41. Ibid. IV. xv. p. 276, n. 45.
42. Cf., (e.g.) Bacchius Isagoge Musica 77 (JanS. 308-9).
43. The Principles of Music IV. xv-xvi.
44. The Principles of Music IV. xv-xvi. pp. 275-283.

these charts, in which the intervallic structures of the middle octave are clearly outlined, can be transcribed into modern symbols as follows, and the computation of the seven octave species can be seen to logically follow from the systematic transposition of the Diezeugmenon system:

Hypodorian	Hypophrygian	Hypolydian	Dorian	Phrygian	Lydian	Mixolydian	Hypermixolydian
							a
						g	g
					f#		
						f	f
				e	e		e
						e	
			d	d	d	d	d
		c#			c#		
			c	c		c	c
	b	b		b	b		b
			b♭			b♭	
a	a	a	a	a	a	a	a M
		g#			g#		
g	g		g	g		g M	g
	f#	f#		f#	f# M		
f			f			f	f
e	e	e	e	e M	e		e
		d#				e♭	
d	d		d M	d	d	d	d
	c#	c# M			c#		
c			c	c		c	c
b	b M	b		b	b		b
			b♭			b♭	
a M	a	a	a	a	a	a	a
		g#			g#		
g	g		g	g		g	
	f#	f#		f#	f#		
f			f				
e	e	e	e	e			
		d#					
d	d		d				
	c#	c					
c							
B	B						
A							

Therefore the seven octave species are contained between a and a' of the Hypodorian through the Mixolydian modes. An eighth mode, the Hypermixolydian, is said to have been added at the top by Ptolemy, for a two octave system will contain eight octave consonances.[45] But the intervallic structure of this eighth octave is identical with the octave species contained in the Hypodorian mode, and Ptolemy actually rejects the validity of this eighth mode on this basis.[46] The comparative pitch of the respective modes is best determined by the mese of each mode, signified by the symbol M in each transposed scale.[47] Thereby Ptolemy's theory of a dynamic or "functional" mese is implicit in Boethius' discussion of modal theory.

The modal theory presented in _The Principles of Music_ is basically identical with that presented in Ptolemy's _Harmonics_. The only ambiguity between the presentations of this theory by Ptolemy and Boethius is found in Boethius' confused numbering of the species of consonances, and the numbers assigned to the various species could in no way

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45. _The Principles of Music_ IV. xvii.
46. _Ibid._, pp. 290-291.
47. _Ibid._, pp. 285-290.

be considered essential to any modal theory. The only aspect of Ptolemy's modal theory which is not specifically present in Boethius' modal theory is that of the dynamic mese, and this theory is clearly implicit in Boethius' text. Boethius probably translated these chapters concerning modal theory from Nicomachus' Introduction to Music, and Nicomachus no doubt based his discussion of modal theory on that of Ptolemy.[48]

The final aspect of Boethius' musical theory which must be discussed is his definition of the musician. Three types of persons concerned with the art of music are discussed in Book I, Chapter XXXIII: the performer, the composer, and the musical judge. The performer cannot be considered a true musician, for he is involved in the mere exercise of a skill and is "totally lacking in thought."[49] The composer likewise cannot be considered a musician, for he "composes songs not so much by thought and reason as by a certain natural instinct."[50] Thus only the judge can be

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48. The Principles of Music I. xxxiii. pp. 100-101.
49. Ibid. I. xxxiv. p. 103.
50. Ibid., p. 104.

considered a musician, for he alone "is devoted totally to reason and thought."[51] *The Principles of Music* thus defines the true musician as follows:

> And that man is a musician who has the faculty of judging the modes and rhythms, as well as the genera of songs and their mixtures, and the songs of the poets, and indeed all things which are to be explained subsequently; and this judgment is based on a thought and reason particularly suited to the art of music.[52]

Thus the full circle has been completed, for Boethius' musical theory has led back to the central idea of Boethius' aesthetics. Music is a discipline whereby reason can come to know the absolute essences determining this art which can affect men with pleasure or pain. The essences of this art consist of numerical proportions which determine musical consonances. After these essences have been mathematically defined, all aspects of the musical art can be objectified: the consonances by proportions, the other intervals by consonances, the systems or scales by these intervals, and the tuning of the systems by application of the proportions of intervals to the musical space of the monochord ruler;

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51. *Ibid.*, p. 104.
52. *Ibid.*

thereafter the pitches of the scale can be objectified by various notations, and, by using notation to represent the systems and their transpositions, the relationship between the modes and the octave species can be objectified. Through this constant drive toward objectification the student of the musical discipline can _know_ every aspect of the musical art: the consonances, the intervals, the genera, the scales or systems, the species of consonances, and the modes. Therefore only the person who has come to a knowledge of these various aspects of the musical discipline through deductive reasoning can properly be considered a musician. Finally, this true musician finds as much pleasure in the harmony of his musical knowledge as his ears find in the harmony of corporeal sound.

CHAPTER V

THE PRINCIPLES OF MUSIC AND THE TRADITION OF WESTERN MUSIC

This concluding chapter of commentary on *The Principles of Music* is an attempt to place this work in relation to the musical tradition which followed it in the same manner that the first four chapters of this commentary have related it to a musical tradition which existed before it. But this chapter can be little more than the beginning of this task. For until an exhaustive study of manuscripts containing this treatise is finished and a new critical edition of the text is compiled including critical studies of charts and various glosses contained in the manuscripts, the full import of the treatise to the Medieval reader will remain unknown. Moreover, until editions of Medieval and Renaissance works on musical theory are documented in relation to this work and others as this work has been documented in relation to its sources, the precise theories contained in *The Principles of Music* which were incorporated into later works cannot be specifically cited. Thus this chapter can be little more than a non-critical outline of what is hoped will form a more complete future study.

The position of The Principles of Music in the early Middle Ages as the principal source for musical theory is unrivaled. Three other sources of early Medieval musical theory can be cited, namely works by Martianus Capella,[1] Cassiodorus,[2] and Isidore of Seville:[3] but all of these works are of a more or less encyclopedic nature, and thus their somewhat limited discussion of musical theory can in no way be compared with Boethius' thorough presentation. The only musical work of the early Middle Ages comparable to The Principles of Music is Augustine's De Musica;[4] but although this work presents the same aesthetic position as Boethius' treatise, its main concern is with the mathematical proportions of various poetic meters, and consequently it does not contain the thorough explication of

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1. Martianus Capella, De Nuptiis Philologiae et Mercurii, ed. A. Dick (Leipzig: Teubner, 1925).

2. Cassiodorus, De Artibus ac Disciplinis Liberalium Litterarum, Migne Patrologia Latina, lxx. (Paris: 1865), 1149-1220.

3. Isidore of Seville, Etymologiarum, ed. W. M. Lindsay (Oxford: The Clarendon Press, 1911).

4. Augustine, De Musica, Migne Patroligia Latina, xxxii. (Paris: 1877), 1081-1194.

musical consonances and scales presented by Boethius. Thus The Principles of Music stands alone in the early Middle Ages as the only source of Greek musical theory discussing a rational theory of consonances and scales.

The reputation this work enjoyed in the Middle Ages could be surmised merely from the authority which the name of Boethius held to the medieval mind. But this reputation is more easily demonstrated by the fact that Boethius' quadrivium formed the basis of mathematical education in the Middle Ages; and no scholarly library was considered complete without a copy of The Principles of Music. Thus the eleventh century library at Corbie contained no less than five copies of the work,[5] the eleventh century chapter-house at Puy contained a copy,[6] the fourteenth century library of the Sorbonne contained at least two copies,[7] the twelfth century library of Christ Church, Canterbury, held at least two copies,[8] and one copy was found in the fourteenth century

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5. Leopold V. Delisle, Cabinet des manuscripts, II (Paris: 1874), p. 429, Nos. 76-78, 81, 87.
6. Ibid., I (Paris: 1886), p. 110.
7. Ibid., III (Paris: 1881), p. 61, Nos. 29, 39.
8. Montague Rhodes James, The Ancient Libraries of Canterbury and Dover (Cambridge: Cambridge University Press, 1903), p. 55, Nos. 433, 348-441.

Peterborough Abbey Library.[9] This list compiled from a very limited number of Medieval library records will surely demonstrate the presence of this work in most of the monastic libraries of Europe.

The popularity of The Principles of Music in the Medieval scholastic community is further seen in the number of surviving manuscripts of the work. The following list, compiled from various sources,[10] is again incomplete, for as yet no full survey of extant manuscripts of The Principles of Music has been completed.

Austria:	Vienna, Nat.	50
		51
		55
		299
		2269
		15470

9. M. R. James, Lists of Manuscripts formerly in Peterborough Abbey (Oxford: Oxford University Press, 1926), p. 46, No. 121.

10. Roger Bragard, "Boethiana: Etudes sur le 'De Institutione Musica' de Boece," Hommage a Charles van den Borren (Anvers: N. V. de Nederlandische Boekhandel, 1945), pp. 84-139, see especially pp. 98-99; H. M. Bannister Monumenti Vaticani di Paleografia musicale Latina (Leipzig: 1913), No. 19, 174, 190, 301, 954-962; G. Reese Music in The Middle Ages (New York: Norton, 1940), p. 135; MSS marked * used by Friedlein.

Belgium:	Brussels, Bib. Royale:	5444-6	(X-XI. c.)
		10144-6	(XI. c.)
		18397	(XIII. c.)
		II-6188	(XV. c.)
	Bruges, Bibl. de la Ville:	531	(XI. c.)
Czechoslovakia:	Prague, Univers.:	1717-IX. C. 6	
France:	Paris Bibl. Nat.:	7181	(X. c.)*
		7185	
		7199	
		7200	(X. c.)*
		7201	
		7202	
		7203	
		7204	
		7205	
		7206	
		7297	
		7361	
		10275	
		13020	
		13955	
		14080	
		16201	
		16652	
		17872	
		18514	
		N.A. 1618	
	Avranches:	236	
		237	
	Chartres:	48	
		498	
	Saumure:	3	
	Orleans:	293	

England:	London, British Museum:	Arundel 77	
		Harley 957	
		Harley 2688	
		Harley 3595	
		Harley 5237	
		Kings 15 B. IX.	
	Lambeth Palace:	67	
	Cambridge:	Ii. III. 12	
	Oxford, Bodleian Library:	Ashmole 1524	
		Rawlinson C. 270	
Germany:	Bamberg:	H. J. IV, 19	(IX. c.)*
	Cologne:	Dombibl: 187	
		Stadtarchiv: X. 331	
	Erlangen:	Univers.: 193	
		2112	
	Leipzig:	1492	
		1493	
	Munich:	367	(XII. c.)*
		6361	(XI. c.)*
		13021	
		14272	
		14465	
		14523	(X. c.)*
		14601	(XII. c.)*
		18478	(XI. c.)*
		18480	(XI. c.)*
	Stuttgart:	XI, 33	
	Wolfenbuettel:	4376	
Italy:	Florence,	Bibl. nation.: I. 406	
		Laurentiana Bibl.: 1051	
	Milan:	Ambr. C. 128 i.	
	Rome:	Bibl. Vatic.: Reg. 1005	
		Pal. 1342	
		Ottob. 2069	
		823	
		954.962	

Switzerland:	Bern:	A. 94
	Einsiedeln:	298 358
	St. Gall, Vidiana:	296
United States:	New York:	Collection of Philip Hofer, 174 East Eighteenth St.: No. 26
		Library of the Late George A. Plimpton, 61 Park Ave.: No. 45
		Library of Grenville Kane, Tuxedo Park: No. 50
	Chicago:	Newberry Library

Thus the first task of future scholarship concerning *The Principles of Music* is an examination of these manuscripts, especially the charts and glosses contained therein; for only thereafter will the meaning of this treatise to the Medieval musician be fully understood.

If a rather limited survey of Medieval library catalogues and surviving manuscripts of *The Principles of Music* implies that the work was read as a source for musical theory in the Middle Ages, a similarly limited survey of Medieval works on music will demonstrate that without this work Medieval musical theory would have been considerably different. Works attributed to Hucbald in Gerbert's

Scriptores Ecclesiastici de Musica[11] draw on Books I, II, IV, and V of *The Principles of Music*,[12] while the treatise attributed to Regino of Pruem,[13] literally quoting passages from Books I-IV, would be reduced to a few pages were the theory from *The Principles of Music* removed. The very language of Boethius' musical treatise is such an integral part of the Regino treatise that even such a Boethian statement as "Credulity must be summoned to the present disputations to support mediocre intelligence" is incorporated into the text.[14] Divisions of the monochord in Medieval treatises draw directly on Boethius' proportional division of the ruler,[15] and some such divisions, quoting Chapters VI-XII

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11. Hucbald, *De Musica* and *Alia Musica*, Gerbert *Scriptores Ecclesiastici de Musica* (Hildesheim: Georg Olms reprint of 1784 edition, 1963), I, pp. 103-152.

12. Concerning Hucbald's use of Boethius' text see W. Brambach, *Die Musiklitteratur des Mittelalters bis zur Bluethe der Reichenauer Saengerschule* (Karlsruhe: 1883), p. 3.

13. Regino of Pruem, *De Harmonica Institutione*, Gerbert *Scriptores*, I, pp. 230-247; concerning Regino's use of Boethius' text see W. Brambach, *Die Musiklitteratur* (cf. n. 12), p. 3.

14. Cf. Regino, *GS*. I, p. 233: "*Interim supra memoratae disputationi sub mediocri intelligentia credulitas adhibenda est*," and Boethius I. xix. (Friedlein, p. 205): "*Interea praesenti disputationi sub mediocri intelligentia credulitas adhibenda est*."

15. Cf. *Anonymi*, Gerbert *Scriptores* I, pp. 330-347.

more or less verbatum, use the exact same numbers and letters presented by Boethius.[16] Thus the intonation presented by Boethius becomes that of the Medieval musical theorists.

When Medieval masters of practical music such as Odo of Cluny and Guido of Arezzo divide the monochord, they present only the divisions of the diatonic genus; but they likewise present their symbols for the notes in conjunction with the division of the monochord (Γ, A, B, C, D, E, F, G, a, b, c, d, e, f, g),[17] just as *The Principles of Music* had introduced the concept of symbols objectifying specific pitches in its division of the monochord. An eleventh century *Antiphonarium Tonale Missarum*[18] uses Boethius' own series of letters from A-P to denote the exact pitches of notes in Gregorian chants. Thus the essentially European

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16. Cf. e.g., Adelboldi, *Musica*, Gerbert *Scriptores* I, pp. 303-312, and Bernelinus, *Cita et Vera Divisio Monochordi*, Gerbert *Scriptores* I, pp. 312-330.

17. Odo of Cluny, *Dialogus de Musica*, Gerbert *Scriptores* I, pp. 251-264, and Guido of Arezzo, *Micrologus*, ed. J. Smits van Waesberghe (Rome: American Institute of Musicology, *CSM* IV, 1955), pp. 96-102.

18. Montpellier, School of Medicine, H. 159; printed in *Paleographie Musicale*, VIII (Tournai: Desclee, Lefebvre & Co., 1901-1905).

concept of a diastematic notation can be considered a logical bi-product of theoretical concepts set forth in *The Principles of Music*.

Musical theorists of the thirteenth century *Ars Antiqua* such as Jerome of Moravia[19] and Walter Odington[20] quote extended passages and diagrams from *The Principles of Music* in compiling their musical treatises; and Franco of Cologne cites Boethius as the authority on musical theory, comparing him with St. Gregory as the authority on ecclesiastical music and Guido as the authority on musical practice.[21] The *Speculum Musicae* of Jacob of Liege, a work written at the beginning of the *Ars Nova*, but which more than any other presents the Medieval scholarly concept of *musica*, draws on Boethius more than any other source in forming its musical speculation.[22]

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19. Jerome of Moravia, *Tractatus de Musica*, Coussemaker *Scriptorum de Musica Medii Aevi*, I (Paris: A Durand, 1864), pp. 1-94.

20. Walter Odington, *De Speculatione Musice*, Coussemaker *Scriptorum de Musica Medii Aevi*, I, pp. 182-250.

21. Franco of Cologne, *Ars Cantus Mensurabilis*, Coussemaker *Scriptorum*, I, pp. 117-136; see specifically p. 117.

22. Jacob of Liege, *Speculum Musicae*, ed. Roger Bragard, 4 vols. (Rome: American Institute of Musicology, *CSM* III, 1955-1965).

The full development of a rational rhythmic notation by the Ars Nova and the Renaissance can be considered an application of Boethius' musical mathematics to the problem of musical rhythm. This is especially true of the fifteenth century development of rhythmic proportions by such theorists as Prosdocimus de Beldemandis,[23] Guilelmus Monachus,[24] Tinctoris,[25] and Gaffurius;[26] for this development is nothing more than an application of the various species of inequality defined in The Principles of Music to temporal space similar to earlier theorists' application of these species of inequality to harmonic space. Tinctoris' definition of a musician clearly reflects the Boethian concept of a musician as one who has mastered music through knowledge rather than mere skill:

> Musicorum est cantorum magna est differentia:
> Illi sciunt, ii dicunt quae componit musica.
> Et qui dicit quod non sapit diffinitur bestia.[27]

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23. Prosdocimus de Beldemandis, Tractatus Practice de Musica Mensurabili, Coussemaker Scriptorum, III (Paris: A. Durand, 1869), pp. 200-228.

24. Guilelmus Monachus, De Preceptis Artis Musice et Practice Compendiosus Libellus, Coussemaker Scriptorum, III, pp. 273-307.

25. Tinctoris, Proportionale Musices, Coussemaker Scriptorum, IV (Paris: A. Durand, 1876), pp. 153-177.

26. Gaffurius, Practica Musicae (Venice: 1496).

27. Tinctoris, Diffinitorium Musicae, Coussemaker Scriptorum, IV, pp. 177-191; see p. 186, "musicus."

The development of the Pythagorean aesthetics into a mathematics whereby music can be known was continued in the Renaissance by such musical humanists as Pietro Aron,[28] Heinrich Glareanus,[29] Nicola Vicentino,[30] Gioseffo Zarlino,[31] and Francisco Salinas.[32] These theorists, however, were no longer dependent upon Boethius' translation of Greek theoretical treatises for their knowledge of Greek theory for through their own knowledge of Greek and through their first hand study of Greek treatises they arrived at an aesthetics, mathematics, and musical theory quite similar to that of Boethius. These theorists thus had access to the concluding part of Ptolemy's Harmonics which Boethius had never completed translating; and the melodic emphasis of Ptolemy's thought gave rise to intonations which departed from the Pythagorean diatonic intonation introduced into Medieval musical thought by Boethius. Nevertheless, these new intonations were justified by the same type of

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28. Pietro Aron, Thoscanello de la musica (Venice: 1523).

29. Heinrich Glareanus, Dodecachordon (Basil: 1547).

30. Nicola Vicentino, L'Antica musica ridotta alla moderna prattica (Rome: 1555).

31. Gioseffo Zarlino, Le Istitutioni harmoniche (Venice: 1558).

32. Francisco Salinas, De musica libri septem (Salamanca: 1577).

mathematical logic and monochord divisions described in *The Principles of Music*. Moreover, despite the direct availability of Greek sources, Boethius remained one of the principal authorities in musical theory. Zarlino presents Boethius' concepts of *musica mundana* and *musica humana* in Book I, Chapters VI-VII of his *Istitutioni harmoniche*, and much of his classification and discussion of numerical proportions follows *The Principles of Music* rather closely.[33] Zarlino's *senario* (1:2:3:4:5:6), the series of numbers from which his theory of consonances and intonation is derived, can be viewed as an elaboration or trope on Boethius' theory of harmonic proportionality (3:4:6), the series of numbers from which the Pythagorean theory of consonances and intonation could be derived. But the stature of Boethius in the minds of sixteenth century theorists is best exemplified by the following terse sentences of Glareanus:

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33. Zarlino, *Istitutioni harmoniche*, Book I, Chapters XXI-XXVIII.

> No one has taught more learnedly and diligently about music than Boethius. Nor do I see anyone who is clearly his equal.[34]

There can be no doubt that Boethius is cited in Medieval and Renaissance theoretical works more than any other single authority on musical theory, and thus his translations of Nicomachus and Ptolemy presented a concept of music which directly influenced European musical thought for over a millenium. In the modern era The Principles of Music has not had such a direct influence on musical thought, but the tendency to approach music as a rational discipline has remained as essential part of Western musical tradition. The musical theory of seventeenth and eighteenth century French rationalism is analogous to the structure of musical thought presented in The Principles of Music. Descartes began French rationalistic musical thought by acknowledging sense perception as the first principle of music, but he developed his musical thought according to the mathematical method, building each successive element in musical theory on the basic induction of sense perception

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34. Glareanus, Dodecachordon, trans. Clement A. Miller, 2 vols. (Rome: American Institute of Musicology, MSD VI, 1965), I, p. 82.

using mathematical axioms to arrive at the numerical essences of each successive proof.[35] This system of musical thought was further developed by Mersenne[36] and brought to its final fruition by Rameau. Rameau's debt to Descartes' philosophical method is expressed in the introduction to his _Démonstration du principe de l'harmonie_:[37]

> _Je compris d'abord qu'il falloit suivre dans mes recherches le même ordre que les chose avaient entre-elles . . . Éclairé par la méthode de Descartes que j'avois heureusement lûe et dont j'avois été frappé, je commençai par descendre en moi-même; j'essayaï des chants, à peu près comme un enfant qui s'exerceroit à chanter; j'examinai ce qui se passoit dans mon esprit et dans mon organe. J'imaginai . . .; et je conclus. . . Je me mis cependant à calculer . . . et je trouvai._

Thus like Descartes, Rameau begins in an ultimately subjective position, but out of this position grows a fully rational system of thought, a system which defines each triad as the mathematical and acoustical result of the overtone series, just as Boethius had defined his concept of harmony as a mixture of two sounds having commensurate frequencies and expressed the relationship of such sounds in mathematical

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35. René Descartes, _Compendium Musicae_, trans. Walter Robert with introduction and notes by Charles Kent (Rome: American Institute of Musicology, MSD VIII, 1961).

36. Martin Mersenne, _Harmonie Universelle_ (Paris: 1635).

37. Jean Phillippe Rameau, _Démonstration du principe de l'harmonie_ (Paris: Durand, 1750), "_Introduction_."

proportions. The complete basis of this rationalistic tradition, like that of The Principles of Music, begins with the premise that music is affective, it gives pleasure and pain, but one should not accept this art as such without asking why; the principles of music must be objectified so they can become known, and the best way to express objectively the "affective" relationships of sounds is in mathematical terms.

Composers such as Paul Hindemith have continued the theoretical tradition of The Principles of Music into the twentieth century; for his whole aesthetics concerned with "Perceiving Music Intellectually" is built around the Pythagorean aesthetics of Augustine and Boethius.[38] Hindemith clearly recognizes the contribution of these authors to the development of Western music, particularly that of the latter:

> It [The Principles of Music] was written in the early sixth century, about one hundred years later than Augustine's De Musica. Unlike the latter work, it was a well-known book, which throughout the following centuries exerted a strong influence on European musical education. Without his influence

38. Paul Hindemith, The Composer's World (Garden City, New York: Doubleday & Company, Inc., 1961), pp. 19-26.

> the organized technique of composition and its underlying theories, up to about 1700, would probably have taken a course different from the one it actually followed.[39]

Hindemith's whole concept of musical composition is built around the premise that music is determined by mathematically objectifiable laws determining various combinations of sounds, and he begins his study of musical composition with a mathematical justification for a modern intonation which can indeed be considered a twentieth century exercise in musical mathematics similar to that of Boethius' sixth century division of the monochord.[40]

The Principles of Music as translated, documented, and discussed in the present study can be seen to be a work which translated Nicomachus and Ptolemy into Latin, and thereby presented an aesthetics, a musical mathematics, and a theory of music to European readers of the work in subsequent centuries. Moreover, this work alone forms the historical link between the Pythagorean musical speculation

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39. *Ibid.*, pp. 7-8.

40. Paul Hindemith, *Craft of Musical Composition*, Book I, Theoretical Part, trans. Arthur Mendel (4th ed; New York: Associated Music Publishers, Inc., 1945), pp. 14-50.

of Greek antiquity and the theoretical musical speculation of the Middle Ages; for there were few men of the Middle Ages who could have read the Greek sources of this theory, if they were indeed available to them, and The Principles of Music is the only musical work of the Middle Ages which presents such a thorough discussion of this aesthetics, mathematics, and musical theory.

Professor Walter Wiora, when defining the general characteristics of Western music, characterizes Western musical art in the following terms:

> Western musical art was impregnated as no other by scholarly and, in the broad sense, scientific theory. In mensural rhythm, in the rules governing tonality, in harmony it was rationalized through and through. The seemingly irrational world of tone was laid down *imperio rationis*--under the command of reason, as was said following Boethius--in concepts and written signs. There took shape systems of relationship and forms of presentation, like the coordinating system of the score, metrical schemes using bar-line and time signature, and the well-tempered keyboard. More than anywhere else music was objective spirit and *scientia musica*.[41]

Thus the most characteristic trait of Western music is its tendency to theorize and objectify, and there can be little

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41. Walter Wiora, *The Four Ages of Music*, trans. M. D. Herter Norton (New York: W. W. Norton and Company, Inc., 1965), p. 127.

doubt that the aesthetics introduced into the Latin world of the Middle Ages by The Principles of Music is one of the main sources if not the prime source of this tendency. This "scientific" approach to music has subsequently remained an essential part of Western musical thought since the time of Boethius; for since that time each age has re-objectified the basic elements of its musical style so that they might be comprehended by reason. The aesthetics of The Principles of Music gave birth to a musical mathematics which best objectified the relationships of various sounds; and consequently the theorists of Western music have repeatedly returned to musical mathematics to express the relationships between the sounds of their age. The mathematical theory of The Principles of Music in turn gave birth to purely musical theories, theories of intonation, of consonance, of notation, and of modes; and thus the contents of subsequent Western theoretical works have been centered around discussions of intonation, of harmony, of notation, and of modes or tonality. The complete extent to which specific theories discussed in Western musical thought are ultimately dependent upon Boethius' transmission of Pythagorean musical thought into the Middle Ages will never be fully known until the theoretical works which have exerted such a strong influence on the development of Western music are completely documented as this study has attempted to document the sources of *The Principles of Music*. Nevertheless, this superficial survey of certain basic premises of Western musical thought has shown that such a study might well place *The Principles of Music* as one of the most influential treatises in the history of music.

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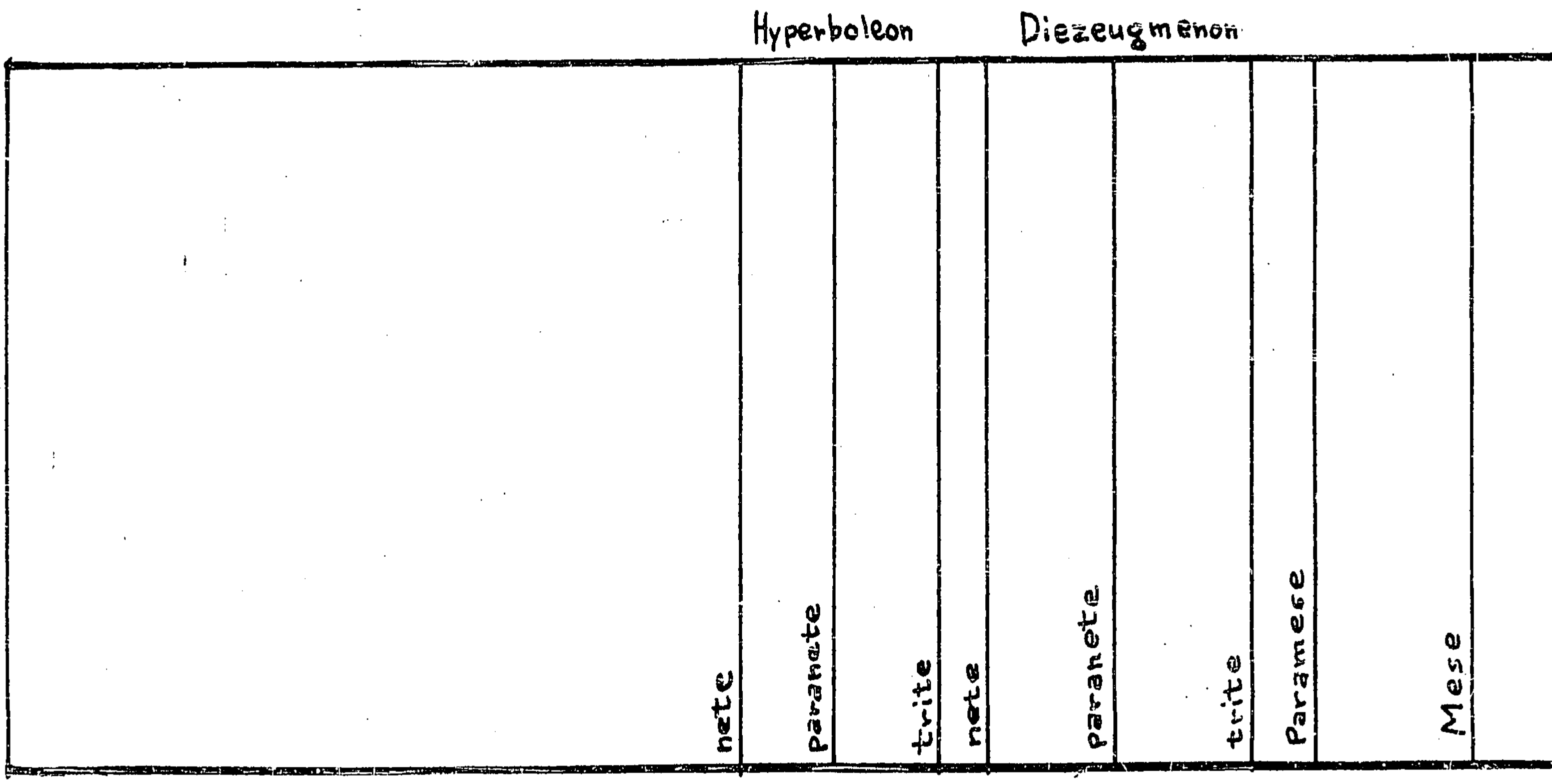
APPENDIX

DIATONIC



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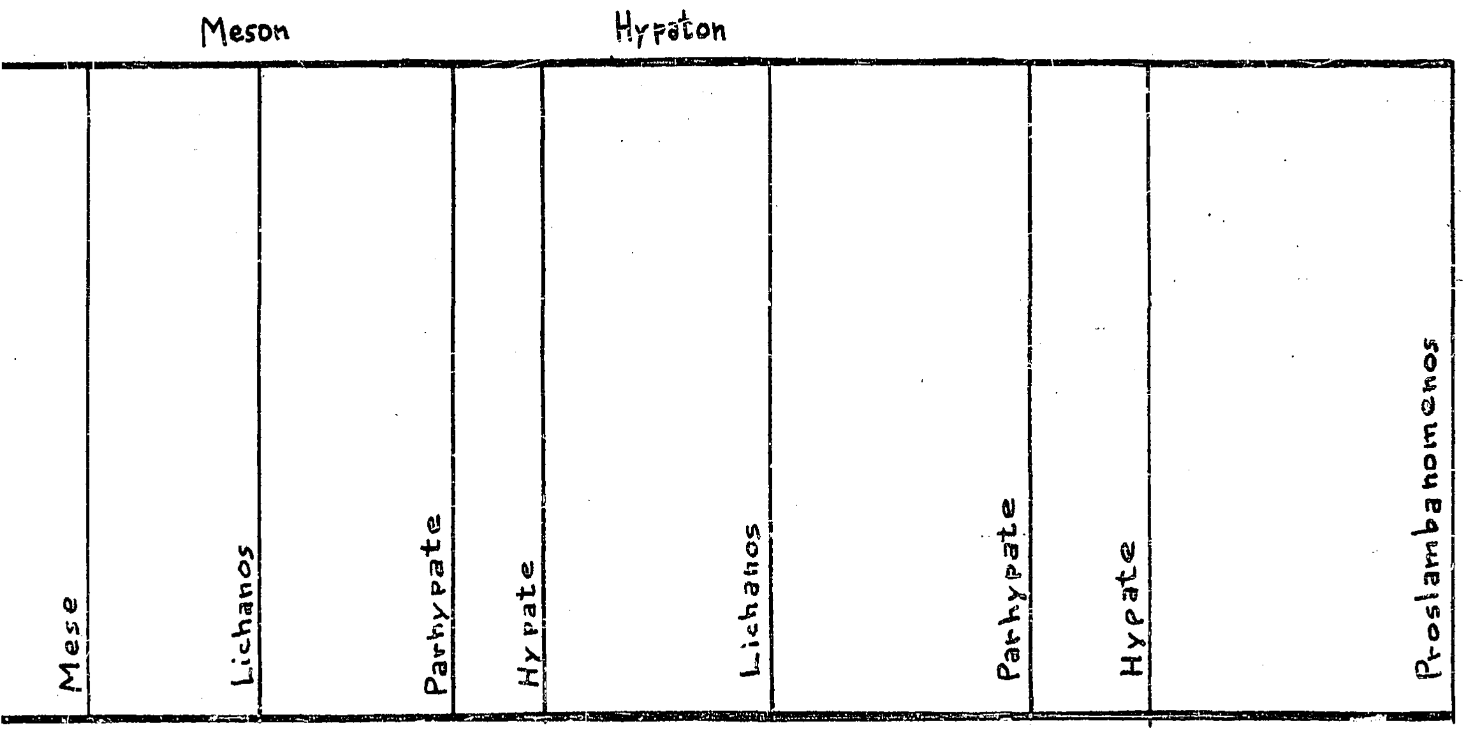
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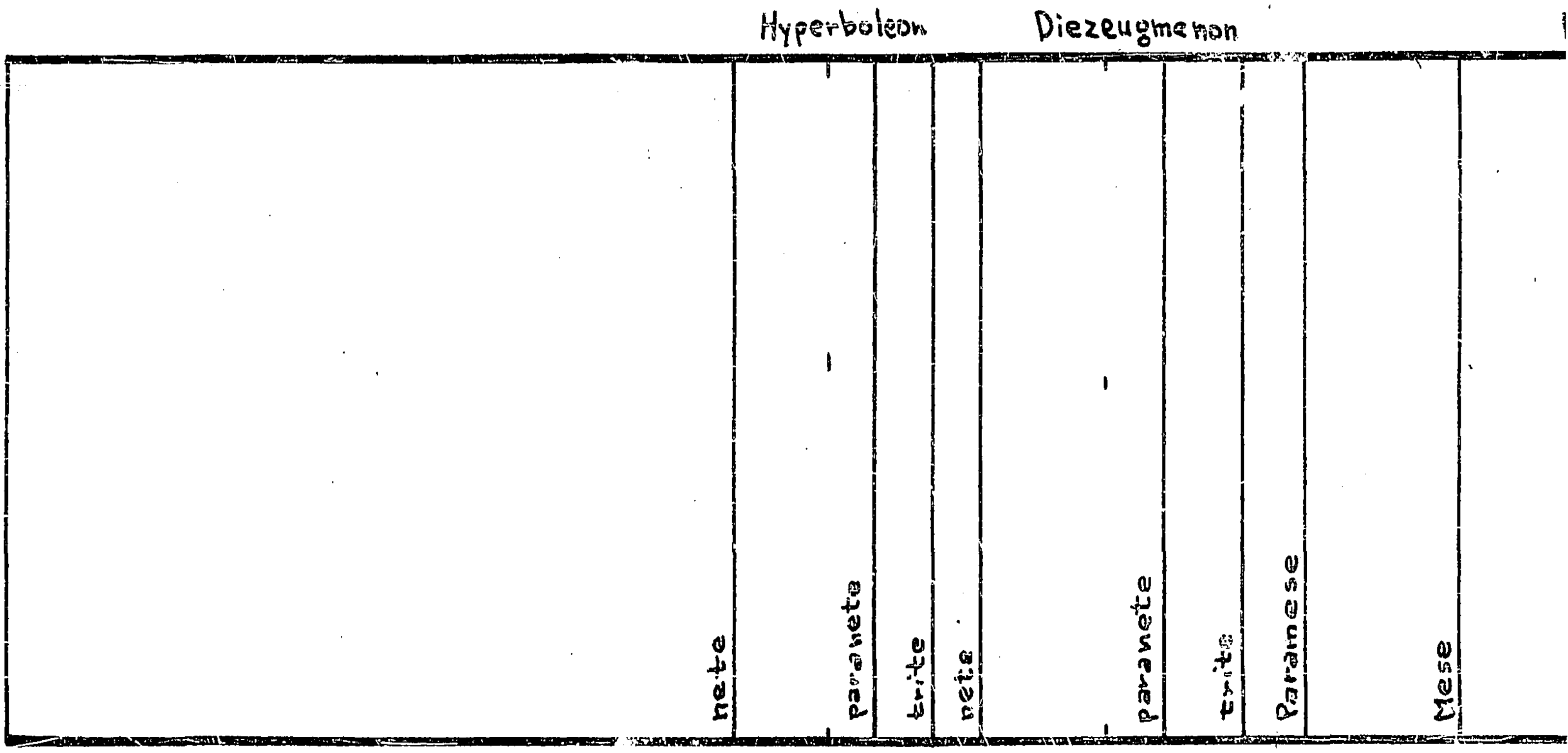
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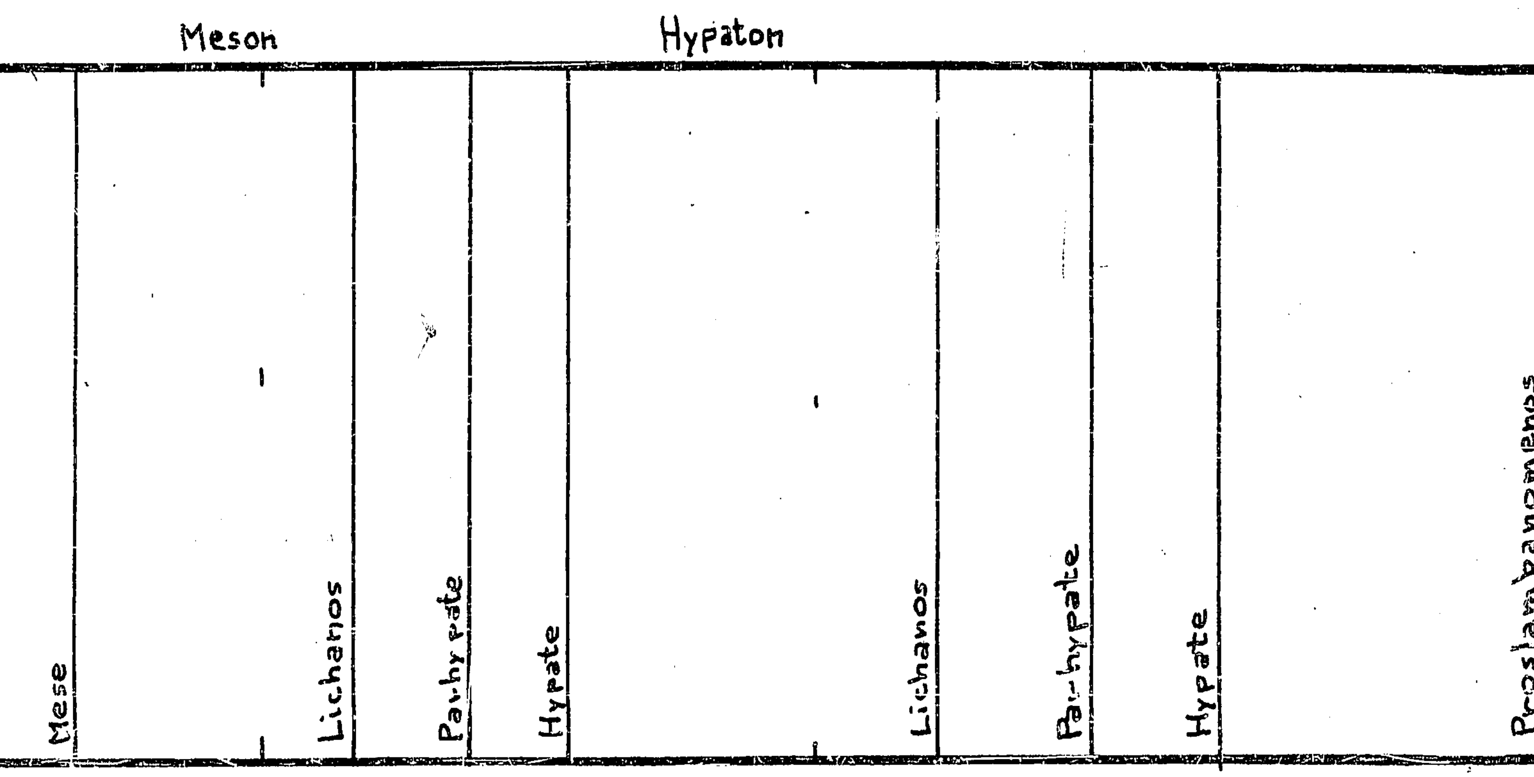
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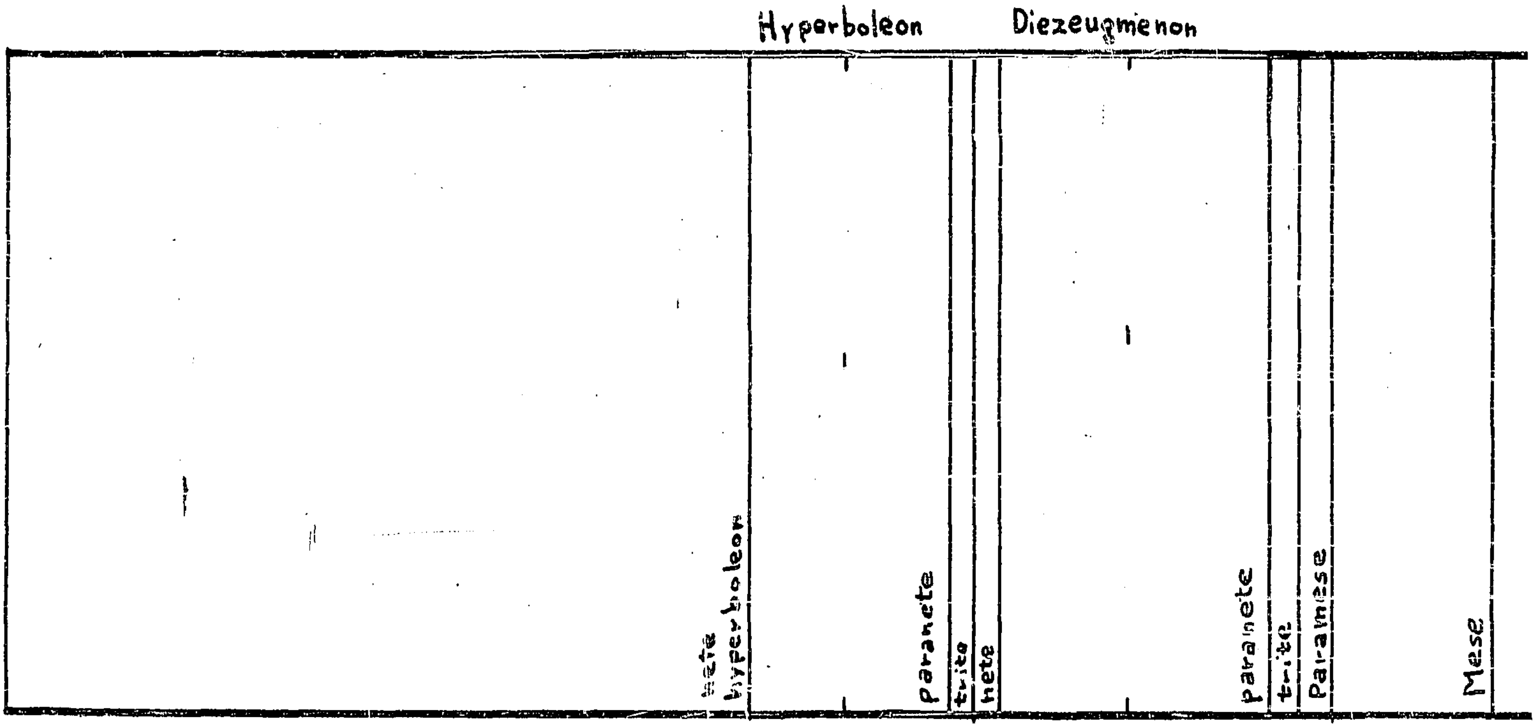

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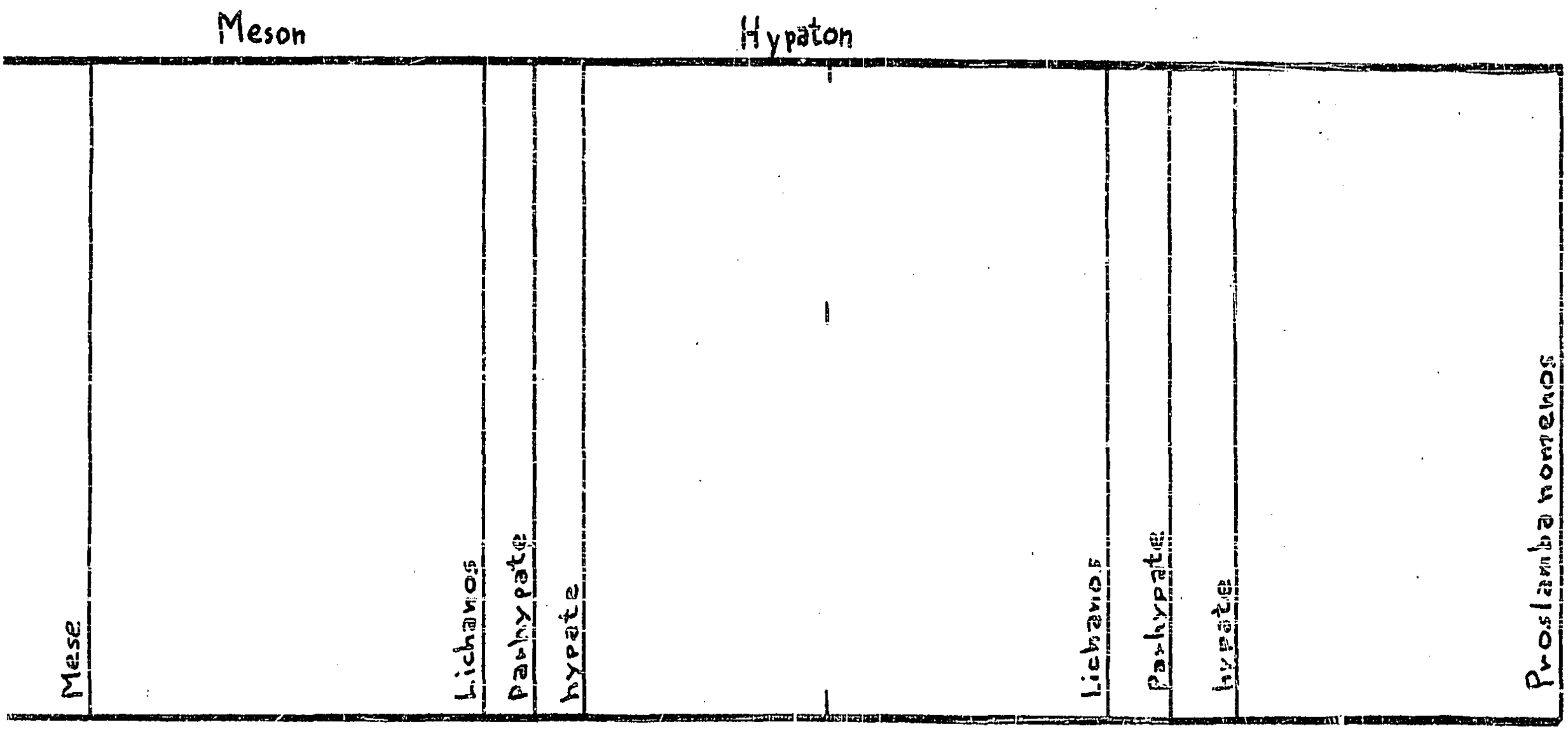
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